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Study of heat and mass transfer in a porous medium with the effects of chemical reaction and radiation absorption

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Abstract

Heat and mass transfer with chemical reaction is important in many engineering and industrial processes such as drying, evaporation, cooling towers, and desert coolers where heat and moisture transfer occur together. These processes have applications in power generation, chemical industries, environmental engineering, and electronic cooling systems. Chemical reactions may occur either throughout the fluid (homogeneous reaction) or only at the boundary surface (heterogeneous reaction). The combined effects of diffusion, chemical reaction, heat generation or absorption, and magnetic field significantly influence fluid flow and heat transfer characteristics. In this study, the flow of a viscous and incompressible fluid through a porous medium in a vertical channel is considered. The channel walls are maintained at different temperature conditions, and thermal buoyancy is produced by a traveling thermal wave. The Boussinesq approximation is applied, and the effects of viscous dissipation and Darcy dissipation are included in the analysis. The study helps to understand the behavior of heat and mass transfer in porous media under the influence of chemical reaction and magnetic field effects.

Keywords: Heat transfer, Mass transfer, Porous medium, chemical reaction, Radiation absorption, Magnetohydrodynamics (MHD), Viscous dissipation

Introduction

Combined heat and mass transfer problems with chemical reaction plays an important role in many engineering systems. In processes like drying evaporation both heat and moisture move at the same time. In cooling towers and desert coolers heat transfer and fluid flow work together to control temperature. These combined effects are commonly used in industries such as chemical processing power generation and environmental engineering. For example, in the power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. Many practical diffusive operations involve the molecular diffusion of a species in the presence of chemical reaction within or at the boundary. There are two types of reactions. A homogeneous reaction is one that occurs uniformly throughout a given phase. In a homogeneous reaction, species are produced throughout the fluid, similar to how heat is generated internally. In contrast a heterogeneous reaction occurs only at a surface or boundary, so it behaves like a boundary condition, similar to applying a constant heat flux. Studying heat and mass transfer with chemical reactions is important because it appears in many scientific and engineering applications. Deka *et al* ^[6] studied the effect of the first order homogeneous chemical reaction on the process of an unsteady flow past an infinite vertical plate with a constant heat and mass transfer. Chamka ^[4] studied the MHD flow of a numerical of uniformly stretched vertical permeable surface in the presence of heat generations absorption and a chemical reaction. It is predominantly attracted in cases of diffusion and chemical reaction occurs at approximately the identical speediness. Once diffusion is to a great extent faster than chemical reaction, then merely chemical reaction influences the rate of chemical reaction; when diffusion is not much quicker than chemical reaction, the diffusion as well as kinetics interacts to construct very dissimilar consequences. The investigation of heat generation or absorption

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consequences in moving fluids is significant, in sight of quite a few substantial problems, they are, and fluids undergo exothermic or else endothermic chemical reaction. Outstanding to the quick development of electronic technology, effectual freezing of electronic apparatus has become certified and freezing of electronic apparatus ranges from own transistors to foremost structure computers and from energy providers to telephone switch panels and thermal diffusion impacts has been exploited for isotopes separation in the combination among gases with extremely low molecular weight (H_2 and He) and average molecular weight. Magnetic field plays an important role in numerous fields such as biological, chemical, mechanical and medical research. Krishna *et al.* [29] investigated the Hall and ion slip effects on the unsteady MHD free convective rotating flow through porous medium past an exponentially accelerated inclined plate.

2. Mathematical formulation of the problem

Consider the motion of viscous, incompressible fluid through a porous medium in a vertical channel bounded by flat walls. The thermal buoyancy in the flow field is created by a traveling thermal wave imposed on the boundary wall at $y = L$ while the boundary at $y = -L$ is maintained at constant temperature T_1 while both the walls are maintained at uniform concentration. The Bossinesq approximation is applied that means density changes are considered only in the buoyancy term, Both viscous and Darcy dissipation effects are included in the energy equation along with conduction and convection. The kinematic viscosity and thermal conductivity are assumed to remain constant. We choose a rectangular Cartesian system $O(x, y)$ with x -axis in the vertical direction and y -axis normal to the walls. The walls of the channel are at $y = \pm L$.

Equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1.1)$$

Equation of linear momentum

$$\rho_e \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \left(\frac{\mu}{k} \right) u \quad (1.2)$$

$$\rho_e \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \left(\frac{\mu}{k} \right) v \quad (1.3)$$

Equation of Energy

$$\rho_e C_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - Q(T - T_e) \quad (1.4)$$

Equation of diffusion

$$\left(\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} \right) = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) - k_1(C - C_e) \quad (1.5)$$

Equation of state

$$\rho - \rho_e = -\beta \rho_e (T - T_e) - \beta^* \rho_e (C - C_e) \quad (1.6)$$

where ρ_e is the density of the fluid in the equilibrium state, T_e , C_e are the temperature and concentration in the equilibrium state, (u, v) are the velocity components along $O(x, y)$ directions, p is the pressure, T, C are the temperature and Concentration in the flow region, ρ is the density of the fluid, μ is the constant coefficient of viscosity, C_p is the specific heat at constant pressure, λ is the coefficient of thermal conductivity, β is the coefficient of thermal expansion, Q is the strength of the constant internal heat source, β^* is the volumetric coefficient expansion with mass fraction. The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$q = \frac{1}{2L} \int_{-L}^L u \, dy \quad (1.7)$$

The boundary conditions for the velocity and temperature fields are

$$u = 0, v = 0, T = T_1, C = C_1 \text{ on } y = -L$$

$$u = 0, v = 0, T = T_2 + (T_1 - T_2) \sin(mx + nt), C = C_2 \text{ on } y = L \quad (1.8)$$

Where $\sin(mx + nt)$ is the imposed traveling thermal wave

In view of the continuity equation we define the stream function ψ as

$$u = -\psi_y, v = \psi_x \quad (1.9)$$

Eliminating pressure p from equations (1.2) & (1.3) and using the equations governing the flow in terms of ψ are,

$$[(\nabla^2 \psi)_t + \psi_x (\nabla^2 \psi)_y - \psi_y (\nabla^2 \psi)_x] = \nu \nabla^4 \psi - \beta g (T - T_e)_y - \beta^* g (C - C_e)_y - \left(\frac{\nu}{k}\right) \nabla^2 \psi \quad (1.10)$$

$$\rho_e C_p \left(\frac{\partial T}{\partial t} - \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} \right) = \lambda \nabla^2 T - Q(T - T_e) \quad (1.11)$$

$$\left(\frac{\partial C}{\partial t} - \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} \right) = D_1 \nabla^2 C - K_1 (C - C_e) \quad (1.12)$$

Introducing the non-dimensional variables in (1.8) & (1.9)

$$x' = mx, y' = y/L, t' = t\nu m^2, \Psi' = \Psi/\nu, \theta = \frac{T - T_2}{T_1 - T_2}, \phi = \frac{C - C_2}{C_1 - C_2} \quad (1.13)$$

the governing equations in the non-dimensional form are

$$\delta R (\delta (\nabla_1^2 \psi)_t + \frac{\partial(\psi, \nabla_1^2 \psi)}{\partial(x, y)}) = \nabla_1^4 \psi + \left(\frac{G}{R}\right) (\theta_y + N \phi_y) - D^{-1} \nabla_1^2 \psi \quad (1.14)$$

$$\delta P (\delta \frac{\partial \theta}{\partial t} - \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y}) = \nabla_1^2 \theta - \alpha \theta \quad (1.15)$$

$$\delta Sc (\delta \frac{\partial \phi}{\partial t} - \frac{\partial \psi}{\partial y} \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial \phi}{\partial y}) = \nabla_1^2 \phi - \gamma_1 \phi \quad (1.16)$$

Where

$$R = \frac{qL}{\nu} \quad (\text{Reynolds number})$$

$$G = \frac{\beta g \Delta T_e L^3}{\nu^2} \quad (\text{Grashof number})$$

$$P = \frac{\mu c_p}{k_1} \quad (\text{Prandtl number}),$$

$$D^{-1} = \frac{L^2}{k} \quad (\text{Darcy parameter}),$$

$$Sc = \frac{\nu}{D_1} \quad (\text{Schmidt number})$$

$$\alpha = \frac{QL^2}{\lambda} \quad (\text{Heat source parameter})$$

$$\gamma_1 = \frac{K_1 L^2}{D_1} \quad (\text{Chemical reaction parameter})$$

$$\delta = mL \quad (\text{Aspect ratio})$$

$$\gamma = \frac{n}{vm^2} \quad (\text{Non-dimensional thermal wave velocity})$$

$$\nabla_1^2 = \delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Boundary conditions are

$$\psi(+1) - \psi(-1) = -1$$

$$\frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial \psi}{\partial y} = 0 \quad \text{at } y = \pm 1 \quad (1.17)$$

$$\theta(x, y) = 1, \quad C = 1 \quad \text{on } y = -1$$

$$\theta(x, y) = \sin(x + \gamma t), \quad C = 0 \quad \text{on } y = +1$$

$$\frac{\partial \theta}{\partial y} = 0, \quad \frac{\partial C}{\partial y} = 0 \quad \text{at } y = 0 \quad (1.18)$$

The value of ψ on the boundary assumes the constant volumetric flow in consistent with the hypothesis (1.7)

3. Analysis of the flow

The main aim of the study is to examine how moving thermal waves at the boundaries disturb a flow that involves both natural and forced convection. Since the aspect ratio is small. A perturbation method is used to simplify and analyze the pressure p .

