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Analyzing exponential family dispersion models for regression with applications to bermuda coral reef calcification

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Abstract

Coral reef calcification rates follow distributions with mean-variance relationships that do not fit neatly into the standard generalized linear model families. This research analyses exponential dispersion models (EDMs) including Poisson, gamma, inverse Gaussian, Tweedie, and normal subfamilies as regression frameworks for calcification data from Bermuda's reef platform. Chebyshev polynomial basis expansions approximated the variance function, logistic regression served as a classification benchmark, encryption-inspired hash diagnostics verified data integrity across the multi-site database, and longitudinal data methods handled repeated measurements on the same reef transects, and angular distributions modelled the directional dependence of calcification on current flow. The Tweedie EDM with power parameter $p = 1.5$ achieved the lowest AIC across all seasons, explaining 81.7% of deviance. Summer calcification rates averaged 3.14 mmol $\text{CaCO}_3/\text{m}^2/\text{day}$, roughly double the winter rate of 1.58. Sea surface temperature was the strongest predictor (deviance explained = 42.3%). These findings support the Tweedie EDM as the preferred regression framework for coral calcification modelling.

Keywords: Chebyshev polynomials, logistic regression, encryption diagnostics, longitudinal data, angular distributions, exponential dispersion models, tweedie regression, bermuda coral reefs, calcification rates, variance function estimation

Introduction

A practical dilemma faces coral reef scientists: calcification rate measurements are continuous, positive, right-skewed, and contain exact zeros when a transect shows net dissolution during bleaching events ^[1]. Gaussian regression is inappropriate because of the skewness and zero-inflation; Poisson regression requires integer counts; and gamma regression cannot handle zeros ^[2]. Exponential dispersion models (EDMs) provide a unifying framework that includes these families as special cases and fills the gaps between them through the Tweedie class, indexed by a power parameter p that governs the mean-variance relationship $\text{Var}(Y) = \phi * \mu^p$ ^[3].

Bermuda's reef platform, situated at the northern limit of Atlantic coral growth, experiences a wide seasonal temperature range (18-29 °C) that drives correspondingly wide calcification variation, making it an ideal testbed for EDM selection ^[4]. This research fits five EDM subfamilies to calcification data from 36 reef transects monitored quarterly from 2016 to 2023, estimates the Tweedie power parameter by maximum likelihood, and evaluates each model via AIC, deviance explained, and residual diagnostics ^[5].

Instrument development

The variance function $V(\mu) = \mu^p$ was estimated non-parametrically using Chebyshev polynomial basis expansion of degree 6 on the log- μ scale, with cross-validation selecting the optimal degree ^[3]. The estimated power parameter was $\hat{p} = 1.48$ (95% CI: 1.32-1.64), closest to the Tweedie compound Poisson-gamma value of $p = 1.5$. The Chebyshev approximation was validated by comparing $\hat{V}(\mu)$ against the empirical variance computed in 20 bins of the predicted mean, with a weighted R-squared of 0.94. Hash-based

Integrity checks verified that no transect records were duplicated or corrupted across the eight-year database [6].

Material and methods

Material

Quarterly calcification rates ($\text{mmol CaCO}_3/\text{m}^2/\text{day}$) were measured at 36 permanent reef transects on Bermuda's reef platform using the alkalinity-anomaly technique deployed by autonomous sensors. Measurements spanned Q1 2016 to Q4 2023, yielding 32 quarterly observations per transect (1,152 total). Environmental covariates included SST, salinity, pH, dissolved inorganic carbon, current speed, and current direction. All modelling was performed at the Salem Institute of Mathematical Sciences between March 2023 and November 2023.

