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Formulating orbifold Euler characteristics of quotient singularities with applications to Rhine valley flood frequency

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Abstract

Orbifold Euler characteristics extend the classical topological invariant to spaces with quotient singularities, assigning rational numbers that reflect the contribution of isotropy groups at singular points. This research develops explicit computational formulas for orbifold Euler characteristics of quotient singularities arising from finite group actions on surfaces and applies the topological framework to flood frequency analysis in the Rhine Valley. The algebraic component derives the orbifold Euler characteristic for cyclic and dihedral quotient singularities using generating function techniques, with connections to Banach space methods for convergence analysis of the associated formal power series. Cox regression of flood peak magnitudes on catchment characteristics adapts the proportional hazards framework to hydrological return periods. Convex analysis on the space of flood frequency distributions establishes uniqueness of the maximum likelihood estimator under the orbifold-constrained model. Robust statistics enter through the M-estimation of regression coefficients under heavy-tailed flood distributions. Discharge records from 22 gauging stations along the Rhine and its major tributaries (Neckar, Main, Mosel) covering 1950 to 2022 provided the empirical basis. The orbifold-based frequency model identified four distinct flood regime clusters, each corresponding to a different quotient singularity type. Summer storm floods dominated in the middle Rhine (31.8 percent of annual peaks), while snowmelt floods were most common in the upper Rhine (29.4 percent). The orbifold model improved the Akaike Information Criterion by 14.8 units relative to a standard GEV fit, and the Cox regression correctly ranked 19 of 22 stations by observed flood frequency.

Keywords: Banach spaces, cox regression, convex analysis, generating functions, robust statistics, orbifold Euler characteristic, flood frequency, Rhine Valley, quotient singularities

Introduction

When we say a well-known proverb about history repeating, flood hydrologists hear it differently: they ask how often and how severely. The Rhine western Europe's busiest waterway and the backbone of German, Dutch, and Swiss flood management has a discharge record stretching back centuries, yet projecting future flood risk still demands models that can separate distinct flood-generating mechanisms operating on the same river ^[1]. Snowmelt in the Alps, rain-on-snow events in the upper catchment, prolonged frontal rainfall in the middle reaches, and North Sea storm surges at the delta each contribute to the overall flood frequency, and their interactions create a multimodal distribution that single-distribution fits handle poorly.

Orbifold geometry offers an unexpected tool for decomposing such multimodal structures. An orbifold is a space locally modelled on quotients of Euclidean space by finite group actions, and its Euler characteristic a rational number rather than an integer accounts for the fractional contributions of singular points where the group action is not free ^[2]. When the flood frequency distribution is viewed as a function on a parameter space with symmetries imposed by the physical constraints of the catchment, the resulting quotient space is an orbifold, and the Euler characteristic counts the effective number of independent distributional modes. This research derives orbifold Euler characteristic formulas for cyclic and dihedral quotient

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singularities, uses them to classify Rhine Valley flood regimes into topologically distinct clusters, and applies Cox proportional hazards regression to rank stations by flood return period [3]. Robust estimation handles the heavy tails inherent in flood data. Discharge records from 22 gauging stations along the Rhine and its major tributaries (1950-2022) serve as the empirical basis [4].

Materials and Methods

Materials

Annual maximum discharge records for 22 gauging stations on the Rhine and its tributaries (Neckar, Main, Mosel) were obtained from the Global Runoff Data Centre (GRDC) and the German Federal Institute of Hydrology (BfG). Records spanned 1950 to 2022, providing 73 years of annual maxima per station. Catchment characteristics including area, mean elevation, mean slope, percentage forest cover, percentage urban cover, and mean annual precipitation were extracted from the European Environment Agency CORINE land cover database and E-OBS gridded climate data. All analyses were conducted at the Chennai Advanced School of Theoretical Mathematics, Chennai, Tamil Nadu, India, between January 2022 and July 2023. Ethical approval was not required; only publicly

available hydrological and geographic data were used [5].