$$\psi(x, y, t) = \psi_0(x, y, t) + \delta \psi_1(x, y, t) + \delta^2 \psi_2(x, y, t) + \dots$$

$$\theta(x, y, t) = \theta_0(x, y, t) + \delta \theta_1(x, y, t) + \delta^2 \theta_2(x, y, t) + \dots$$

$$\phi(x, y, t) = \phi_0(x, y, t) + \delta \phi_1(x, y, t) + \delta^2 \phi_2(x, y, t) + \dots \quad (1.19)$$

Using (1.19) in (1.14) - (1.16) and separating the like powers of δ the equations and respective conditions to the zeroth order are

$$\psi_{0,yyyy} - M_1^2 \psi_{0,yy} = -\frac{G}{R}(\theta_{0,y} + NC_{0,y}) \quad (1.20)$$

$$\theta_{0,yy} - \alpha \theta_0 = 0 \quad (1.21)$$

$$\phi_{0,yy} - \gamma_1 \phi_0 = 0 \quad (1.22)$$

With

$$\psi_0(+1) - \psi_0(-1) = -1, \quad \psi_{0,y} = 0, \quad \psi_{0,x} = 0 \quad \text{at } y = \pm 1 \quad (1.23)$$

$$\begin{aligned} \theta_0 &= 1, & \phi_0 &= 1 & \text{on } y &= -1 \\ \theta_0 &= \sin(x + \gamma t), & \phi_0 &= 0 & \text{on } y &= 1 \end{aligned} \quad (1.24)$$

The first order are

$$\psi_{1,yyyy} - M_1^2 \psi_{1,yy} = -\frac{G}{R}(\theta_{1,y} + N \phi_{1,y}) + R(\psi_{0,y} \psi_{0,xyy} - \psi_{0,x} \psi_{0,yyy}) \quad (1.25)$$

$$\theta_{1,yy} - \alpha \theta_0 = P(\psi_{0,x} \theta_{0,y} - \psi_{0,y} \theta_{0x}) \quad (1.26)$$

$$\phi_{1,yy} - \gamma_1 \phi_0 = Sc(\psi_{0,x} \phi_{0,y} - \psi_{0,y} \phi_{0x}) \quad (1.27)$$

With

$$\begin{aligned} \psi_{1(+1)} - \psi_{1(-1)} &= 0 \\ \psi_{1,y} &= 0, \psi_{1,x} = 0 \text{ at } y = \pm 1 \\ \theta_{1(\pm 1)} &= 0 \quad \phi_{1(\pm 1)} = 0 \quad \text{at } y = \pm 1 \end{aligned} \quad (1.28)$$

The equations to the second order are

$$\begin{aligned} \psi_{2,yyy} - M_1^2 \psi_{2,yy} &= -\frac{G}{R}(\theta_{2y} + N \phi_{2,y}) + R(\psi_{0,yyt} + \psi_{0,x} \psi_{1,yy} + \\ &+ \psi_{1,x} \psi_{0,yy} - \psi_{0y} \psi_{1,xy} - \psi_{1,y} \psi_{0,xy}) \end{aligned} \quad (1.29)$$

$$\theta_{2,yy} - \alpha \theta_2 = P(\theta_{0,t} + \psi_{0,x} \theta_{1,y} - \psi_{0,y} \theta_{1x} - \psi_{1,y} \theta_{0,x} + \psi_{1,x} \theta_{0,y}) \quad (1.30)$$

$$\phi_{2,yy} - \gamma_1 \phi_2 = Sc(\phi_{0,t} + \psi_{0,x} \phi_{1,y} - \psi_{0,y} \phi_{1x} - \psi_{1,y} \phi_{0,x} + \psi_{1,x} \phi_{0,y}) \quad (1.31)$$

With

$$\begin{aligned} \psi_{2(+1)} - \psi_{2(-1)} &= 0 \\ \psi_{2,y} &= 0, \psi_{2,x} = 0 \text{ at } y = \pm 1 \\ \theta_{2(\pm 1)} &= 0 \quad \phi_{2(\pm 1)} = 0 \quad \text{at } y = \pm 1 \end{aligned} \quad (1.32)$$

4. Solution of the problem

Solving the equations (1.2) - (1.13) subject to the relevant boundary conditions we obtain

$$\phi_0 = 0.5 \left(\frac{Ch(\beta_1 y)}{Ch(\beta_1)} - \frac{Sh(\beta_1 y)}{Sh(\beta_1)} \right)$$

$$\theta_0 = a_3 Ch(\beta_2 y) + a_4 Sh(\beta_2 y) + a_1 Ch(\beta_1 y) + a_2 Sh(\beta_1 y)$$

$$\psi_0 = a_5 Ch(M_1 y) + a_6 Sh(M_1 y) + a_7 y + a_8 f_2(y)$$

$$f_2(y) = a_{13} Sh(\beta_1 y) + a_{14} Ch(\beta_1 y) + a_{15} Sh(\beta_2 y) + a_{16} Ch(\beta_2 y)$$

$$\begin{aligned} \phi_1 &= b_1 (Sh(\beta_1 y) - Sh(\beta_1)) \frac{Sh(\beta_2 y)}{Sh(\beta_2)} + b_2 (Sh(\beta_3 y) - Sh(\beta_3)) \frac{Sh(\beta_2 y)}{Sh(\beta_2)} + b_3 (Sh(\beta_4 y) - Sh(\beta_4)) \frac{Sh(\beta_2 y)}{Sh(\beta_2)} \\ &+ b_4 (Sh(\beta_5 y) - Sh(\beta_5)) \frac{Sh(\beta_2 y)}{Sh(\beta_2)} + b_5 (Sh(\beta_6 y) - Sh(\beta_6)) \frac{Sh(\beta_2 y)}{Sh(\beta_2)} + b_6 (Ch(\beta_1 y) - Ch(\beta_1)) \frac{Ch(\beta_2 y)}{Ch(\beta_2)} \\ &+ b_7 (Ch(\beta_3 y) - Ch(\beta_3)) \frac{Ch(\beta_2 y)}{Ch(\beta_2)} + b_8 (Ch(\beta_4 y) - Ch(\beta_4)) \frac{Ch(\beta_2 y)}{Ch(\beta_2)} + b_9 (Ch(\beta_5 y) - Ch(\beta_5)) \frac{Ch(\beta_2 y)}{Ch(\beta_2)} \end{aligned}$$

$$\begin{aligned} &+ b_{10} (Ch(\beta_6 y) - Ch(\beta_6)) \frac{Ch(\beta_2 y)}{Ch(\beta_2)} + 0.5 b_{11} (1 - y^2) + b_{12} (Ch(2\beta_1 y) - Ch(2\beta_1)) \frac{Ch(\beta_2 y)}{Ch(\beta_2)} + b_{13} (Sh(2\beta_1 y) - \\ &- Sh(2\beta_1)) \frac{Sh(\beta_2 y)}{Sh(\beta_2)} + b_{14} (y Sh(\beta_1 y) - Sh(\beta_1)) \frac{Ch(\beta_2 y)}{Ch(\beta_2)} + b_{15} (y Ch(\beta_1 y) - Ch(\beta_1)) \frac{Sh(\beta_2 y)}{Sh(\beta_2)} \end{aligned}$$

$$\theta_1 = d_1 Sh(\beta_1 y) + d_2 Ch(\beta_1 y) + f_3(y)$$

$$f_3(y) = b_{60}Sh(\beta_1 y) + b_{61}Sh(\beta_2 y) + b_{62}Sh(\beta_3 y) + b_{63}Sh(\beta_4 y) + b_{64}Sh(\beta_5 y) + b_{65}Sh(\beta_6 y) + b_{66}Sh(\beta_7 y) + b_{67}Sh(\beta_8 y) + b_{68}Ch(\beta_1 y) + b_{69}Ch(\beta_2 y) + b_{70}Ch(\beta_3 y) + b_{71}Ch(\beta_4 y) + b_{72}Ch(\beta_5 y) + b_{73}Ch(\beta_6 y) + b_{74}Ch(\beta_7 y) + b_{75}Ch(\beta_8 y) + b_{76}Sh(2\beta_1 y) + b_{77}Ch(2\beta_1 y) + b_{78}Sh(2\beta_2 y) + b_{79}Ch(2\beta_2 y) + b_{80}ySh(\beta_1 y) + b_{81}yCh(\beta_1 y) + b_{82}ySh(\beta_2 y) + b_{83}yCh(\beta_2 y) + b_{84}y^2$$

$$\theta_{00} = 0.5 \left(\frac{Ch(h\eta)}{Ch(h)} - \frac{Sh(h\eta)}{Sh(h)} \right)$$

$$\theta_{00} = 0.5(1 - \eta)$$

$$\psi_0 = a_6 ch(\beta_1 \eta) + a_7 sh(\beta_1 \eta) + a_9 + \Phi_1(\eta)$$

Similarly the solutions to the first order are

$$\theta_1 = a_{21}\eta^3 + a_{22}\eta^4 + a_{23}\eta^5 + a_{24}\eta^6 + a_{25}Ch(\beta_1 \eta) + a_{26}\eta Ch(\beta_1 \eta) + a_{27}\eta^2 Ch(\beta_1 \eta) + a_{28}Sh(\beta_1 \eta) + a_{29}\eta Sh(\beta_1 \eta) + a_{30}\eta^2 Sh(\beta_1 \eta) + a_{31}\eta + a_{32}$$

