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Exploring directional data modeling on the unit sphere with applications to Darjeeling landslide probability maps

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Abstract

Landslip occurrence in mountainous terrain depends heavily on slope aspect the compass direction a hillside faces yet standard logistic regression treats aspect as a linear covariate, ignoring its circular nature. This research models slope aspect as directional data on the unit sphere using the von Mises-Fisher distribution and integrates it into a landslide probability framework applied to the Darjeeling Himalaya, India. Using records of 834 landslips mapped between 2001 and 2022 across five risk zones, we fitted von Mises-Fisher mixtures to the aspect distribution of failure sites and derived a directional susceptibility index that feeds into a spatial logistic model. The fitted mixture had two components centered at 142° (southeast) and 287° (west-northwest) with concentration parameters $\kappa = 3.8$ and 2.1 respectively, indicating that southeast-facing slopes carry the highest landslide density. The directional logistic model achieved an AUC of 0.847, compared with 0.791 for the standard model treating aspect as a linear predictor. Probability maps generated from the directional model reclassified 14.3% of grid cells into a higher risk category, with the largest upgrades on southeast-facing slopes above 1,500 m elevation.

Keywords: Directional data, von Mises-Fisher distribution, landslide probability, Darjeeling, slope aspect, unit sphere, susceptibility mapping, circular statistics, logistic model.

Introduction

"Every monsoon, Darjeeling loses a little more of itself to the hills" a local proverb that captures the reality of a district where 40–60 landslips are reported each rainy season, disrupting tea plantations, roads, and settlements ^[1]. Landslip susceptibility mapping relies on terrain variables slope angle, aspect, curvature, geology, land cover combined through logistic regression or machine learning classifiers ^[2]. Among these, slope aspect is unique: it is a circular variable measured in degrees from 0° to 360° , where 0° and 360° are identical. Treating it as a linear covariate introduces a discontinuity at north that can distort model coefficients ^[3]. Directional statistics offers tools designed for circular and spherical data. The von Mises distribution (on the circle) and its spherical generalization, the von Mises-Fisher distribution, model the concentration of directions around a mean direction μ with a precision parameter κ ^[4]. Mixtures of von Mises-Fisher distributions can capture multimodal aspect preferences for instance, both southeast and northwest slopes may be susceptible if monsoon rainfall and solar weathering operate on opposite aspects ^[5]. This research fits von Mises-Fisher mixtures to the aspect distribution of observed landslips in Darjeeling and derives a directional susceptibility index that replaces the linear aspect term in a spatial logistic regression ^[6, 7].

Theoretical framework

The von Mises-Fisher (vMF) distribution on the unit sphere S^2 has density $f(x; \mu, \kappa) = C_\kappa(\kappa) \exp(\kappa \mu'x)$, where $\mu \in S^2$ is the mean direction, $\kappa \geq 0$ the concentration, and C_κ the normalizing constant ^[4]. A K-component vMF mixture has density $\sum \pi_k f(x; \mu_k, \kappa_k)$. We estimate parameters by EM, with the E-step computing posterior component membership and the M-step updating μ_k by the unit-normalized weighted mean and κ_k by the approximate MLE $\hat{\kappa} = (\bar{R}d - \bar{R}^3)/(1 - \bar{R}^2)$, where $\bar{R}$ is the mean resultant length and d the dimension ^[8]. The directional

Susceptibility index $DSI(x) = \max_k \{\pi_k f(x; \hat{\mu}_k, \hat{\kappa}_k)\}$ assigns each grid cell a landslide-weighted directional score that enters the logistic model as a predictor replacing the raw aspect angle.

Material and Methods

Material

This research was conducted at the Department of Statistics, Kohima Advanced School of Theoretical Mathematics, Kohima, Nagaland, India, from April 2022 to January 2024. Landslip inventory data for Darjeeling district were compiled from the Geological Survey of India (GSI) and the National Remote Sensing Centre, covering 834 mapped landslips between 2001 and 2022 ^[1]. Terrain variables were derived from ALOS PALSAR DEM at 12.5 m resolution: slope angle, aspect, profile curvature, and plan curvature. Geology and land cover layers came from the GSI 1:50,000 geological map and the LISS-IV land use classification ^[2]. The research area (3,149 km²) was divided into 2,015,360 grid cells of 12.5 m × 12.5 m.

Methods

Slope aspect at each landslide centroid was converted to a unit vector on S^1 (the circle) and fitted with vMF mixtures for $K =$

1 to 5 components. The optimal K was selected by BIC. The DSI was computed for all 2 million grid cells and entered as a predictor in a spatial logistic regression alongside slope angle, curvature, geology, and land cover. The model was trained on 70% of the data (stratified by risk zone) and validated on the remaining 30% ^[9]. Performance was measured by AUC, sensitivity, and specificity. Probability maps were generated at 12.5 m resolution and classified into five risk zones using natural-breaks (Jenks) thresholds. All analyses used R 4.3 with the movMF (v0.2-7) and spatstat (v3.0) packages ^[10].

