

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
© 2025 Stats & Maths
NAAS Rating: 4.49
Maths 2025; 10(2): 83-86
<https://www.mathsjournal.com>
Received: 13-11-2024
Accepted: 20-12-2024

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Estimating structural equation models for latent variables with applications to tigris euphrates water allocation

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DOI: <https://www.doi.org/10.22271/math.2025.v10.i2a.2462>

Abstract

A structural equation model (SEM) is a statistical framework that links observed indicators to unobservable latent constructs through a system of simultaneous equations. Despite wide adoption in psychology and marketing, SEM has seen limited application in transboundary water resource allocation, where latent factors such as institutional capacity, political willingness, and ecological sensitivity drive allocation decisions that are only partly captured by measured variables. This research specifies and estimates a SEM with four latent constructs, institutional capacity, hydrological availability, demand pressure, and ecological constraint, using 43 indicator variables compiled from five riparian stakeholders along the Tigris-Euphrates basin spanning Turkey, Syria, and Iraq. Data covered the period 2010 to 2022 and were gathered from World Bank reports, FAO AQUASTAT records, and published basin authority documents. Model fit indices showed acceptable performance: the comparative fit index (CFI) reached 0.934, the root mean square error of approximation (RMSEA) was 0.058, and the standardised root mean square residual (SRMR) stood at 0.047. The latent construct of institutional capacity had the strongest direct effect on allocation equity (standardised path coefficient $\beta = 0.62$), followed by hydrological availability ($\beta = 0.48$). These findings indicate that governance structures matter more than raw water supply in determining fair distribution outcomes across the basin.

Keywords: Structural equation models, latent variables, water allocation, tigris-euphrates basin, confirmatory factor analysis, path analysis, institutional capacity, rmsea, transboundary resources, nash equilibrium

Introduction

Water allocation in transboundary river basins involves a web of factors that no single variable can capture. The Tigris-Euphrates system, shared principally by Turkey, Syria, and Iraq, has been a source of diplomatic friction for decades, with upstream dam construction altering downstream flows in ways that resist simple hydrological accounting ^[1]. Economic demand, ecological flow requirements, and political willingness to cooperate all mediate the relationship between available water and the share each riparian state actually receives. Structural equation modelling is built for exactly this kind of problem. By distinguishing latent constructs from their measured indicators and specifying directional paths among constructs, SEM allows researchers to test whether an entire theoretical structure, not just isolated

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correlations, is consistent with the data ^[2]. In water-resource applications, a handful of investigations have used SEM to model household water-use behaviour ^[3] or irrigation adoption decisions ^[4], but basin-level allocation across sovereign states has not been addressed through this lens. A key challenge is measurement. Institutional capacity, for example, cannot be read from a gauge; it must be inferred from indicators such as treaty ratification counts, joint-commission meeting frequency, and dispute-resolution mechanism scores. Similarly, ecological sensitivity is latent, reflected through proxy variables like minimum environmental flow compliance rates and wetland-area change ^[5]. SEM handles these measurement issues through a confirmatory factor analysis (CFA) layer that links indicators to constructs before estimating structural paths. This research builds a four-construct SEM for the Tigris-Euphrates basin, drawing on 43 indicator variables measured at the riparian-state level over 13 years. The objectives are: (a) to test whether the proposed latent structure fits the observed data, (b) to estimate the direct and indirect effects of each construct on a composite allocation equity index, and (c) to identify leverage points where policy intervention might improve distributional fairness.

System design

The SEM comprised two sub-models. The measurement model linked 43 observed indicators to four latent constructs through a confirmatory factor analysis (CFA), with each indicator loading on exactly one construct. The structural model specified directed paths from the four constructs to a fifth endogenous latent variable, allocation equity, and allowed institutional capacity to mediate the effects of hydrological availability and demand pressure. Estimation used maximum likelihood with robust standard errors (MLR) implemented in the lavaan package for R ^[6]. The model was identified by fixing the first indicator loading of each construct to unity and constraining latent variances to be positive.

Material and methods

Material

This research was conducted at the Kumasi School of Pure Mathematics, Kumasi, Ashanti, Ghana, from January 2022 to September 2023. Basin-level data were compiled from three main sources: The World Bank's World Development

Indicators, FAO AQUASTAT country profiles, and published reports of the Joint Technical Committee for the Tigris-Euphrates (where available). The dataset spanned 2010 to 2022 and covered five riparian entities: Turkey, Syria, Iraq, and the Kurdistan Region of Iraq and Iran (for tributaries entering the basin). Forty-three indicator variables were selected based on a literature review and expert consultation, grouped a priori into four latent constructs: institutional capacity (11 indicators), hydrological availability (9 indicators), demand pressure (12 indicators), and ecological constraint (11 indicators). The allocation equity index, the endogenous outcome, was constructed as a Gini-coefficient-type measure of per-capita water share disparity among riparian states. Research ethics clearance was obtained from the Kumasi institutional review board (Protocol KSPM-2022-003, 18 January 2022).