$$\psi_1 = b_{49}Cosh(\beta_1 \eta) + b_{50}Sinh(\beta_1 \eta) + b_{51}\eta + b_{52} + \phi_2(\eta)$$

$$\begin{aligned} \phi_2(\eta) = & b_{22} + b_{23}\eta + 3b_{24}\eta^2 + 4b_{25}\eta^3 + 5b_{26}\eta^4 + 6b_{27}\eta^5 + 7b_{28}\eta^6 + b_{29}\beta_1 Sh + b_{30}(Ch + \beta_1 \eta Sh) + \\ & + b_{31}(2\eta Ch + \beta_1 \eta^2 Sh) + b_{32}(5\eta^2 Ch + \beta_1 \eta^3 Sh) + b_{33}(4\eta^3 Ch + \beta_1 \eta^4 Sh) + b_{34}(5\eta^4 Ch + \beta_1 \eta^5 Sh) + \\ & b_{35}(6\eta^5 Ch + \beta_1 \eta^6 Sh) + b_{36}\beta_1 Ch + b_{37}(Sh + \beta_1 \eta Ch) + b_{38}(2\eta Sh + \beta_1 \eta^2 Ch) + b_{39}(3\eta^2 Sh + \beta_1 \eta^3 Ch) \\ & + b_{40}(4\eta^3 Sh + \beta_1 \eta^4 Ch) + b_{41}(5\eta^4 Sh + \beta_1 \eta^5 Ch) + b_{42}(6\eta^5 Sh + \beta_1 \eta^6 Ch) + b_{43}(2\beta_1)Sh(2\beta_1 \eta) + \\ & b_{44}(2\beta_1)Ch(2\beta_1 \eta) \end{aligned}$$

Where $a_1, a_2, \dots, a_{16}, b_1, b_2, \dots, b_{84}, k_1, k_2, \dots, k_{93}$ are constants.

5. Nusselt number and sherwood number

The heat transfer rate at the walls is evaluated using the Nusselt number, which represents the local heat transfer coefficient.

$$Nu = \frac{1}{\theta_m - \theta_w} \left(\frac{\partial \theta}{\partial y} \right)_{y=\pm 1}$$

Where

$$\theta_m = 0.5 \int_{-1}^1 \theta dy$$

And the corresponding expressions are

$$(Nu)_{y=+1} = \frac{(d_9 + \delta d_{11})}{(\theta_m - \sin(x + \pi))} \quad (Nu)_{y=-1} = \frac{(d_8 + \delta d_{10})}{(\theta_m - 1)},$$

$$\text{Where } \theta_m = d_{14} + \delta d_{15}$$

The mass transfer rate at the walls is determined using the Sherwood number, which represents the local mass transfer coefficient.

$$Sh = \frac{1}{C_m - C_w} \left(\frac{\partial C}{\partial y} \right)_{y=\pm 1}$$

Where

$$C_m = 0.5 \int_{-1}^1 C dy$$

And the corresponding expressions are

6. Discussion of the Numerical results

This study looks at how heat and mass move together in a vertical channel when both temperature differences (Soret effect) and concentration differences (Dufour effect) are involved, especially under a magnetic field, and solves the complex equations using an approximate numerical method (Galerkin finite element method).

Figures 1.1–1.7 display the axial velocity (u) for various values of G , M , D^{-1} , Sc , N , S_0 , Du , and γ . The Grashof number G is used to denote u in Fig. 1.1. It is discovered that $u < 0$ indicates a reversal flow and that the true axial flow is vertically down ward. When $G > 0$, the magnitude of u grows, and when $G < 0$, it depreciates. For $G < 0$, u shows a reversal flow, and as $|G|$ increases, the reversal flow region grows. For $|G|$, the greatest $|u|$ is at $y = -0.2$. Figures 1.2 and 1.3 show that the magnitude of u in the whole flow zone decreases as the Lorentz force increases and the porous medium's permeability decreases. When the molecular buoyancy force outweighs the thermal buoyancy force, $|u|$ decreases when the buoyancy forces act in the same direction, and for forces acting in opposing directions, $|u|$ increases in the flow region, as seen by the fluctuation of u with buoyancy ratio N (fig.1.4). It is evident from the Schmidt number Sc that $|u|$ increases with greater $Sc \geq 1.3$ and decreases with increasing $Sc \leq 0.6$ (fig.1.5). u is represented by Soret parameters S_0 in. From fig 1.6 increasing soret number makes the flow velocity higher, From fig 1.7 increasing the Dufour effect also increases the velocity, From fig 1.8 stronger chemical reactions ($g > 0$) further increase the flow speed.

Fig 1.9 through display the non-dimension temperature for various parametric parameters. For every variation the non-dimensional temperature is determined to be +ve. This suggests that the temperature on $y = +1$ is always higher than the real temperature which is T_2 .

Fig. 1.9 uses θ with G . It is evident that when $G > 0$ the actual temperature increases, and when $G < 0$, it decreases. The real temperature throughout the flow zone decreases as the Lorentz force increases and the porous medium's permeability decreases (figs. 1.10 & 1.11). Regarding the buoyancy ratio N , we discover that when the buoyancy forces act in the same direction, the real temperature increases with $N > 0$, and when the forces work in opposing directions, the temperature decreases in the area (fig.1.12). Figure 1.13 shows that the real temperature decreases when the molecule diffusivity ($Sc \leq 1.3$) decreases, and the total region's θ increases as the molecular diffusivity ($Sc \geq 2.01$) decreases. While a drop in the Dufour parameter Du causes a depreciation in θ , a rise in the Soret parameter so raises the actual temperature (figs. 1.14 & 1.15). Therefore, the actual temperature in the flow zone increases as the thermo-diffusion/diffusion thermal effect increases.

7. Graphs

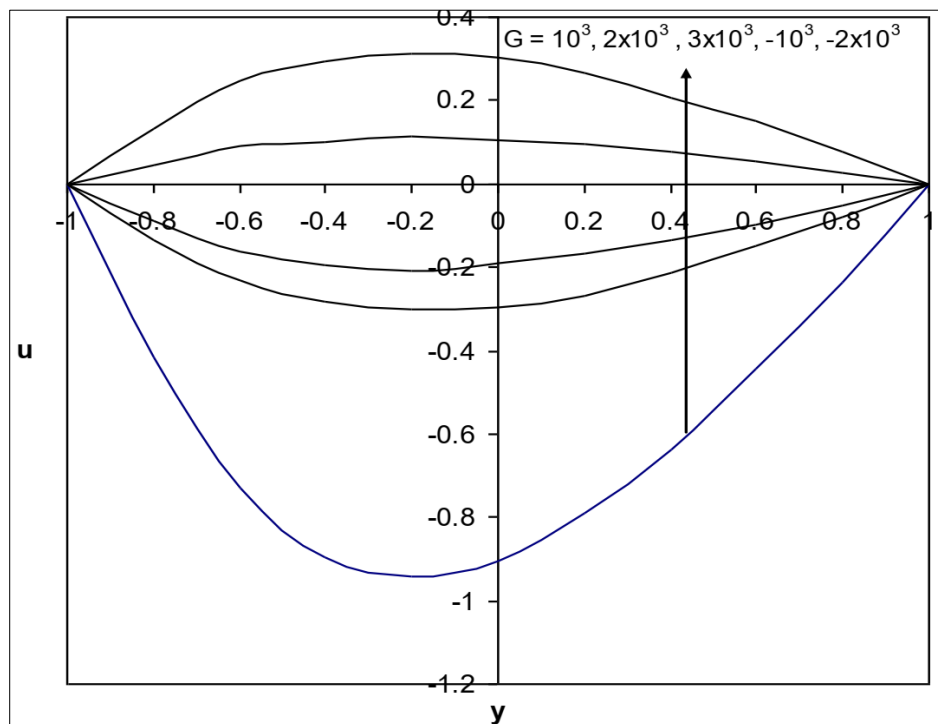


Fig 1.1: Variation of u with G $M = 2$, $D^{-1} = 10^2$, $N = 1$, $Sc = 1.3$, $S_0 = 0.01$, $Du = 0.1$