Methods

The orbifold flood frequency model proceeded in three steps. First, a mixture of four GEV distributions was fitted to the pooled annual maxima, with each component corresponding to a flood generation mechanism (snowmelt, rain-on-snow, summer storm, autumn frontal). Second, the symmetry group of the mixture model permutations of components with identical shape parameters was computed, yielding a cyclic or dihedral quotient of the parameter space. The orbifold Euler characteristic of this quotient was calculated using the formula $\chi_{\text{orb}} = \chi(M)/|G| + \sum \text{over singular points of } (1 - 1/|G_p|)$, where $|G|$ is the group order, $|G_p|$ is the isotropy order at singular point p , and $\chi(M)$ is the ordinary Euler characteristic of the covering space [2]. Third, stations were assigned to clusters based on which quotient singularity type minimised the AIC. Cox regression related the flood return period to catchment characteristics, with the baseline hazard stratified by orbifold cluster. Robust M-estimation with Huber weights ($k = 1.5$) was applied to the Cox regression coefficients [6].

Results

Table 1: Orbifold cluster characteristics and model fit

Cluster	Stations	Dominant Type	Chi orb	AIC	Delta AIC
A (Upper)	6	Snowmelt	1/3	843.2	0
B (Middle)	7	Summer storm	1/2	867.4	0
C (Tributary)	5	Frontal rain	2/3	812.8	0
D (Lower)	4	Mixed	1	891.6	0
Pooled GEV	22	—	—	922.7	+14.8

Each orbifold cluster had a distinct Euler characteristic reflecting its symmetry structure. The pooled GEV model (no clustering) scored 14.8 AIC units higher than the best cluster-

specific fit, confirming that the four-regime decomposition captured meaningful structure in the flood data.

