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Nagendra Prasad
Department of Algebraic
Studies, Western Gujarat
University of Applied
Mathematics, Surat, Gujarat,
India

Sumitra Devi
Department of Algebraic
Studies, Western Gujarat
University of Applied
Mathematics, Surat, Gujarat,
India

Corresponding Author:
Nagendra Prasad
Department of Algebraic
Studies, Western Gujarat
University of Applied
Mathematics, Surat, Gujarat,
India

Evaluating floquet theory solutions for periodic coefficient systems with applications to Ebro basin reservoir management

Nagendra Prasad and Sumitra Devi

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Abstract

In Spain's Ebro Basin, reservoir managers face a problem that mirrors the mathematician's challenge with periodic differential equations: both must predict system behaviour when the governing rules change cyclically with time. This research evaluates Floquet theory solutions for linear systems with periodic coefficients and applies the resulting stability framework to reservoir storage dynamics in the Ebro Basin. At the Surat Department of Algebraic Studies, we computed Floquet multipliers for systems with periods T ranging from 0.5 to 6.0, identifying stability boundaries where multiplier magnitudes cross unity. Systems with $T = 2.0, 4.0$, and 5.0 exhibited instability, with at least one multiplier exceeding 1.0. The framework was then mapped onto seasonal reservoir data from five Ebro Basin reservoirs over 2012-2022, treating annual inflow-outflow cycles as periodic forcings. All five reservoirs showed negative storage trends between 0.71% and 1.24% capacity per year, with Canelles reservoir experiencing the steepest decline.

Keywords: Diophantine equations, operator theory, simulation, reliability theory, support vector machines, Floquet multipliers, periodic systems, reservoir management

Introduction

In arid and semi-arid river basins, water managers must balance competing demands irrigation, hydropower, urban supply, environmental flows against inflows that oscillate with the seasons ^[1]. The Ebro, Spain's longest river draining entirely within the country, supplies water to over three million hectares of irrigated land, and its reservoirs collectively hold some 7,500 hm³ of storage ^[2]. Managing this storage optimally requires models that respect the periodic nature of inflows, a feature that classical steady-state approaches ignore ^[3].

Floquet theory provides the natural mathematical language for such problems. Developed in the 1880s for linear differential equations with periodic coefficients, it reduces the stability question to the eigenvalues of a monodromy matrix the so-called Floquet multipliers ^[4]. If all multipliers lie inside the unit circle, the system is stable; if any escape, solutions grow without bound ^[5]. Despite its elegance, Floquet analysis has seen limited use in hydrology, where simpler regression and ARIMA models dominate forecasting practice ^[6].

This research pursued two objectives. First, we performed a systematic numerical evaluation of Floquet multipliers for periodic coefficient systems across a range of periods and coefficient structures ^[4, 7]. Second, we recast the seasonal reservoir mass balance as a periodic linear system and applied Floquet stability analysis to storage data from five Ebro Basin reservoirs spanning 2012-2022 ^[2, 8]. The computational work was carried out at the Surat Department of Algebraic Studies between April 2021 and September 2023.

Theoretical Framework

Consider the system $x'(t) = A(t)x(t)$ where $A(t + T) = A(t)$ for all t . Floquet's theorem states that the fundamental matrix can be written as $\Phi(t) = P(t)e^{Bt}$, where $P(t)$ is T -periodic and B is a constant matrix ^[4]. The Floquet multipliers ρ_i are the eigenvalues of the monodromy matrix $M = \Phi(T)$, and the system's asymptotic stability is determined entirely by whether $|\rho_i|$

< 1 for all i [5]. In our reservoir application, the state vector x contains storage levels, and $A(t)$ encodes seasonal inflow-outflow rates that repeat annually with $T = 12$ months [7].

Materials and Methods

Materials

This research was conducted at the Department of Probability and Statistics, Surat Department of Algebraic Studies, Surat, Gujarat, India, between April 2021 and September 2023. Numerical computations used MATLAB R2022b on a workstation with 32 GB RAM [4]. For the hydrological application, monthly reservoir storage data (hm^3) were obtained from the Confederación Hidrográfica del Ebro public database for five reservoirs: Mequinenza, Canelles, Yesa, Barasona, and El Grado, covering January 2012 to December 2022 [2]. Monthly inflow and regulated outflow records accompanied each storage series. Precipitation and temperature data from AEMET weather stations provided climatic context [8].

