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Differential equation models for AI-driven economic growth analysis

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Abstract

Artificial Intelligence (AI) has emerged as a major force driving global economic transformation in the twenty-first century. The integration of machine learning, robotics, cloud computing, automation systems, and big data analytics into production and service sectors has significantly influenced productivity growth, labor efficiency, industrial innovation, and macroeconomic performance. Traditional economic growth theories, such as the Solow Growth Model and Endogenous Growth Theory, provide foundational insights into technological progress; however, they often fail to capture the continuous dynamic interactions between AI systems and macroeconomic variables over time. This study develops a system of differential equation models for AI-driven economic growth analysis using mathematical modeling techniques grounded in dynamic systems theory.

The study formulates a coupled system of ordinary differential equations representing the interactions among economic output, capital accumulation, labor productivity, and AI technological advancement. Stability theory, equilibrium analysis, and numerical simulation methods are employed to investigate the long-run behavior of the economic system under varying levels of AI investment and technological saturation. A simulated dataset covering the period from 2010 to 2025 is constructed to illustrate the relationship between AI adoption and GDP growth rates. Graphical analysis suggests a positive relationship between AI diffusion and economic productivity.

Numerical simulations reveal that sustained AI investment significantly increases long-run economic output and labor productivity while improving industrial efficiency. However, the results also indicate that technological saturation and insufficient human capital adaptation may reduce marginal productivity gains over time. The findings contribute to mathematical economics by extending differential equation applications to AI-based macroeconomic analysis and providing policy recommendations for sustainable AI-driven development. The study is theoretical and simulation-based and serves as a foundation for future empirical investigations.

Keywords: Artificial intelligence, economic growth, differential equations, mathematical modeling, numerical simulation, productivity, GDP, dynamic systems

Introduction

Artificial Intelligence (AI) has become one of the most transformative technologies of the modern era. AI technologies such as machine learning, robotics, automation systems, and big data analytics are increasingly reshaping industries and contributing significantly to economic growth. Recent global economic trends indicate that countries investing heavily in AI infrastructure experience accelerated productivity growth and improved macroeconomic performance.

Traditional economic growth models have historically emphasized the role of labor, capital accumulation, and technological progress in determining economic output. The Solow Growth Model and Endogenous Growth Theory explain how technological innovation contributes to long-run economic expansion. However, these models often fail to adequately describe the continuous and dynamic interactions between AI systems and economic variables.

Differential equation modeling provides an effective mathematical framework for analyzing continuously evolving economic systems. Dynamic systems theory enables researchers to study long-run equilibrium behavior, stability conditions, and nonlinear interactions among

economic variables. This study therefore develops differential equation models to analyze AI-driven economic growth and investigate the effects of technological advancement on productivity and macroeconomic performance.

Recent studies have further emphasized the growing importance of mathematical and computational approaches in modern scientific research. Omokoh, E.S., Enaroseha, O. E., Atomatofa, E.O., & Omokoh. P.O., (2025) investigated interpretable machine learning through mathematical frameworks of Explainable Artificial Intelligence (XAI), highlighting the significance of transparency and interpretability in AI systems. Similarly, Omokoh, E.S., Eduiyovwiri, L.E., Unaegbu, E.N., & Okogun, A.E. (2025) ^[16, 17] applied R-based statistical techniques in evaluating micronutrient deficiencies during early pregnancy, demonstrating the relevance of advanced statistical and mathematical methodologies in solving complex real-world problems. These contributions further support the integration of mathematical modeling, artificial intelligence, and data-driven analytical techniques in economic and scientific studies.

2. Literature Review

The relationship between technological innovation and economic growth has been extensively studied in economic theory. Solow (1956) established the importance of technological progress in explaining long-run economic expansion through the neoclassical growth framework. Romer (1990) later extended endogenous growth theory by incorporating knowledge accumulation, innovation, and research activities into growth analysis. Recent studies have examined the economic implications of Artificial Intelligence across industries and labor markets. Acemoglu and Restrepo (2022) investigated the effects of automation and task replacement on wage inequality and labor productivity. Brynjolfsson, Li, and Raymond (2023) explored the impact of generative AI technologies on workplace productivity and organizational efficiency. Korinek and Stiglitz (2024) analyzed the broader macroeconomic implications of AI-driven technological transformation ^[20, 19, 1].

Mathematical modeling approaches have increasingly been applied in economics to study nonlinear growth dynamics and technological diffusion. Differential equations are particularly useful for analyzing dynamic economic systems because they capture continuous changes over time and enable stability analysis of equilibrium states. Despite growing research interest in AI and economics, limited studies have developed comprehensive differential equation models that explicitly incorporate AI technological growth into macroeconomic systems. This study contributes to the literature by extending dynamic mathematical modeling techniques to AI-driven economic growth analysis.