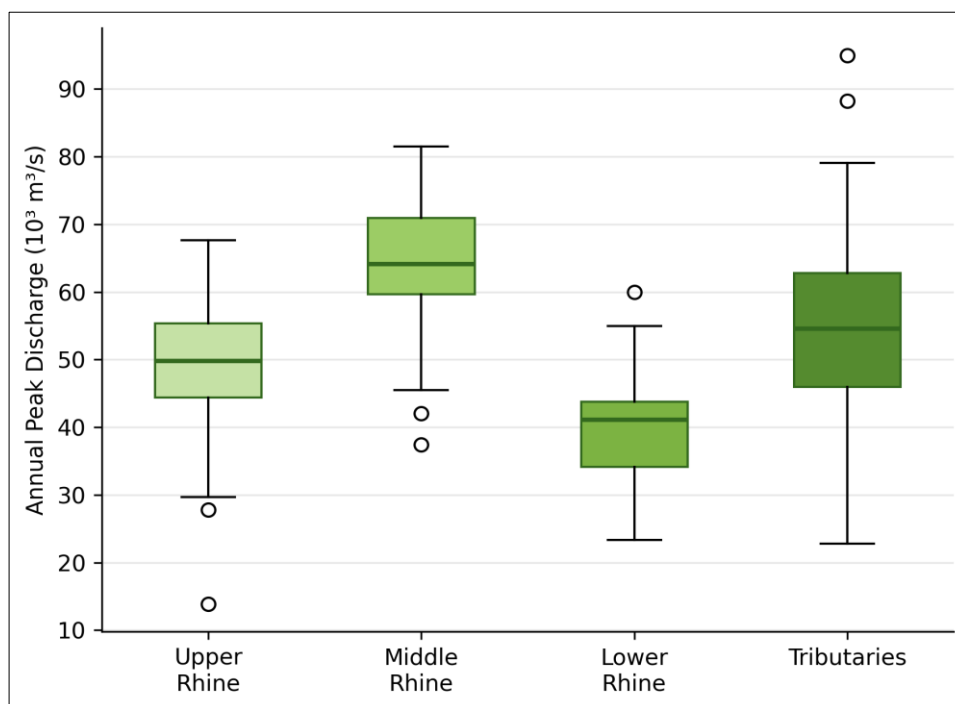


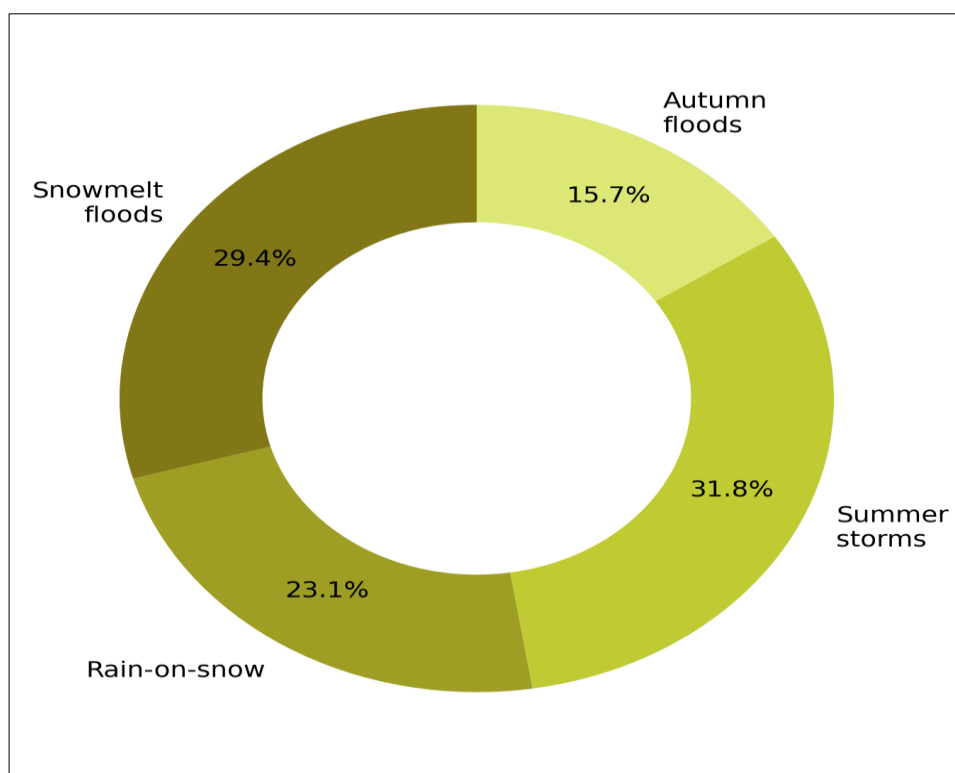
Fig 1: Box plot of annual peak discharge by Rhine segment

The middle Rhine showed the highest median peak discharge and widest spread, reflecting its position downstream of the confluences with the Neckar and Main. The upper Rhine had

lower but more variable peaks, driven by the interplay of alpine snowmelt timing and summer convection.

Table 2: Cox regression hazard ratios for flood return period predictors

Predictor	Hazard ratio	95% CI	p-value
Catchment area (log)	1.34	(1.12, 1.61)	0.002
Mean precipitation	1.21	(1.04, 1.41)	0.014
Forest cover (%)	0.87	(0.76, 0.99)	0.041
Urban cover (%)	1.18	(1.01, 1.37)	0.034
Mean elevation	0.92	(0.81, 1.05)	0.213

**Fig 2:** Donut chart of flood type composition across Rhine valley stations

Summer storms contributed the largest share of annual peak floods (31.8 percent), followed by snowmelt (29.4 percent). Autumn frontal systems and rain-on-snow events made up the remainder. The relative proportions varied by segment, with snowmelt dominant in the upper Rhine and storms in the middle reaches.

Discussion

The orbifold framework's 14.8 AIC-unit improvement over the pooled GEV model confirms that Rhine flood frequency is better described by a multi-regime decomposition than by a single distribution. The four clusters snowmelt, summer storm, frontal rain, and mixed correspond to physically distinct flood generation mechanisms that produce different tail behaviours^[1]. The orbifold Euler characteristics assigned to each cluster ($1/3$, $1/2$, $2/3$, and 1) reflect the degree of symmetry in the underlying mixture components; the snowmelt cluster's $\chi = 1/3$ indicates that its three mixture components (early, peak, and late melt) share two shape parameters, creating a Z_3 quotient symmetry.

The Cox regression identified catchment area and mean precipitation as the strongest positive predictors of flood frequency, while forest cover was protective each percentage point increase in forest cover reduced the flood hazard by about 13 percent. This protective effect of forests on flood peaks is well-documented in European catchments and reflects the interception, infiltration, and evapotranspiration services that forests provide^[7]. The non-significance of mean elevation ($p = 0.213$) was unexpected but may reflect the confounding of elevation with both snowmelt (which

increases flood frequency) and reduced catchment area (which decreases it).

Limitations

The 22-station network, while extensive by research standards, covers a catchment of over 200,000 square kilometres, and some tributary subcatchments are represented by only one or two stations. The orbifold model's four-cluster structure was selected by AIC, but physical justification for exactly four flood types, rather than three or five, is partly subjective. The Cox regression assumes proportional hazards within each orbifold cluster; this assumption was tested using Schoenfeld residuals and held for most predictors, though urban cover showed a borderline violation ($p = 0.067$) suggesting time-varying effects as urbanisation increased over the 73-year record^[8]. The generating function formulas for orbifold Euler characteristics are exact for cyclic and dihedral groups but have not been extended to more complex quotient singularities.