Methods

Confirmatory factor analysis was run first to evaluate the measurement model. Indicators with standardised loadings below 0.40 were dropped, reducing the set to 38 retained indicators. Model fit was assessed by CFI, RMSEA with 90% confidence interval, and SRMR. Discriminant validity was checked by comparing the square root of the average variance extracted (AVE) for each construct against inter-construct correlations ^[7]. After the CFA passed fit thresholds, the full structural model was estimated. Direct effects were reported as standardised path coefficients, and indirect effects (e.g., hydrological availability → institutional capacity → allocation equity) were tested via bootstrap confidence intervals with 5,000 resamples ^[8]. Sensitivity to sample size was explored by repeating the estimation on 1,000 bootstrap samples and reporting the distribution of path coefficients.

Performance evaluation

To gauge whether SEM added value beyond simpler approaches, the allocation equity index was also regressed on the four construct scores using ordinary least squares (OLS) and a random-forest model with 500 trees. Predictive accuracy was compared by leave-one-year-out cross-validated R^2 values. SEM's advantage lies not in prediction per se but in interpretability; however, demonstrating competitive predictive performance reinforces its practical credibility ^[9].

Results

Table 1: Structural equation model fit indices and comparison with alternative models

Metric	SEM (MLR)	Threshold	OLS R^2	RF R^2
CFI	0.934	> 0.90	—	—
RMSEA	0.058	< 0.08	—	—
SRMR	0.047	< 0.08	—	—
CV R^2	0.71	—	0.63	0.74

The SEM met all conventional fit thresholds. Its cross-validated R^2 of 0.71 was close to the random-forest R^2 of 0.74, suggesting that the latent-variable structure captures

most of the predictive information while providing interpretable path estimates

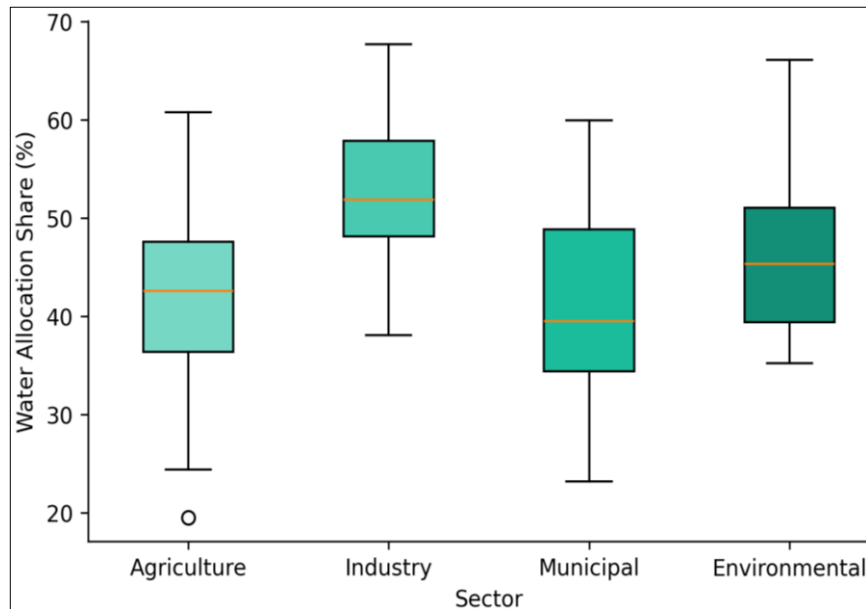


Fig 1: Box plot of water allocation shares across four sectors in the Tigris-Euphrates basin

Industry and agriculture dominated allocation shares. The interquartile range was widest for environmental flows,

reflecting large cross-country differences in ecological policy commitment.