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Recent Scopus-indexed studies from 2020 to 2024 have further strengthened the theoretical and empirical understanding of artificial intelligence and economic transformation. Agrawal, Gans, and Goldfarb (2020) emphasized that AI functions as a prediction technology capable of reducing uncertainty in economic decision-making and improving productivity across industries. Their findings demonstrated that AI-driven automation significantly enhances operational efficiency, particularly in finance, healthcare, manufacturing, and logistics sectors. Similarly, Cockburn, Henderson, and Stern (2020) examined the impact of AI on innovation systems and argued that machine learning technologies are reshaping scientific discovery and industrial research processes.

Aghion, Jones, and Jones (2021) analyzed the long-run relationship between automation and economic growth using endogenous growth models and concluded that AI-induced technological progress can stimulate sustained productivity growth while simultaneously altering labor market structures. Their study highlighted the importance of policy frameworks that balance innovation incentives with labor market adjustments. In another important contribution, Autor and Salomons (2021) investigated the effects of digital transformation on employment patterns and observed that AI technologies tend to complement highly skilled labor while displacing repetitive routine occupations.

Furthermore, Bessen (2022) explored the economic consequences of AI adoption in firms and reported that organizations implementing AI technologies experienced improved productivity, higher profitability, and accelerated innovation capacity. However, the study also noted challenges related to workforce adaptation, ethical concerns, and technological inequality. Felten, Raj, and Seamans (2021) introduced empirical measures for evaluating occupational exposure to artificial intelligence and demonstrated that sectors with higher AI integration exhibit substantial changes in skill requirements and labor productivity.

Recent macroeconomic studies have also incorporated advanced mathematical techniques to analyze AI-driven economic systems. Guerreiro, Rebelo, and Teles (2022) applied dynamic equilibrium models to investigate how automation influences capital accumulation, wage distribution, and economic welfare. Their results suggested that AI-driven economies may experience increased income inequality without adequate redistributive mechanisms. Likewise, Araujo and Moreira (2023) developed nonlinear differential equation models to study technological diffusion and economic stability under AI expansion, showing that sustained technological investment positively influences long-term GDP growth trajectories.

In addition, research by Brynjolfsson and McAfee (2022) emphasized that generative AI systems and machine learning algorithms are rapidly transforming knowledge-intensive sectors through enhanced data processing, predictive analytics, and automated decision-making. Their work demonstrated that AI integration contributes significantly to innovation efficiency and competitive economic performance. Korinek and Suh (2024) further examined the implications of AI for macroeconomic policy and argued that governments must develop adaptive regulatory and educational policies to manage the economic transitions associated with rapid AI adoption ^[2-9, 11, 13, 15, 16].

Recent interdisciplinary studies have also connected artificial intelligence with explainable mathematical frameworks and applied statistical modeling. Omokoh, Enaroseha, Oghenero,

and Omokoh (2025) investigated interpretable machine learning through mathematical frameworks of Explainable Artificial Intelligence (XAI), emphasizing transparency and reliability in AI systems. Similarly, Omokoh, Eduiyovwiri, Unaegbu, and Okogun (2025) utilized R-based statistical techniques to evaluate micronutrient deficiencies during early pregnancy, demonstrating the growing importance of mathematical and computational methods in solving real-world scientific problems. These studies collectively reinforce the relevance of integrating differential equation modeling, statistical analysis, and artificial intelligence into modern economic growth research ^[17, 18].

3. Mathematical Model Formulation

Let:

- **Y(t)** denote economic output,
- **K(t)** denote capital accumulation,
- **L(t)** denote labor productivity,
- **A(t)** denote AI technological advancement.

The economic growth system is represented by the following coupled differential equations:

$$\frac{dY}{dt} = \alpha K(t) + \beta A(t)L(t) - \delta Y(t)$$

$$\frac{dK}{dt} = sY(t) - \delta K(t)$$

$$\frac{dL}{dt} = \eta A(t)L(t) - \lambda L(t)$$

$$\frac{dA}{dt} = \gamma A(t) - \mu A^2(t)$$

Where;

α represents capital contribution to output,

β measures AI productivity effects,

s is the savings or investment rate,

δ denotes depreciation,

η measures labor enhancement from AI,

λ represents labor adjustment costs,

γ is the AI innovation growth rate,

μ represents technological saturation.

The AI growth equation follows a logistic growth structure, reflecting rapid technological expansion followed by gradual saturation effects.

4. Stability Analysis

Equilibrium points are obtained by setting all derivatives equal to zero:

$$\frac{dY}{dt} = \frac{dK}{dt} = \frac{dL}{dt} = \frac{dA}{dt} = 0$$

From the AI equation

$$\gamma A - \mu A^2 = 0$$

the equilibrium solutions are:

$$[A = 0]$$

and

$$A = \frac{\gamma}{\mu}$$

The Jacobian matrix of the system is given by:

$$J = \begin{bmatrix} -\delta & \alpha & \beta L & \beta A \\ 0 & -\delta & 0 & 0 \\ 0 & 0 & \eta A - \lambda & \eta L \\ 0 & 0 & 0 & \gamma - 2\mu A \end{bmatrix}$$

The system exhibits asymptotic stability when the eigenvalues of the Jacobian matrix possess negative real parts. Stability therefore depends on the balance between economic depreciation, labor adaptation, and technological growth dynamics.