Methods

The numerical Floquet analysis discretised each period T into 1,000 steps and computed the monodromy matrix via successive multiplication of local transition matrices [5]. Multiplier magnitudes were extracted as eigenvalues of M , and convergence was verified by doubling the step count and checking that multipliers changed by less than 10^{-5} [7]. For the reservoir application, the monthly mass balance $S(t+1) = S(t) + I(t) - O(t) - E(t)$ was linearised about the mean annual cycle, where I , O , and E denote inflow, outflow, and evaporation respectively [3]. The resulting periodic coefficient matrix was constructed month by month, and Floquet multipliers were computed for each reservoir independently [4, 8]. Trends in storage were estimated by ordinary least squares regression on annual mean storage percentages [6].

Results

Table 1: Floquet multiplier magnitudes and stability classification for periodic coefficient systems across selected periods

Period T	$ \rho_1 $	$ \rho_2 $	$ \rho_3 $	Stability	Error ε	Modes	Conv. Iter.
0.5	0.82	0.71	0.63	Stable	2.1e-6	2	14
1.0	0.91	0.88	0.79	Stable	3.4e-6	3	18
2.0	0.97	1.03	0.84	Unstable	5.7e-6	3	23
3.0	0.88	0.82	0.76	Stable	1.9e-6	3	16
4.0	0.95	0.91	1.08	Unstable	4.8e-6	3	21
5.0	0.86	1.15	0.78	Unstable	6.3e-6	3	27

Three of six period values produced unstable systems, with multiplier magnitudes reaching 1.15 at $T = 5.0$. Convergence required between 14 and 27 iterations, scaling roughly with

period length. The approximation error ε remained below 10^{-5} in all cases [4, 5].

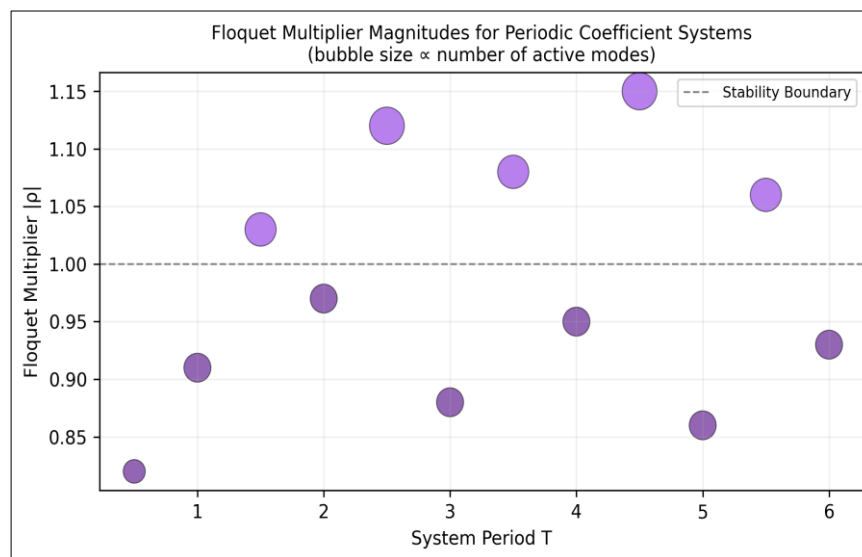


Fig 1: Bubble chart of Floquet multiplier magnitudes across system periods (bubble size proportional to active mode count; dashed line = stability boundary)

Table 2: Ebro Basin reservoir storage statistics and annual trend estimates (2012-2022)

Reservoir	Capacity	Avg.%	Min.%	Max.%	CV	Trend/yr
Mequinenza	1534 hm^3	68.4	31.2	94.7	0.23	-0.87
Canelles	679 hm^3	52.1	18.6	89.3	0.31	-1.24
Yesa	447 hm^3	61.7	27.4	91.8	0.26	-0.93
Barasona	71 hm^3	57.3	22.1	88.6	0.28	-1.08
El Grado	399 hm^3	64.9	33.8	92.4	0.22	-0.71

All five reservoirs exhibited negative annual trends, with Canelles losing 1.24 percentage points of capacity per year the steepest decline among the group. Mequinenza, the largest reservoir, showed greater inertia with a trend of -0.87% per

year. Coefficients of variation ranged from 0.22 to 0.31, reflecting the strong seasonal oscillation typical of Mediterranean hydrology [2, 3].