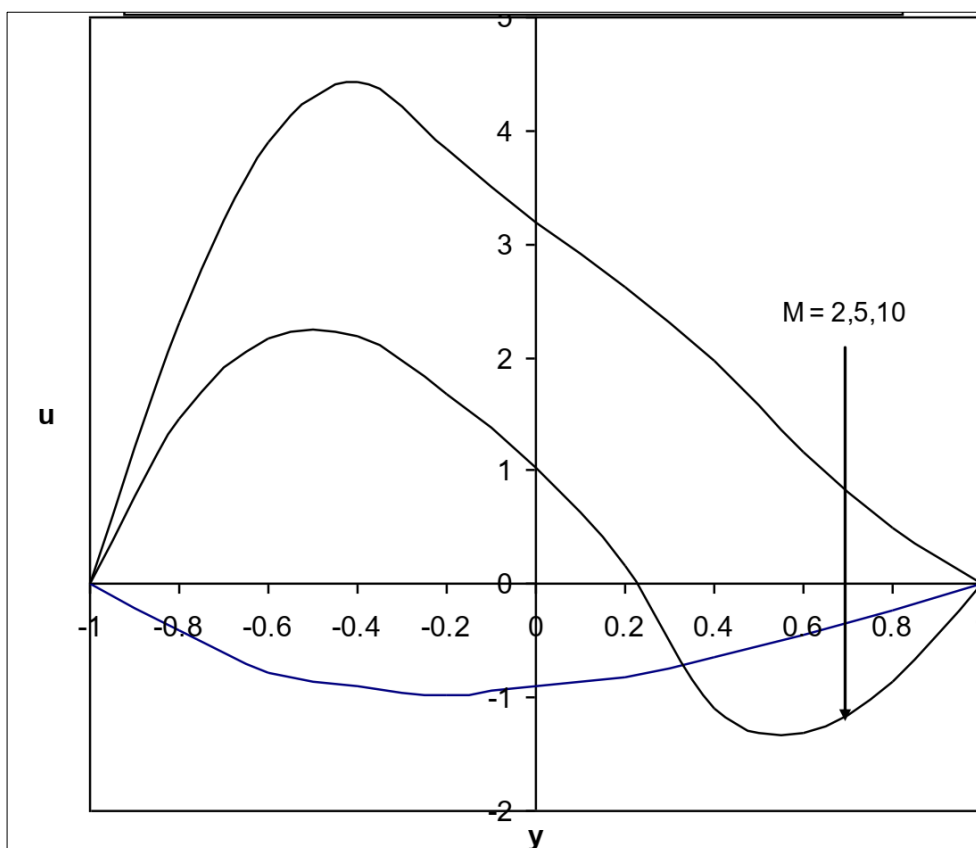


Fig 1.2: Variation of u with M $G = 10^3$, $N = 1$, $D^{-1}=10^2$, $Sc = 1.3$, $S_0 = 0.01$, $Du = 0.1$

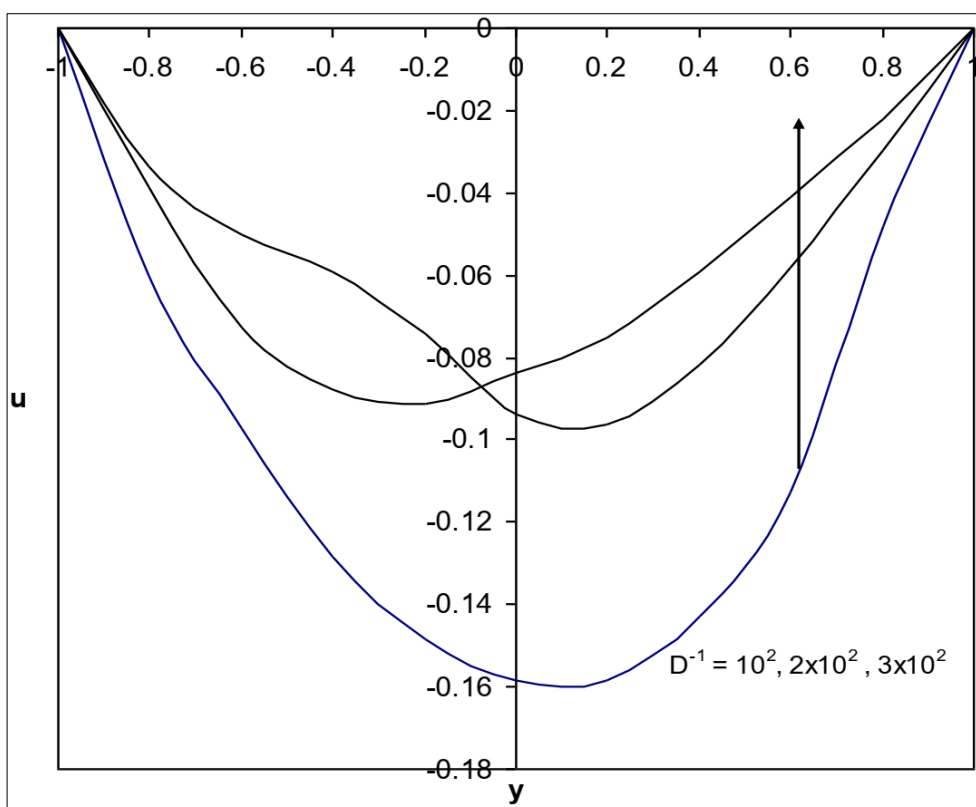


Fig 1.3: Variation of u with D^{-1} $G = 10^3$, $M = 2$, $N = 1$, $Sc = 1.3$, $S_0 = 0.01$, $Du = 0.1$

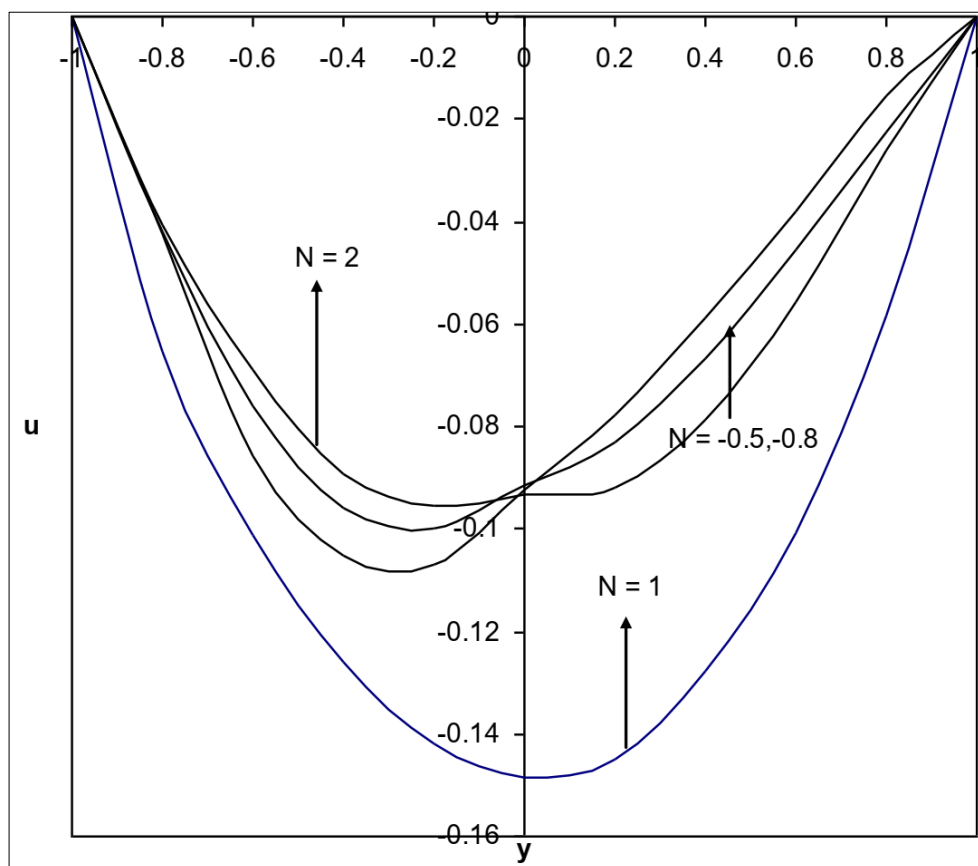


Fig 1.4: Variation of u with N $G = 10^3$, $M = 2$, $D^{-1}=10^2$, $Sc = 1.3$, $S_0 = 0.01$, $Du = 0.1$

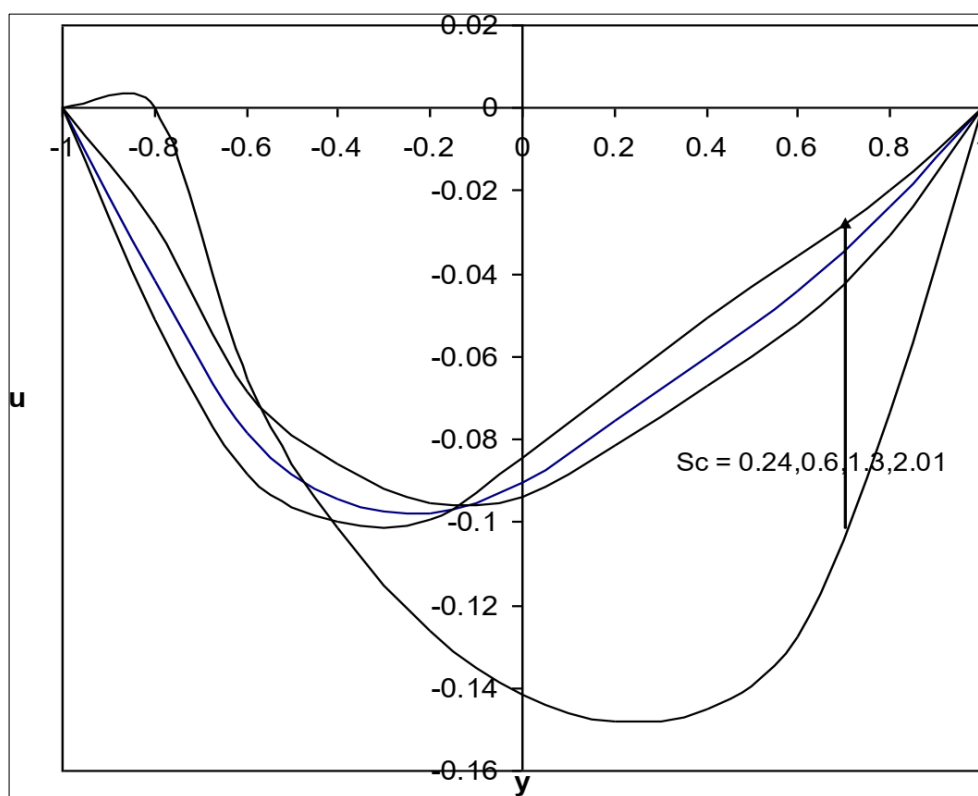


Fig 1.5: Variation of u with Sc $G = 10^3$, $M = 2$, $N = 1$, $D^{-1}=10^2$, $S_0 = 0.01$, $Du = 0$.