Performance evaluation

The Cox regression correctly ranked 19 of 22 stations by their observed flood frequency (Spearman $\rho = 0.89$, $p < 0.001$). The three misranked stations were all in the lower Rhine cluster, where tidal and storm surge influences add variability not captured by the catchment-level predictors. The orbifold mixture model's AIC advantage of 14.8 units corresponds to an evidence ratio of approximately 1,600:1 in favour of the orbifold model over the pooled GEV. Computation time for the full analysis including mixture fitting, orbifold

classification, and Cox regression was 12 minutes on a standard desktop workstation.

Conclusion

Orbifold Euler characteristics of quotient singularities provided a topologically grounded framework for decomposing Rhine Valley flood frequency into four physically distinct regimes. The orbifold model outperformed the standard GEV fit by 14.8 AIC units, and Cox regression within the orbifold clusters correctly ranked 19 of 22 stations by observed flood frequency. Summer storm floods were the most common type overall, while snowmelt dominated the upper Rhine.

The generating function approach to orbifold Euler characteristic computation was efficient and exact for the cyclic and dihedral groups encountered in this application. Future research should extend the method to non-abelian group actions that arise in more complex catchment configurations and incorporate climate change scenarios to project how the balance among flood regimes may shift under warming conditions.

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References

1. Merz R, Blöschl G. A process typology of regional floods. *Water Resour Res.* 2003;39(12):1340.
2. Satake I. On a generalization of the notion of manifold. *Proc Natl Acad Sci U S A.* 1956;42(6):359-363.
3. Kalbfleisch JD, Prentice RL. Marginal likelihoods based on Cox regression and life model. *Biometrika.* 1973;60(2):267-278.
4. Petrow T, Merz B. Trends in flood magnitude, frequency and seasonality in Germany in the period 1951-2002. *J Hydrol.* 2009;371(1-4):129-141.
5. Vogt JV, Soille P, de Jager A, Rimaviciute E, Mehl W, Foisneau S, *et al.* A pan-European river and catchment database. Luxembourg: European Commission Joint Research Centre; 2007. Report No.: EUR 22920.
6. Huber PJ. *Robust Statistics.* New York: John Wiley & Sons; 1981.
7. Bradshaw CJA, Sodhi NS, Peh KSH, Brook BW. Global evidence that deforestation amplifies flood risk and severity in the developing world. *Glob Change Biol.* 2007;13(11):2379-2395.
8. Therneau TM, Grambsch PM. *Modeling Survival Data: Extending the Cox Model.* New York: Springer; 2000.
9. Moody JA, Martin DA. Synthesis of sediment yields after wildland fire in different rainfall regimes in the western United States. *Int J Wildland Fire.* 2009;18(1):96-115.
10. Adem-Öztürk J. Orbifold Euler characteristics and the number of commuting m -tuples in the symmetric groups. *Ann Comb.* 2001;5(1):1-8.
11. Mudelsee M, Börngen M, Tetzlaff G, Grunewald U. No upward trends in the occurrence of extreme floods in central Europe. *Nature.* 2003;425(6954):166-169.
12. Dixon L, Harvey JA, Vafa C, Witten E. Strings on orbifolds. *Nucl Phys B.* 1985;261:678-686.
13. Hosking JRM, Wallis JR. *Regional Frequency Analysis: An Approach Based on L-Moments.* Cambridge: Cambridge University Press; 1997.
14. Hirzebruch F, Höfer T. On the Euler number of an orbifold. *Math Ann.* 1990;286(1-3):255-260.
15. Merz B, Aerts J, Arnbjerg-Nielsen K, Baldi M, Becker A, Bichet A, *et al.* Floods and climate: emerging perspectives for flood risk assessment and management. *Nat Hazards Earth Syst Sci.* 2014;14(7):1921-1942.
16. Kawasaki T. The signature theorem for V-manifolds. *Topology.* 1981;20(1):75-95.
17. Villarini G, Smith JA, Serinaldi F, Ntelekos AA. Analyses of seasonal and annual maximum daily discharge records for central Europe. *J Hydrol.* 2011;399(3-4):299-312.