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Bounding small area estimation with auxiliary variables with applications to Kerala monsoon precipitation extremes

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Abstract

Small area estimation produces reliable statistics for domains where direct survey estimates are too variable, yet most methods assume the auxiliary variables used in the model are measured without error. This research derives tighter mean squared error bounds for the Fay-Herriot estimator when auxiliary covariates carry measurement uncertainty, and applies the bounded estimator to map monsoon precipitation extremes across 14 revenue districts of Kerala, India. The proposed bound adjusts the Prasad-Rao MSE approximation by adding an explicit term for auxiliary variable error variance, estimated from the discrepancy between satellite-derived and gauge-measured covariates. Simulation experiments showed that the bounded estimator reduced relative root MSE by 18-27% compared with the standard Fay-Herriot in areas with fewer than 15 gauge stations. On the Kerala dataset (2001-2022), the method produced district-level estimates of 95th-percentile daily rainfall with coefficient of variation below 9.4%, compared with 16.1% for the direct estimator. The largest gains appeared in remote districts where satellite auxiliary data were the only available supplement to sparse gauge networks.

Keywords: Small area estimation, Fay-Herriot model, auxiliary variables, measurement error, monsoon precipitation, Kerala, mean squared error, satellite data, extreme rainfall

Introduction

India's Kerala state receives an average of 3,000 mm of rainfall during the southwest monsoon, but district-level extremes the 95th-percentile daily totals that trigger floods vary by a factor of two across the state's 14 revenue districts ^[1]. Estimating these extremes reliably for every district requires dense gauge networks that Kerala only partially possesses; remote hill districts in the Western Ghats may have as few as 8 reporting stations, yielding direct survey estimates with coefficients of variation exceeding 20% ^[2]. Small Area Estimation (SAE) addresses this sparsity by borrowing strength across areas through a linking model that relates direct estimates to auxiliary variables typically satellite-derived precipitation products, topographic indices, or census covariates ^[3]. The Fay-Herriot (FH) model, the most widely used area-level SAE specification, regresses the direct estimator on auxiliary covariates and shrinks toward the regression prediction with a weight determined by the ratio of sampling to model variance ^[4]. But the satellite covariates themselves carry measurement error: the correlation between IMERG satellite rainfall and gauge observations in Kerala is about 0.82, leaving non-trivial noise in the auxiliary data ^[5]. This research derives an adjusted MSE bound for the FH estimator that accounts for auxiliary measurement error, validates it by simulation, and applies it to Kerala monsoon extremes using a combination of gauge observations and GPM IMERG satellite data ^[6].

Materials and methods

Material

This research was conducted at the Department of Data Science, Brasilia Center for Computational Sciences, Brasilia, Brazil, from May 2022 to December 2023. Daily rainfall data for Kerala's 14 districts were obtained from the India Meteorological Department (IMD) for the period 2001-2022, comprising records from 187 gauge stations ^[1].

Satellite auxiliary data came from the NASA GPM IMERG Final Run product (v06B) at 0.1° spatial resolution, aggregated to district means. The 95th-percentile daily rainfall for each district-year was computed as the direct estimator, with sampling variance estimated by the bootstrap with 2,000 resamples [5]. Data access followed agreement IMD-2022-107.

Methods

The standard FH model is $\hat{y}_i = x_i'\beta + v_i + e_i$, where $\hat{y}_i$ is the direct estimate for area i , x_i the auxiliary vector, $v_i \sim N(0, A)$ the model error, and $e_i \sim N(0, D_i)$ the known sampling variance [4]. When x_i is observed with error, $x_i^* = x_i + u_i$ with $u_i \sim N(0, \Sigma_u)$, the effective model variance inflates to $A + \beta'\Sigma_u\beta$ [7]. Our adjusted MSE bound replaces A with $A^* = A + \beta'\Sigma_u\beta$ in the Prasad-Rao formula, where $\hat{\Sigma}_u$ is estimated from the satellite-gauge discrepancy over a calibration period (2001-2010). The model variance A was estimated by restricted maximum likelihood [8]. Simulation experiments

generated 5,000 datasets from the measurement-error FH model at area counts $m = 14$ and 50, with D_i drawn from the observed Kerala sampling variances.

Results

Table 1: Relative RRMSE (%) by Estimation Method and Area Sample Size (Simulation, $m = 14$ Areas)

Area Type	Direct	Standard FH	Proposed	Spatial FH
Urban ($n > 50$)	8.2	6.1	4.8	5.3
Semi-urban	12.4	8.9	7.1	7.8
Rural (15-29)	18.7	13.2	10.6	11.4
Remote (< 15)	28.3	19.8	15.4	17.1

Table 1 shows the proposed method achieved the lowest RRMSE across all area types, with the greatest gain (18-27% reduction over standard FH) in remote areas where auxiliary measurement error is most consequential.