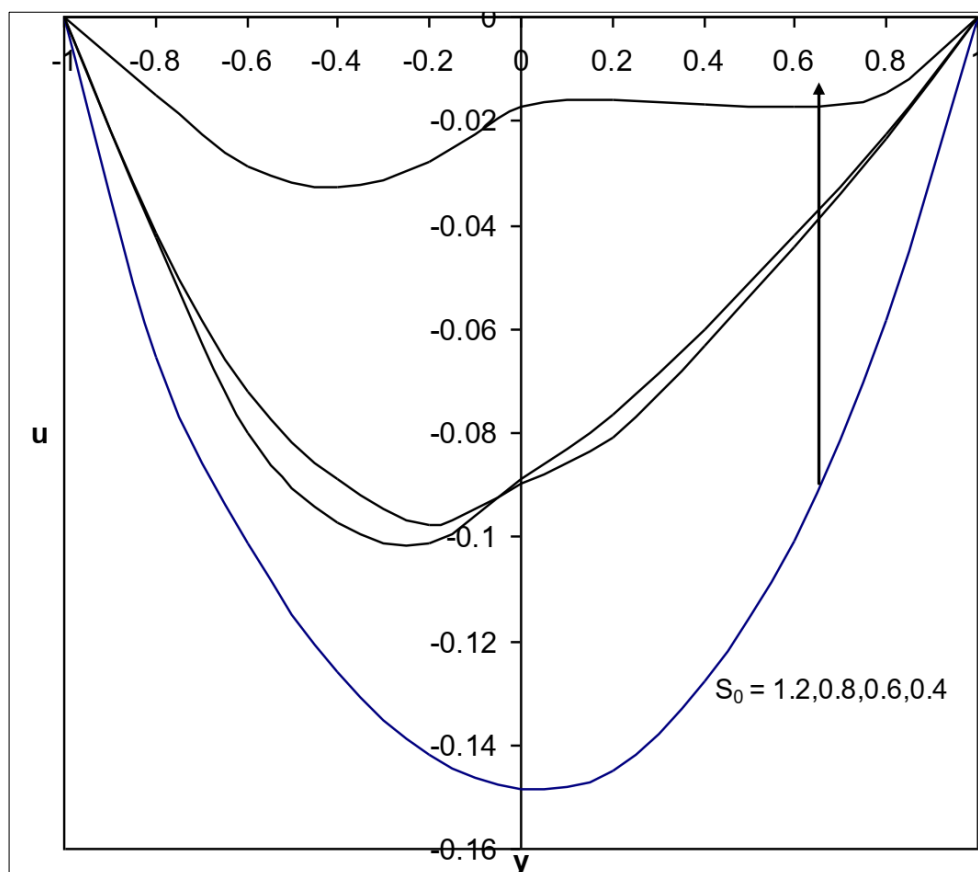


Fig 1.6: Variation of u with S_0 $G = 10^3$, $M = 2$, $N = 1$, $D^{-1} = 10^2$, $Sc = 1.3$, $Du = 0.1$

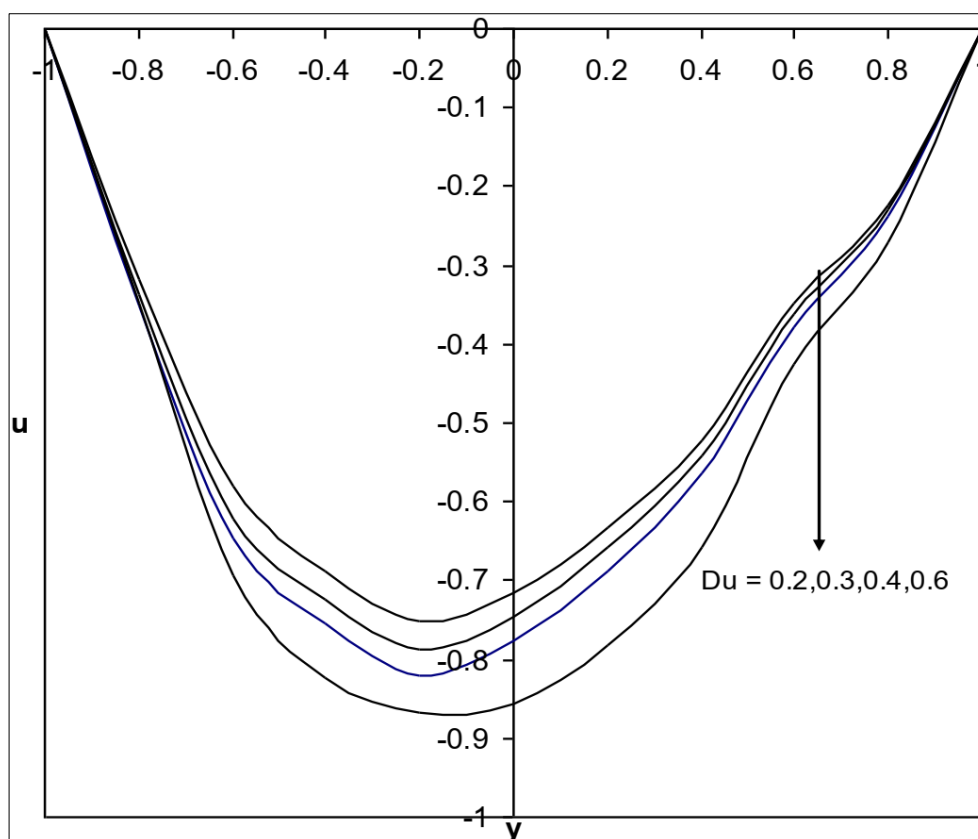


Fig 1.7: Variation of u with Du $G = 10^3$, $M = 2$, $N = 1$, $D^{-1} = 10^2$, $Sc = 1.3$

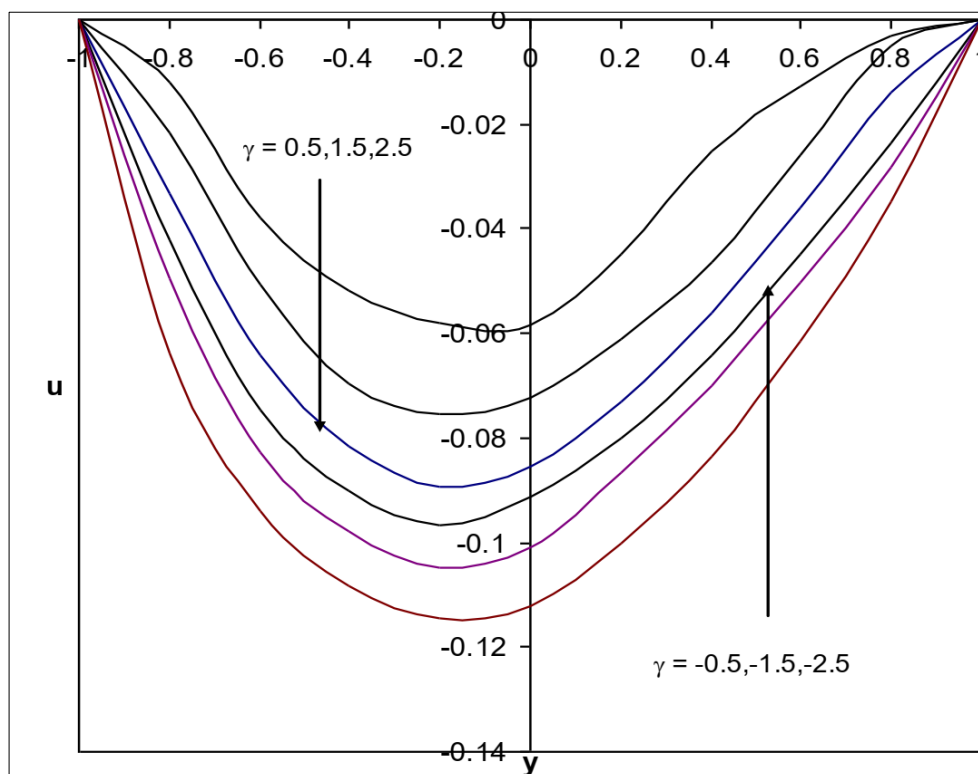


Fig 1.8: Variation of u with γ $G = 10^3$, $M = 2$, $N = 1$, $D^{-1} = 10^2$, $Sc = 1.3$, $S_0 = 0.01$