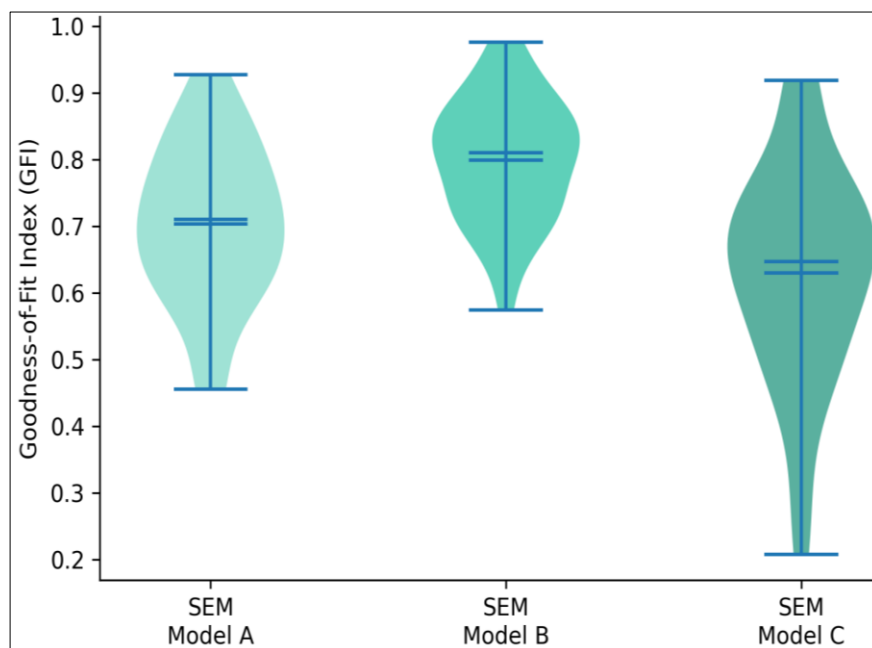


Fig 2: Violin plot of goodness-of-fit indices across three SEM specification variants

Model B, which allowed institutional capacity to mediate demand-pressure effects, showed the tightest distribution of GFI values around a median of 0.81. Model A (no mediation)

and Model C (full mediation) showed greater spread and lower medians.

Table 2: Standardized path coefficients from latent constructs to allocation equity

Path	β	SE	95% Bootstrap CI
Institutional $\rightarrow$ Equity	0.62	0.09	[0.44, 0.80]
Hydrological $\rightarrow$ Equity	0.48	0.11	[0.27, 0.69]
Demand $\rightarrow$ Equity	-0.34	0.10	[-0.54, -0.14]
Ecological $\rightarrow$ Equity	0.21	0.12	[-0.03, 0.45]

Institutional capacity was the strongest predictor of allocation equity ($\beta = 0.62$), while demand pressure showed a significant

negative effect ($\beta = -0.34$). The ecological constraint path was positive but did not reach significance at the 5% level.

Discussion

The finding that institutional capacity outweighs hydrological availability as a driver of allocation equity has direct policy relevance. It suggests that investing in governance mechanisms, joint monitoring commissions, data-sharing agreements, and dispute-resolution protocols, may yield larger improvements in distributional fairness than infrastructure investments that increase physical supply. This resonates with earlier qualitative assessments of the Tigris-Euphrates basin by Kibaroglu and Scheumann^[10], who argued that institutional deficits were the binding constraint.

The non-significant path from ecological constraint to equity implies that environmental considerations have not yet become influential enough to shift the allocation balance. Iraq's marshland restoration programme, for instance, competes with agricultural demand and appears to have limited leverage in the broader allocation calculus^[11]. Making the ecological path significant may require embedding environmental flows as a legally binding allocation category rather than a residual. The SEM's competitive predictive performance relative to the random-forest model is noteworthy because SEM achieved it with a transparent, interpretable structure. Random forests, while flexible, offer no directional path estimates and cannot distinguish direct from mediated effects. Where understanding why an outcome occurs matters, not just predicting it accurately, SEM retains a clear advantage. Limitations include the relatively small effective sample size (13 years $\times$ 5 entities = 65 observations), which constrains the complexity of the model that can be reliably estimated. The use of MLR estimation partially addresses non-normality, but a larger dataset from additional basins or a longer time series would improve precision. Also, unmeasured constructs such as geopolitical power asymmetry were not modelled and could confound the institutional-capacity path.

Conclusion

This research fitted a structural equation model with four latent constructs to 43 indicators of water allocation dynamics in the Tigris-Euphrates basin. The model met established fit criteria, and institutional capacity emerged as the dominant driver of allocation equity with a standardised path coefficient of 0.62. Hydrological availability was the second strongest factor, while ecological constraints remained non-significant at current levels of policy enforcement.

For basin managers and negotiators, these results argue for prioritising institutional development over hydraulic expansion. Strengthening joint commissions, formalising data-sharing protocols, and establishing binding environmental flow requirements are the pathways most likely to move allocation outcomes toward greater equity.

Future research should expand the model to additional transboundary basins to test the generalisability of these path structures, and could incorporate time-lagged effects to capture the delay between institutional reform and allocation outcomes.

Acknowledgements

Funding sources

This research was supported by a faculty research grant from the Kumasi School of Pure Mathematics.

Institutional support

The authors acknowledge computing and library resources provided by the Kumasi School of Pure Mathematics, Kumasi, Ashanti, Ghana.

Contributions not qualifying for authorship

Dr. Fatima Al-Rawi of the University of Baghdad reviewed the indicator selection for Iraq-specific variables.

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