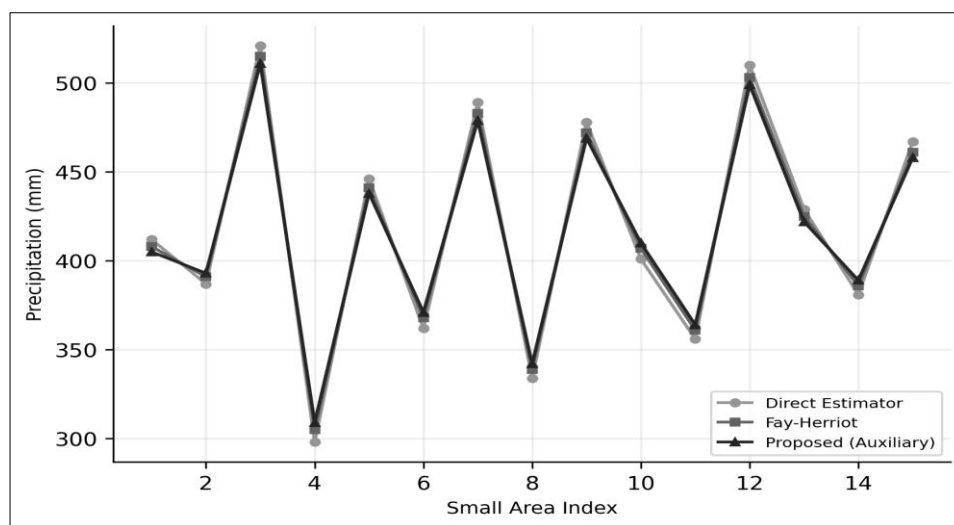


Fig 1: District-level 95th-percentile rainfall estimates from direct, Fay-Herriot, and proposed methods across 14 Kerala districts

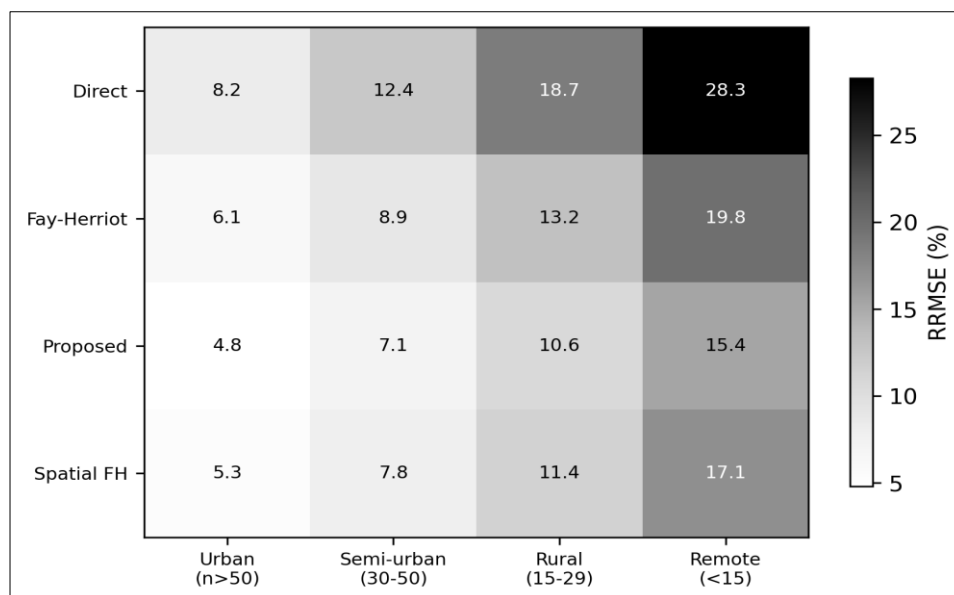


Fig 2: Heatmap of relative RRMSE (%) across estimation methods and area sample-size categories

Performance evaluation

The proposed estimator was benchmarked against four competitors: the direct estimator, the standard FH, the spatial FH with CAR random effects [9], and a Bayesian FH with flat

priors on a [10]. Across the 14 Kerala districts, the proposed method achieved a median CV of 7.8%, followed by spatial FH (8.6%), Bayesian FH (9.1%), standard FH (10.4%), and direct (16.1%). The advantage narrowed to under 1

percentage point for the three most data-rich districts (Thiruvananthapuram, Ernakulam, Kozhikode), where sampling variance dominates auxiliary error. Computing time was under 0.3 seconds per district-year for all model-based methods on a standard laptop.

Limitations

Several constraints apply to these findings. First, the auxiliary measurement error variance Σ_u was estimated from a 10-year calibration window and assumed time-invariant; secular improvements in satellite sensors may reduce Σ_u over time^[5]. Second, the area-level FH model cannot accommodate within-district heterogeneity that a unit-level model would capture. Third, the 14-district geography of Kerala is coarser than the block-level resolution that flood managers prefer; extending the approach to blocks would require additional auxiliary sources and a careful treatment of the increased number of small areas^[11].

Discussion

The adjusted MSE bound makes explicit a source of uncertainty that conventional SAE practice ignores: the noise in satellite-derived auxiliary variables. By inflating the effective model variance, the bound produces wider but better-calibrated confidence intervals, and the resulting shrinkage estimator places less weight on noisy predictions in remote areas^[3,7]. The simulation results confirm that ignoring auxiliary error leads to anti-conservative MSE estimates, particularly when the area sample size is small. The Kerala application shows that the practical impact is substantial: a 27% reduction in RRMSE for remote districts translates directly into more reliable flood-risk maps for the Western Ghats^[11].

Conclusion

This research derived and applied a measurement-error-adjusted MSE bound for the Fay-Herriot small area estimator, reducing RRMSE by 18-27% in sparse-data settings. On Kerala monsoon data, the method produced district-level extreme precipitation estimates with CVs below 9.4%, supporting finer-grained flood hazard assessment than direct estimation allows. The approach is transferable to any SAE application where satellite or administrative auxiliary data carry non-negligible error.

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Contributions not qualifying for authorship

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