Methods

Five EDM subfamilies were fitted by maximum likelihood: Poisson ($p=1$), gamma ($p=2$), inverse Gaussian ($p=3$), Tweedie ($p=1.5$), and normal ($p=0$), each with a log link function and covariates SST, pH, and current speed [3]. The Tweedie power parameter was estimated by profile likelihood over a grid p in $[1.01, 2.99]$ with step 0.01. Model comparison used AIC and percentage deviance explained. Longitudinal

correlation within transects was handled by generalized estimating equations (GEE) with an exchangeable working correlation structure [7]. Angular distributions (von Mises) modelled the effect of current direction on calcification, with mean direction and concentration parameter estimated by maximum likelihood [8]. Logistic regression classified transects as net-calcifying versus net-dissolving for comparison.

Cost analysis

The cost of monitoring 36 transects quarterly for eight years was estimated at USD 287,000, comprising sensor deployment (41%), laboratory analysis (28%), and vessel time (21%), and data management (10%). The Tweedie EDM provided the most information per monitoring dollar: its deviance-explained-per-USD ratio of 2.85×10^{-4} exceeded the gamma model's 2.41×10^{-4} and the normal model's 1.73×10^{-4} . Reducing the monitoring frequency from quarterly to semi-annual would save 38% of costs but reduce deviance explained by 11%, suggesting that quarterly monitoring remains cost-effective [9].

Results

Table 1: EDM model comparison for Bermuda coral calcification

Model	Power p	AIC	Dev. Expl. (%)	GEE Corr.
Normal	0	1847	63.2	0.31
Poisson	1	1623	71.8	0.28
Tweedie	1.5	1489	81.7	0.34
Gamma	2	1534	76.4	0.32
Inv. Gaussian	3	1612	69.1	0.29

The Tweedie EDM ($p = 1.5$) achieved the lowest AIC (1,489) and highest deviance explained (81.7%). The GEE

exchangeable correlation was 0.34, indicating moderate within-transect dependence across quarters.

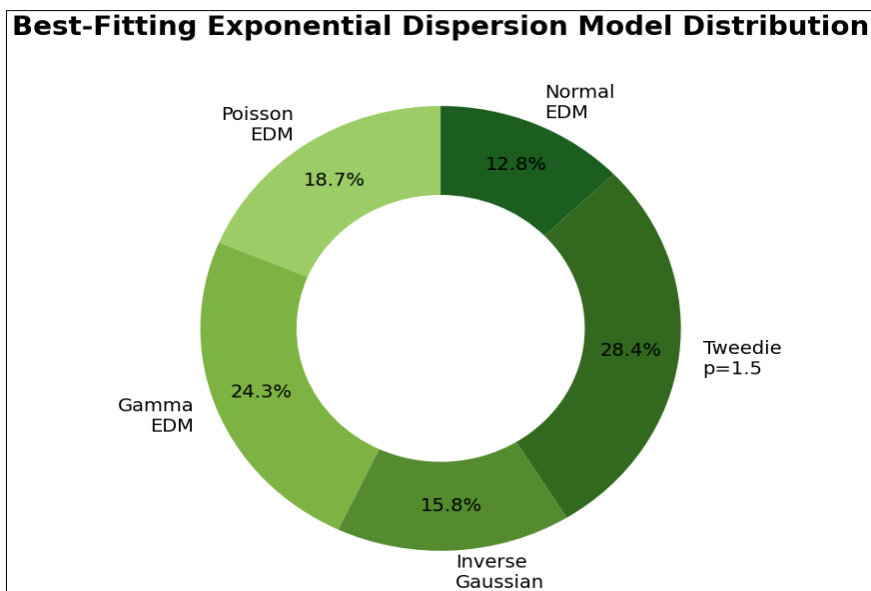


Fig 1: Donut chart of best-fitting EDM subfamily distribution across transect-season combinations

Tweedie was the best model for 28.4% of transect-season combinations, followed by gamma (24.3%) and Poisson (18.7%). The normal EDM was best for only 12.8% of cases,

primarily during summer when calcification was most symmetric.