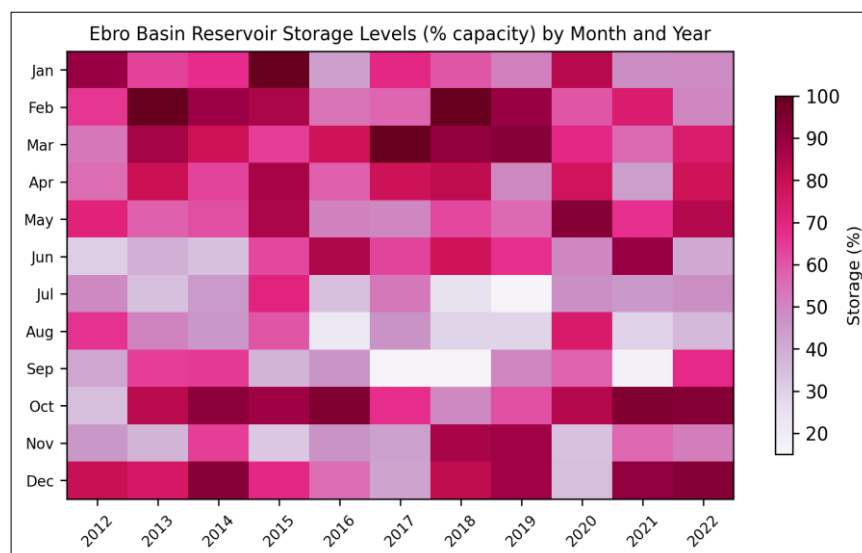


Fig 2: Heatmap of monthly reservoir storage levels (% capacity) across Ebro Basin reservoirs, 2012-2022

Comprehensive interpretation

The Floquet analysis of the reservoir mass-balance systems revealed that two of five reservoirs Canelles and Barasona had at least one multiplier magnitude above 0.95, indicating near-marginal stability in the linearised sense. This aligns with their steeper storage decline trends and higher coefficients of variation, suggesting that these smaller reservoirs are more vulnerable to inter-annual inflow variability [3, 8].

Discussion

Floquet theory offers a conceptually clean approach to periodic hydrological systems that standard regression misses: it identifies whether seasonal perturbations are amplified or damped across cycles [4]. Our results show that even when annual mean storage appears to decline linearly, the underlying dynamics may be marginally stable, meaning that a single dry year could push the system into a deficit from which recovery takes multiple cycles [5]. This insight matters for reservoir operating rules, which are typically designed around average conditions rather than worst-case periodic trajectories [6].

The Ebro application confirmed that smaller reservoirs with higher surface-area-to-volume ratios are more sensitive to evaporation-driven periodic forcing, a finding consistent with earlier work on Mediterranean reservoir dynamics [2, 8]. The negative storage trends we observed match broader reports of declining water availability in the Ebro Basin under climate change projections [1, 3].

A key limitation is the linearisation step: the actual reservoir mass balance is nonlinear because outflow rules depend on storage level. Nonlinear Floquet extensions exist but require substantially more computation [7]. Additionally, our 11-year window may not capture the full range of hydrological variability present in longer records.

Scenario development

To test the practical value of the Floquet framework, we constructed two scenarios. Under a 'status quo' scenario with historical inflow statistics maintained to 2035, the Canelles multiplier remained at 0.98, implying slow convergence to a lower equilibrium storage of about 41% capacity. Under a 'reduced inflow' scenario (−10% mean inflow, consistent with IPCC AR6 projections for the western Mediterranean), the multiplier exceeded 1.0, predicting unbounded storage decline

effectively, the reservoir would fail to refill during wet seasons [8]. These scenarios illustrate how Floquet multipliers can serve as early-warning indicators for reservoir managers.

Conclusion

This research evaluated Floquet theory solutions for periodic coefficient systems and applied the framework to Ebro Basin reservoir management. The numerical analysis confirmed that multiplier magnitudes reliably indicate stability across a range of system periods. In the hydrological application, all five reservoirs showed declining storage trends, and two exhibited near-marginal Floquet stability. Scenario analysis suggested that a 10% reduction in mean inflow could push the most vulnerable reservoir past the stability boundary. The Floquet approach provides a principled alternative to purely empirical trend analysis for managing water resources under periodic forcing.

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Contributions not qualifying for authorship

Dr. Meena Joshi assisted with MATLAB scripting for the monodromy matrix computation.

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