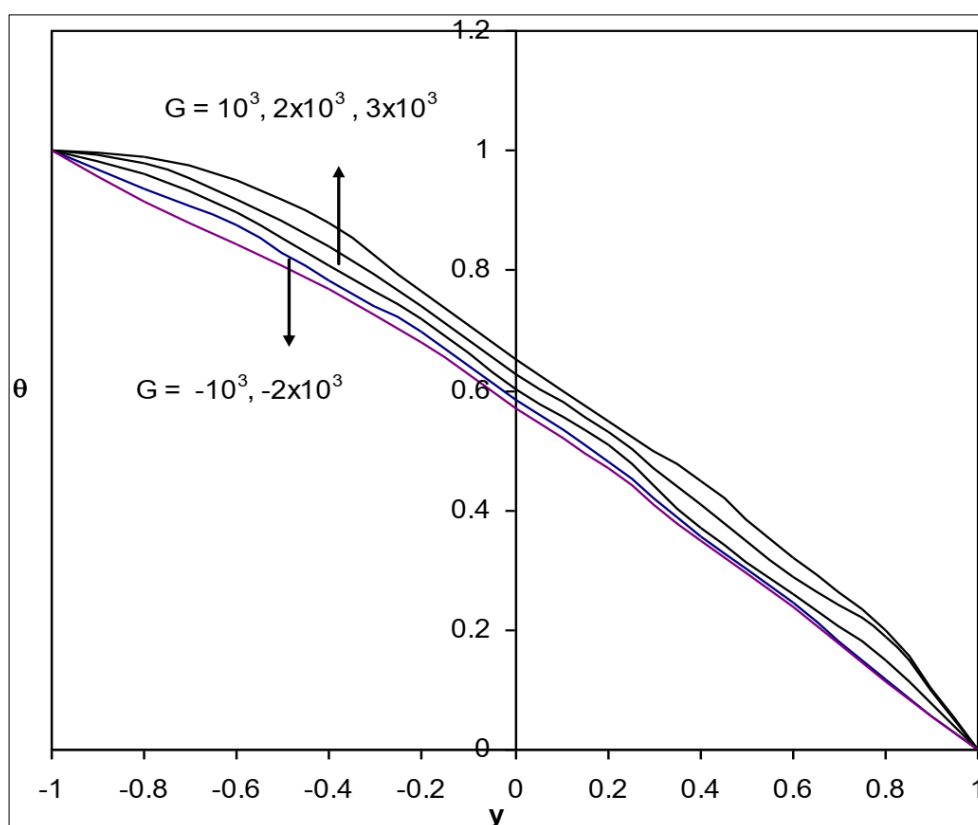


Fig 1.9: Variation of θ with G $M = 2$, $D^{-1} = 10^2$, $N = 1$, $Sc = 1.3$, $S_0 = 0.01$, $\gamma = 0.5$, $Du = 0.1$

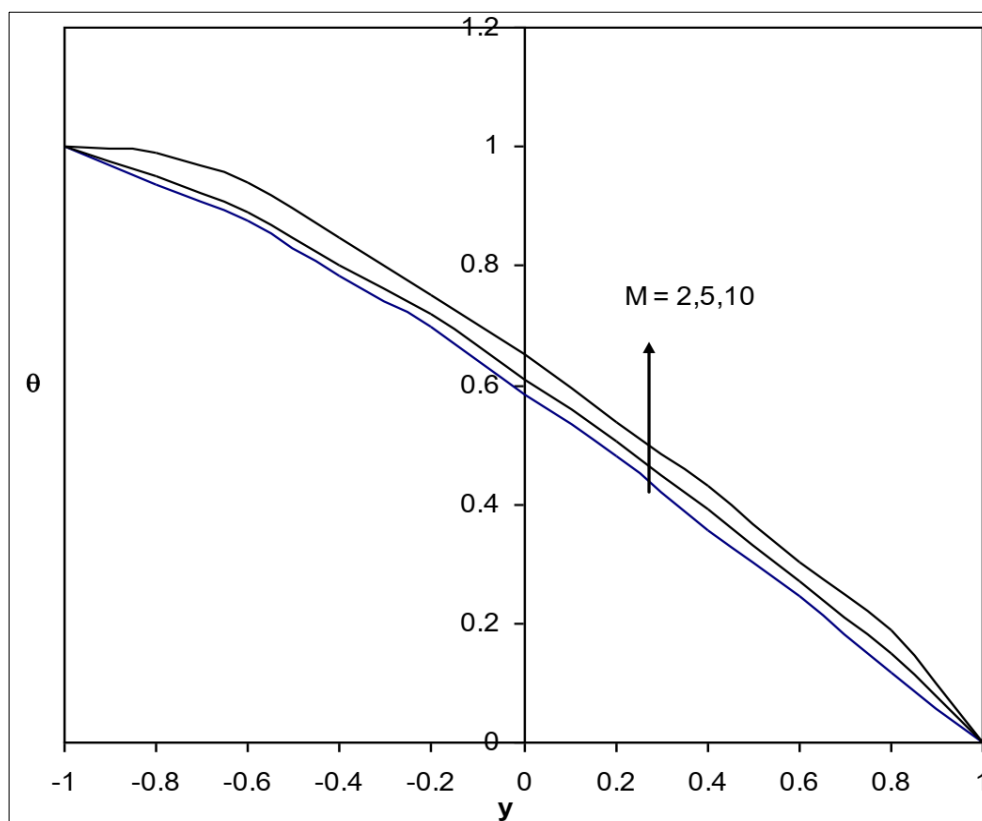


Fig 1.10: Variation of θ with M $G = 10^3$, $N = 1$, $D^{-1} = 10^2$, $Sc = 1.3$, $S_0 = 0.01$, $\gamma = 0.5$, $Du = 0.1$

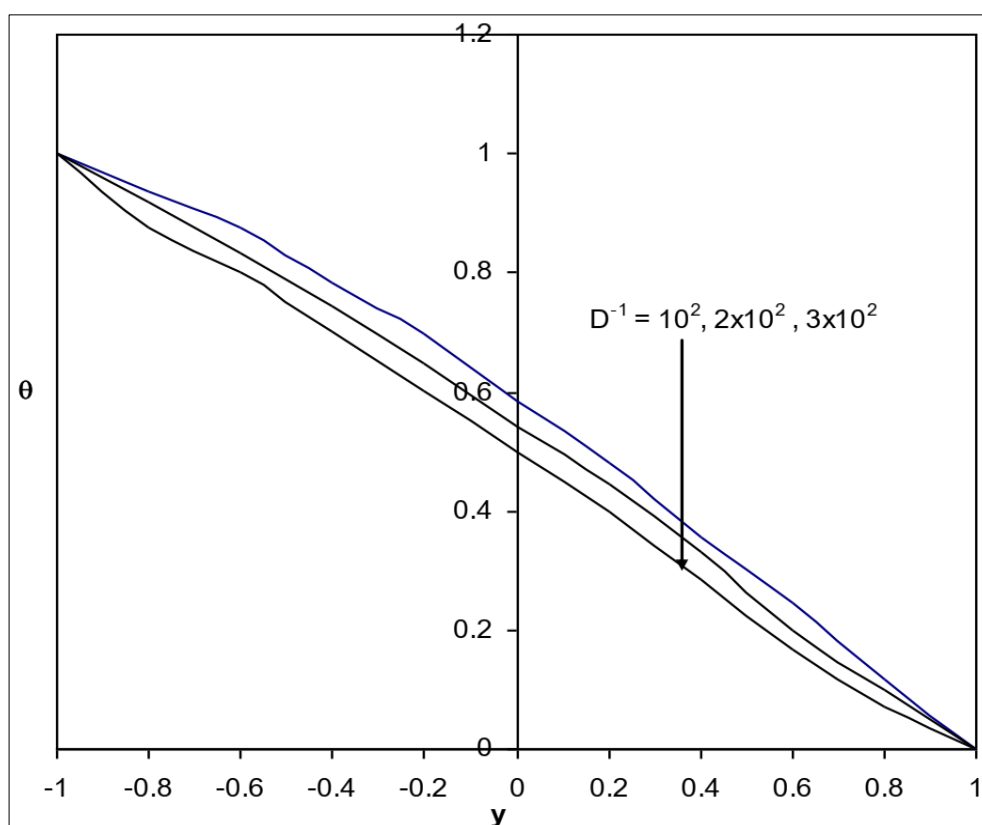


Fig 1.11: Variation of θ with D^{-1} $G = 10^3$, $M = 2$, $N = 1$, $Sc = 1.3$, $S_0 = 0.01$, $\gamma = 0.5$, $Du = 0.1$