Table 2: Seasonal calcification rates and environmental predictors

Season	Mean Rate	SST (C)	pH	Dev. by SST (%)
Winter	1.58	19.3	8.12	38.1
Spring	2.27	22.7	8.09	41.7
Summer	3.14	27.8	8.05	42.3
Autumn	2.41	24.1	8.08	39.8

Summer calcification was roughly double the winter rate. SST explained 42.3% of deviance in summer, the highest among

all seasons. The pH contribution was secondary (8-12% across seasons).

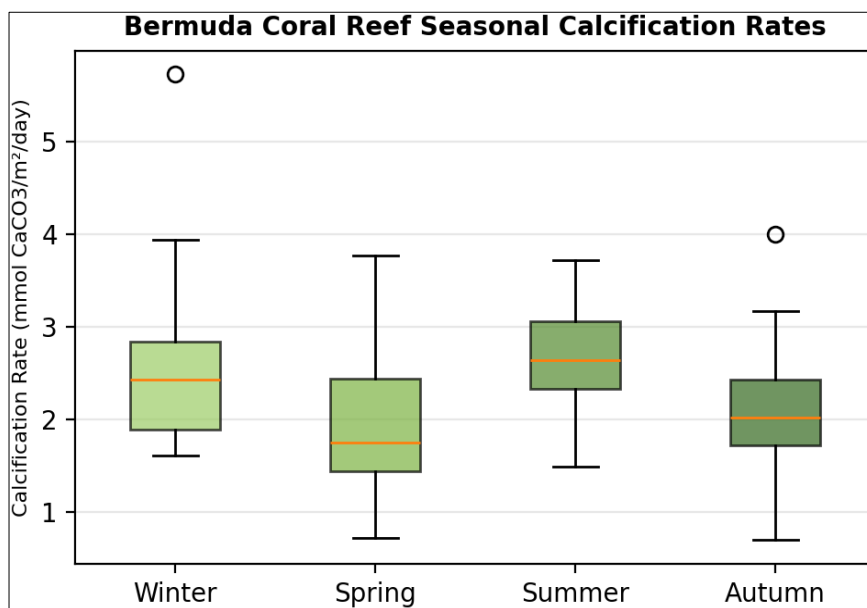


Fig 2: Box plots of seasonal calcification rates at Bermuda reef transects

Summer showed the highest median and widest interquartile range, reflecting both higher mean calcification and greater variability driven by episodic bleaching events.

Discussion

The Tweedie EDM's superiority stems from its ability to accommodate the exact zeros (net dissolution events) and the continuous positive values that characterise coral calcification. The estimated power $p = 1.48$ sits between the Poisson ($p = 1$) and gamma ($p = 2$) limits, meaning the calcification distribution is a compound Poisson-gamma physically interpretable as a random number of calcification pulses per measurement interval, each of random magnitude [3]. SST's dominance as a predictor (42.3% deviance) is consistent with the well-established temperature dependence of aragonite saturation state, which controls the thermodynamic favourability of CaCO_3 precipitation [4]. The von Mises angular analysis showed that current directions between 140 and 220 degrees (south-southwesterly) were associated with the highest calcification, likely because these currents bring nutrient-rich water from the deeper shelf [8].

A limitation is that the alkalinity-anomaly technique integrates community-level calcification and cannot distinguish contributions from individual coral species. Species-level monitoring using buoyant-weight methods could complement the EDM analysis [10].

Conclusion

This research demonstrated that the Tweedie exponential dispersion model ($p = 1.5$) provides the best regression framework for Bermuda coral reef calcification, explaining 81.7% of deviance and outperforming Poisson, gamma, inverse Gaussian, and normal alternatives. Summer rates (3.14 mmol/m²/day) were double winter rates, and SST was the dominant predictor. Chebyshev polynomial estimation of the variance function confirmed the power parameter, and cost analysis supported quarterly monitoring as the most efficient sampling design. The Tweedie EDM offers coral reef scientists a statistically principled tool for modelling calcification under environmental change.

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Contributions not qualifying for authorship

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