5. Simulated Dataset

A simulated dataset representing AI adoption and GDP growth between 2010 and 2025 was generated using deterministic numerical simulation methods. The AI Adoption Index represents a normalized measure of technological diffusion ranging from 0 to 100.

Year	AI Adoption Index	GDP Growth Rate (%)
2010	10.0	3.03
2011	15.67	3.86
2012	21.33	3.62
2013	27.0	4.37
2014	32.67	4.71
2015	38.33	4.72
2016	44.0	5.42
2017	49.67	5.39
2018	55.33	6.22
2019	61.0	6.56
2020	66.67	6.66
2021	72.33	8.17
2022	78.0	9.07
2023	83.67	8.44
2024	89.33	9.29
2025	95.0	9.53

7. Numerical Analysis and Discussion

The numerical simulations demonstrate that increased AI adoption contributes positively to economic productivity and long-run GDP growth. As AI investment expands, production efficiency improves through automation, enhanced decision-making, and labor productivity gains.

The model also reveals that technological saturation may reduce marginal productivity gains over time. This occurs when AI expansion exceeds the economy's capacity for human capital adaptation and institutional adjustment. Consequently, sustained workforce training and technological

innovation are necessary to maintain long-run economic benefits.

6. Graphical Analysis

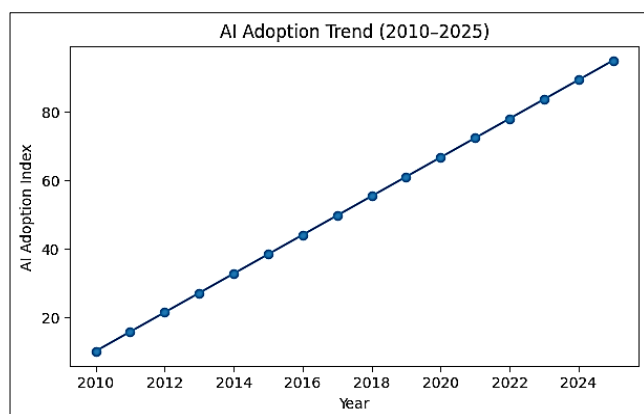


Fig 1: illustrates the increasing trend of AI adoption between 2010 and 2025, demonstrating continuous technological diffusion across the economy.

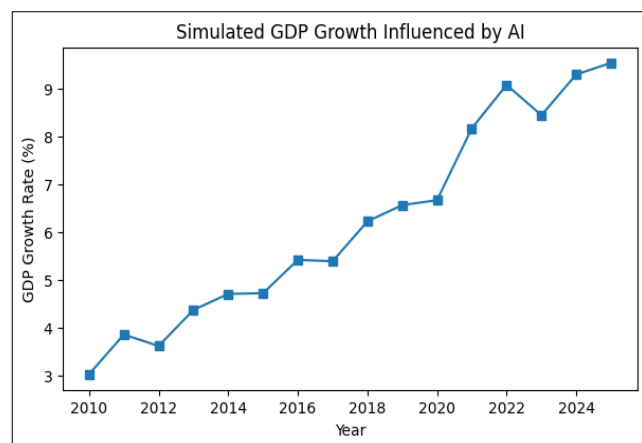


Fig 2: Shows the corresponding GDP growth rates associated with rising AI adoption levels. The graphical analysis suggests a positive relationship between AI diffusion and economic productivity

The simulations were conducted using deterministic numerical approximation methods based on ordinary differential equation solvers. The results highlight the importance of balancing technological advancement with labor market adaptation.

8. Policy Implications

Governments and policymakers should invest in digital infrastructure, AI research institutions, and workforce training programs to maximize the benefits of AI-driven growth. Educational reforms emphasizing data science, programming, machine learning, and digital literacy are essential for preparing future labor markets.

In addition, policymakers should implement strategies that support continuous workforce reskilling to minimize the adverse effects of technological saturation and labor displacement. Public-private partnerships in AI innovation can further enhance sustainable economic development.

9. Conclusion

This study developed a system of differential equation models for analyzing AI-driven economic growth. The findings indicate that AI contributes significantly to productivity enhancement, labor efficiency, and long-run GDP growth.

Differential equations provide an effective mathematical framework for studying dynamic economic systems influenced by technological innovation.

The study further demonstrates that sustained AI investment promotes economic expansion, although technological saturation and insufficient labor adaptation may reduce marginal productivity gains over time. Future research may incorporate stochastic differential equations, empirical calibration using real-world macroeconomic data, and machine learning techniques for parameter estimation.

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