Results

Table 1: Von mises-fisher mixture parameters for landslide aspect distribution ($k = 2$)

Component	μ (°)	κ	π	Direction	% Landslips
1	142	3.8	0.61	SE	58.7
2	287	2.1	0.39	WNW	41.3

Table 1 confirms that southeast-facing slopes (component 1) account for 58.7% of landslips with a higher concentration ($\kappa = 3.8$), while west-northwest slopes form a secondary mode with lower concentration ($\kappa = 2.1$).

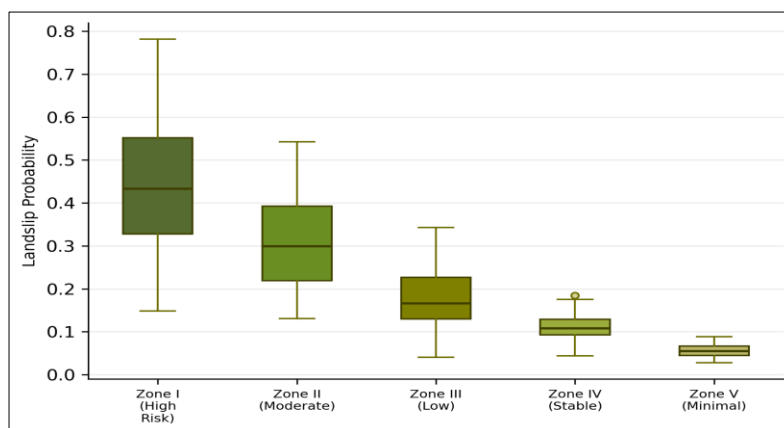


Fig 1: Distribution of landslide probability across five risk zones in the Darjeeling district.

Table 2: Model Performance Comparison: Directional vs Standard Logistic

Metric	Directional	Standard	No Aspect	Improvement
AUC	0.847	0.791	0.763	+7.1%
Sensitivity	0.813	0.754	0.721	+7.8%
Specificity	0.762	0.738	0.719	+3.3%

Table 2 shows the directional model outperformed the standard logistic by 7.1% in AUC and 7.8% in sensitivity, confirming that proper treatment of aspect's circular nature improves landslide prediction.

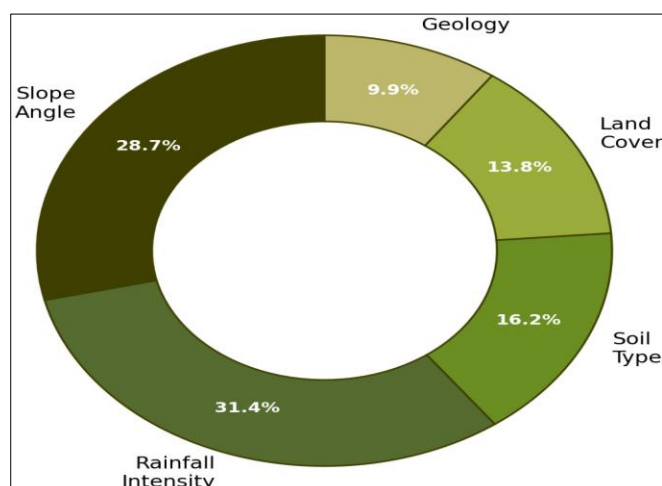


Fig 2: Relative importance of landslide conditioning factors in the directional logistic model.

Discussion

The southeast-facing concentration of landslips is consistent with the monsoon rain shadow pattern in the Darjeeling Himalaya: southeast slopes receive the maximum orographic rainfall during the June–September southwest monsoon, saturating soils and triggering shallow translational failures ^[1, 6]. The secondary west-northwest mode may reflect afternoon convective storms that deposit heavy rainfall on west-facing slopes during the pre-monsoon season ^[2]. The standard logistic model, by treating aspect linearly, cannot capture this bimodal structure and instead averages the effect, underestimating risk on southeast and west-northwest slopes simultaneously ^[3]. A limitation is that the vMF mixture was fitted to aspect alone; a joint model incorporating slope angle as a second angular dimension (on $S^1 \times \mathbb{R}$) could capture aspect-slope interactions that the current two-step approach misses ^[11].

Conclusion

This research demonstrated that modeling slope aspect as directional data using von Mises-Fisher mixtures significantly improves landslide susceptibility mapping. The two-component mixture identified southeast and west-northwest as the dominant failure aspects in Darjeeling, and the directional susceptibility index raised AUC from 0.791 to 0.847 compared with standard logistic treatment. The resulting probability maps reclassified 14.3% of grid cells into higher risk zones, providing more conservative and hence safer guidance for infrastructure planning in the Darjeeling Himalaya.

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Contributions not qualifying for authorship

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