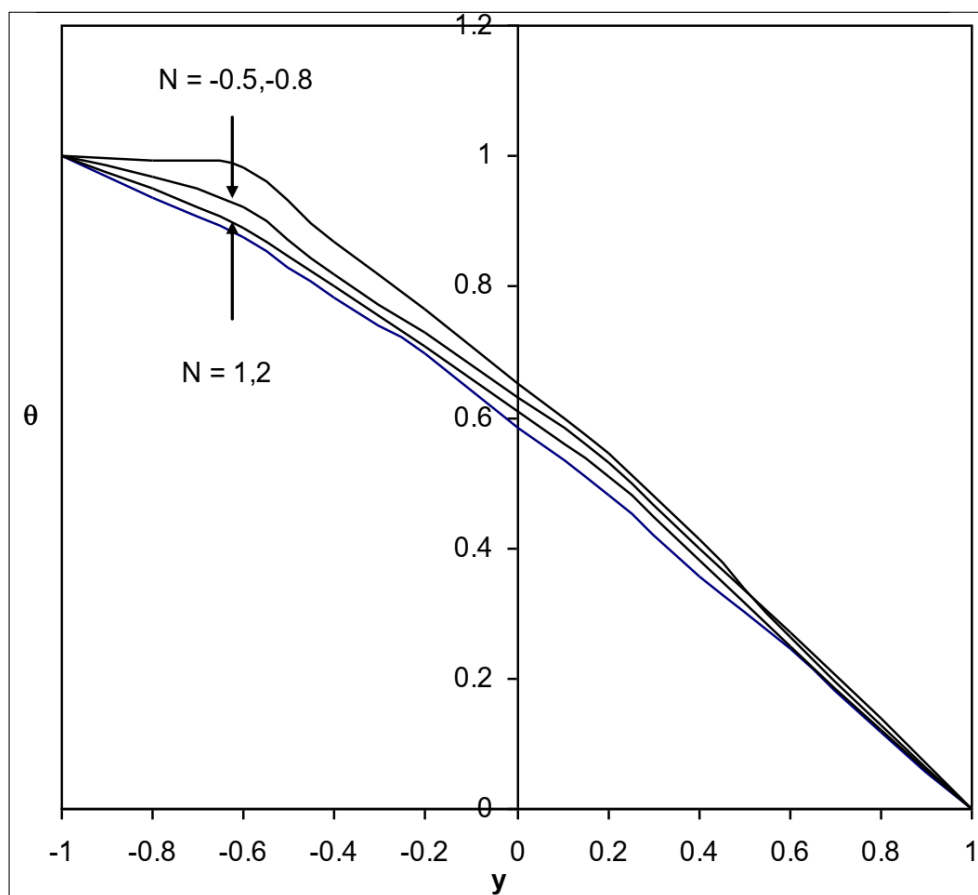


Fig 1.12: Variation of θ with $NG = 10^3$, $M = 2$, $D^{-1}=10^2$, $Sc = 1.3$, $S_0 = 0.01$, $\gamma=0.5$, $Du = 0.1$

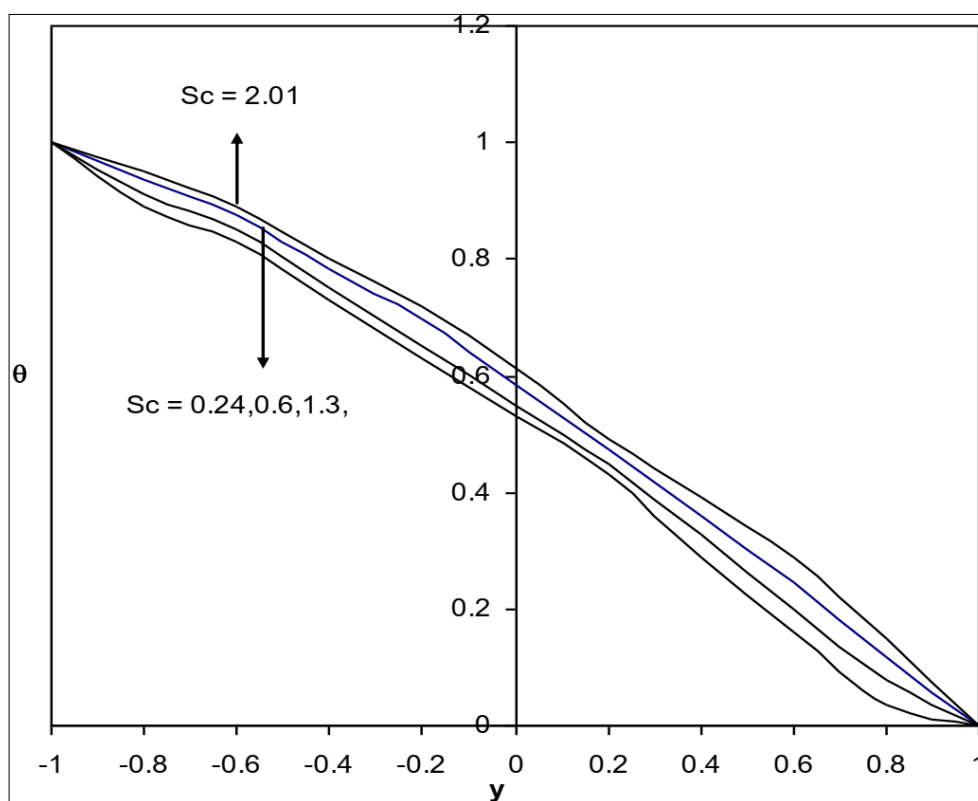


Fig 1.13: Variation of θ with $Sc G = 10^3$, $M = 2$, $N = 1$, $D^{-1}=10^2$, $S_0 = 0.01$, $\gamma=0.5$, $Du = 0.1$

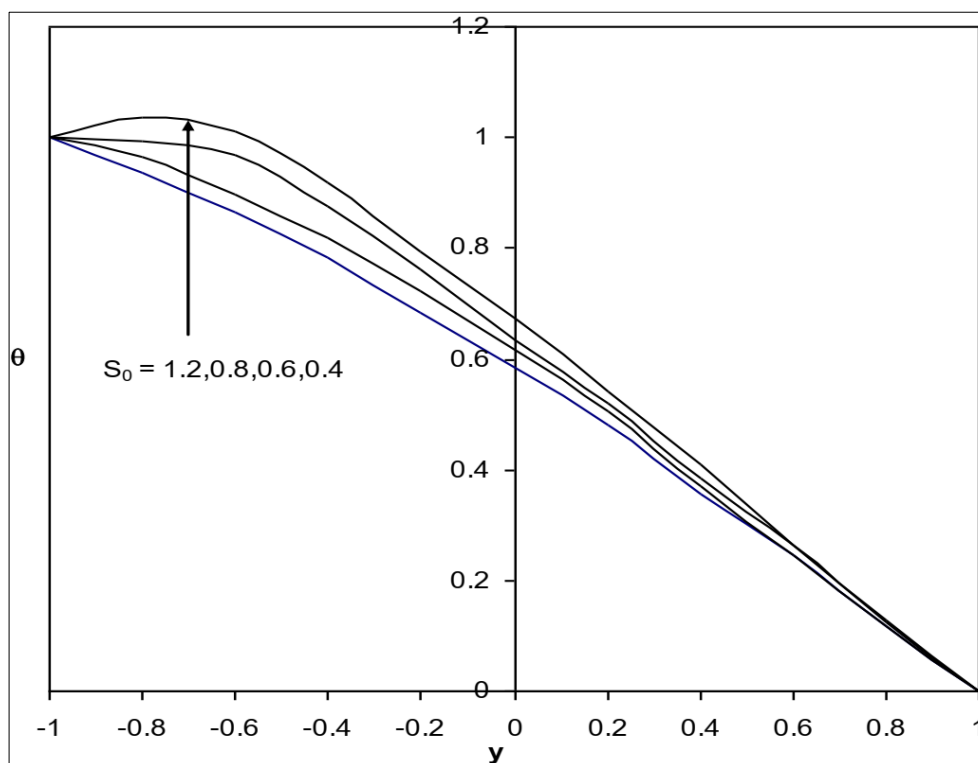


Fig 1.14: Variation of θ with S_0 $G = 10^3$, $M = 2$, $N = 1$, $D^{-1} = 10^2$, $Sc = 1.3$, $\gamma = 0.5$, $Du = 0.1$

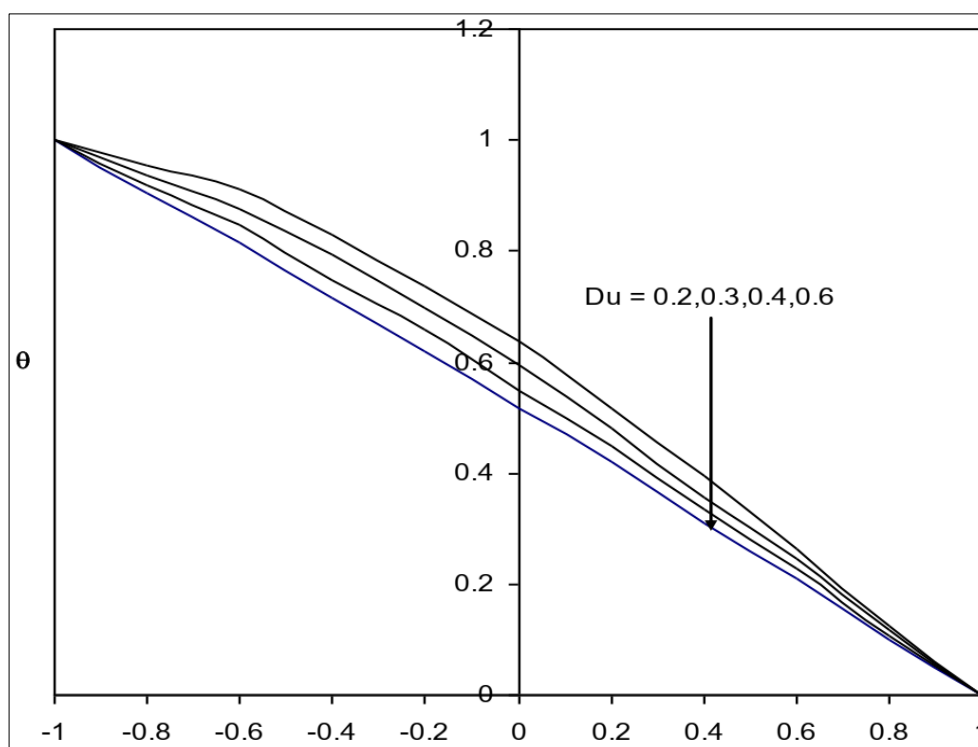


Fig 1.15: Variation of θ with Du $G = 10^3$, $M = 2$, $N = 1$, $D^{-1} = 10^2$, $Sc = 1.3$, $S_0 = 0.01$, $\gamma = 0.5$, $Du = 0.1$

8. Numerical Values

The average Nusselt number (Nu) which represents the rate of heat transfer at $y = \pm 1$ is shown in tables.1.1-1.6 for different values of G, D^{-1} , N, α , γ_1 and $x + \gamma t$. The variation of Nu with D^{-1} shows that lesser the permeability of the porous medium larger the magnitude of Nu at $y = +1$ and at $y = -1$ smaller $|Nu|$ and for further lowering of the permeability larger $|Nu|$ at both the walls. The variation of Nu with α shows that the rate of heat transfer depreciates with $\alpha < 4$ and enhancea with higher $\alpha \geq 6$ at both the walls. Lesser the molecular diffusivity larger $|Nu|$ at $y = +1$, smaller $|Nu|$ at $y = -1$ for $G > 0$ and for $G < 0$ smaller $|Nu|$ at $y = +1$ and larger $|Nu|$ at $y = -1$. An increase in the chemical reaction parameter $\gamma_1 \leq 4$ reduces $|Nu|$ and enhances with higher $\gamma_1 \geq 6$ in the heating case at both the walls. An increase in $x + \gamma t$ reduces the rate of heat transfer at $y = +1$ and fluctuates at $y = -1$. The Sherwood Number (Sh) which measures the rate of mass transfer at $y = \pm 1$ is shown in tables.1.7-1.10 for different variations. The variation of Sh with D^{-1} shows that lesser the permeability of the porous medium smaller the rate of mass transfer at $y = 1$ and larger at $y = -1$. The variation of Sh with α shows that an increase in $\alpha \leq 4$ enhances the rate of mass transfer and depreciates it with higher $\alpha \geq 6$ at $y = 1$

while at $y=-1$, it depreciates with $\alpha \leq 4$ and enhances with $\alpha \geq 6$ and for $G < 0$ a reversed effect is observed in the behaviour of Sh . The variation of Sh with Sc shows that lesser the molecular diffusivity larger $|Sh|$ at $y=\pm 1$ in the heating case while in the cooling case it enhances at $y=1$ and reduces at $y=-1$. The rate of mass transfer reduces at $y=+1$ and enhances at $y=-1$ when the buoyancy forces act in the same direction and for the forces acting in opposite directions $|Sh|$ enhances at $y=1$ and depreciates at $y=-1$. An increase in the chemical reaction parameter γ_1 leads to an enhancement in $n|Sh|$ at both the walls. With reference to the phase $x+\gamma t$ we find that the rate of mass transfer enhances with increase in $x+\gamma t \leq \pi$ and depreciates with $x+\gamma t \geq 2\pi$ at both the walls.

Table 1.1: Nusselt Number Nu_1 at $y = 1$

G								
10^3	-4.0534	-4.441	-5.0488	0.88614	3.0053	-4.0528	-4.053	-4.0538
3×10^3	-4.0625	-4.444	-5.0508	0.23058	2.8179	-4.0622	-4.0625	-4.0627
10^3	-4.0908	-4.4534	-5.0567	3.8822	3.4135	-4.0913	-4.0912	-3.8904
3×10^3	-4.0812	-4.4502	-5.0546	8.7482	3.6362	3.8815	-4.0814	-4.08104
D^{-1}	10^2	2×10^2	3×10^2					
α				4	6			
Sc						0.24	0.6	2.01

Table 1.2: Nusselt Number Nu_1 at $y = 1$

G							
10^3	-4.0534	-3.9856	-4.1793	-4.2058	-3.3133		-3.77302
3×10^3	-4.0625	-3.9935	-4.1906	-4.2176	-3.4967		-3.94741
10^3	-4.0908	-4.0184	-4.2265	-4.2545	-4.2923		-4.68446
3×10^3	-4.0812	-4.01002	-4.214	-4.2419	-3.9738		-4.39321
N	1	2	-0.5	-0.8			
γ_1					4		6

Table 1.3: Nusselt Number Nu_1 at $y = 1$

G								
10^3	-4.0534	-3.61125	-5.31303	-4.8331	-3.9683	-4.0534	-4.0545	-4.0546
3×10^3	-4.0625	-3.61975	-5.325	-4.8434	-4.0189	-4.0625	-4.0631	-4.0631
10^3	-4.0908	-3.6462	-5.3606	-4.8758	-4.1854	-4.0908	-4.0896	-4.0896
3×10^3	-4.0812	-3.6372	-5.3484	-4.8648	-4.1272	-4.0812	-4.0806	-4.0806
$x+\gamma t$	$\pi/4$	$\pi/2$	π	2π				
P					0.01	0.71	7	10

Table 1.4: Nusselt Number Nu_2 at $y = -1$

G								
10^3	7.7348	7.1854	7.5377	0.1104	-3.0962	7.7354	7.7352	7.7345
3×10^3	7.7519	7.1883	7.5377	1.44509	-2.6	7.7522	7.7521	7.7517
10^3	7.8051	7.1971	7.5375	-5.98946	-4.1767	7.8045	7.8047	7.8055
3×10^3	7.787	7.1941	7.5376	-5.8964	4.5664	7.7867	7.7868	7.7872
D^{-1}	10^2	2×10^2	3×10^2					
α	2	2	2	4	6			
Sc						0.24	0.6	2.01

Table 1.5: Nusselt Number Nu_2 at $y = -1$

G							
10^3	7.7348	7.6106	7.9655	8.0138	6.1046		6.7681
3×10^3	7.7519	7.6256	7.9866	8.0359	6.4066		7.0264
10^3	7.8054	7.6724	8.0527	8.105	7.717		8.1177
3×10^3	7.787	7.6565	8.0303	8.0815	7.1924		7.6864
N	1	2	-0.5	-0.8			
γ_1					4		6

Table 1.6: Nusselt Number Nu_2 at $y = 1$

G								
10^3	8.0848	8.0885	8.1997	8.1115	7.8041	8.0848	8.0885	8.0884
3×10^3	8.1019	8.1045	8.2218	8.131	7.9581	8.1019	8.1038	8.1036
10^3	8.1554	8.1543	8.2912	8.1921	8.4646	8.1551	8.1512	8.1511
3×10^3	8.137	8.1374	8.2675	8.1713	8.2883	8.137	8.1351	8.1349
$x+\gamma t$	$\pi/4$	$\pi/2$	π	2π				
P					0.01	0.71	7	10

Table 1.7: Sherwood number (Sh) at $y = 1$

G								
10^3	-0.0443	0.09263	0.09693	-0.08581	-0.0084	0.06502	0.02789	-0.11752
3×10^3	-0.04536	0.09266	0.09698	1.29679	-0.4288	0.06482	0.0274	-0.11916
10^3	-0.04866	0.09274	0.09715	-0.18578	0.05138	0.06421	0.02588	-0.12426
3×10^3	-0.04754	0.09271	0.0971	-0.41848	-0.02843	0.06442	0.02639	-0.12253
D^{-1}	10^2	2×10^2	3×10^2					
α				4	6			
Sc						0.24	0.6	2.01

Table-1.8: Sherwood number (Sh) at $y = 1$

G									
10^3	-0.2943	-0.28389	-0.31362	-0.3176	6.74864	10.43639	-0.30233	-0.3343	-0.295
3×10^3	-0.29536	-0.28481	-0.31497	-0.3190	-3.147	12.54275	-0.30341	-0.3359	-0.296
10^3	-0.29866	-0.28768	-0.31915	-0.3234	-0.61564	-1.10815	-0.30676	-0.3408	-0.3
3×10^3	-0.29754	-0.28671	-0.31773	-3.3219	-0.83107	-1.73405	-0.30563	-0.3393	-0.298
N	1	2	-0.5	-0.8					
$\gamma 1$					4	6			
$x + \gamma t$							$\pi/2$	π	2π

Table 1.9: Sherwood number (Sh) at $y = -1$

G								
10^3	-1.03231	5.0548	47.9692	-0.4181	-0.8794	-1.0188	-1.0289	-1.0333
3×10^3	-1.03277	4.7936	42.574	0.39408	-0.6552	-1.0263	-1.0311	-1.0332
10^3	-1.0341	5.1281	31.7422	-1.6799	-1.34006	-1.0453	-1.0372	-1.03313
3×10^3	-1.03366	4.3325	34.7018	-2.1798	-1.5767	-1.0395	-1.0352	-1.03317
D^{-1}	10^2	2×10^2	3×10^2	10^2	10^2	10^2	10^2	10^2
α	2	2	2	4	6			
Sc						0.24	0.6	2.01

Table 1.10: Sherwood number (Sh) at $y = -1$

G									
10^3	-1.0323	-1.0248	-1.0435	-1.0456	-1.8953	-2.985	-1.3342	-1.7775	-0.9089
3×10^3	-1.0327	-1.0253	-1.0439	-1.0459	-1.8515	-3.1447	-1.3322	-1.7718	-0.9104
10^3	-1.0341	-1.0268	-1.0449	-1.0469	-1.6594	-5.0327	-1.3266	-1.7555	-0.9146
3×10^3	-1.0336	-1.2763	-1.0446	-1.0466	-1.7366	-3.9481	-1.3284	-1.7609	-0.9132
N	1	2	-0.5	-0.8					
$\gamma 1$					4	6			
$x + \gamma t$							$\pi/2$	π	2π

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