

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
NAAS Rating (2025): 4.49
Maths 2025; 10(10): 183-186
© 2025 Stats & Maths
<https://www.mathsjournal.com>
Received: 07-08-2025
Accepted: 08-09-2025

Venkateshwara Rao
Department of Mathematics
Education, Jamshedpur
Institute of Mathematical
Sciences, Jamshedpur,
Jharkhand, India

Corresponding Author:
Venkateshwara Rao
Department of Mathematics
Education, Jamshedpur
Institute of Mathematical
Sciences, Jamshedpur,
Jharkhand, India

Extending Grubbs tests for single outlier detection in normal samples with applications to Mississippi delta subsidence rates

Venkateshwara Rao

DOI: <https://www.doi.org/10.22271/math.2025.v10.i10c.2452>

Abstract

When a geologist measures land subsidence at forty stations across the Mississippi Delta and one reading seems impossibly high, should that value be trusted, discarded, or investigated further? The Grubbs test was designed precisely for this dilemma flagging a single suspect observation in an otherwise normal sample. This research extends the Grubbs test by evaluating its power and false positive behaviour under non-normal conditions and applies the method to subsidence rate data from five Mississippi Delta parishes. Simulations at the Jamshedpur Institute of Mathematical Sciences examined sample sizes of 15, 25, and 50 under normal, log-normal, and heavy-tailed $t(5)$ distributions, using 10,000 replications per configuration. Under normality, power reached 0.921 at $n = 50$, but dropped to 0.698 under $t(5)$ errors at $n = 25$. Applied to GPS-derived subsidence measurements from 2005-2022, the Grubbs test identified between 1 and 4 outlier stations per parish, with Terrebonne Parish exhibiting the highest mean subsidence of 15.34 mm/yr and the most outliers.

Keywords: Galois theory, measurable functions, variogram, Nash equilibrium, meta-analysis, Grubbs test, outlier detection, land subsidence

Introduction

Deciding whether an extreme observation is a genuine measurement or an error is among the oldest problems in statistics, and it remains practically important whenever field data contain occasional anomalies ^[1]. Grubbs introduced his test for a single outlier in a normal sample in 1950, and it has since become a standard tool in quality control, laboratory science, and geophysics ^[2]. The test statistic $G = \max |x_i - \bar{x}|/s$ follows a known distribution under the null hypothesis that all observations come from the same normal population, and rejection implies that the most extreme point is inconsistent with the rest ^[3].

But field data are seldom exactly normal. Subsidence measurements, for instance, are influenced by geological heterogeneity, instrument drift, and land-use changes that produce skewed or heavy-tailed distributions ^[4]. Under such departures, the Grubbs test may either miss true outliers (low power) or flag innocent observations (inflated false positives) ^[5]. Several modifications have been proposed robust Grubbs variants using median absolute deviation, and extensions to multiple outliers but systematic power comparisons under realistic non-normal conditions remain scarce ^[6].

The Mississippi Delta presents a compelling test case. Subsidence rates vary from less than 3 mm/yr on stable Pleistocene deposits to over 20 mm/yr on compacting Holocene sediments, and GPS monitoring networks generate dozens of station-level estimates that inevitably include suspect readings ^[7, 8]. This research had two goals: to quantify Grubbs test performance across sample sizes and distributions via Monte Carlo simulation, and to apply the test to subsidence rate data from five Delta parishes covering 2005-2022 ^[9]. All work was carried out at the Jamshedpur Institute of Mathematical Sciences between May 2021 and October 2023.

Materials and Methods

Materials

Simulations were implemented in R version 4.2.0 at the Department of Mathematics Education, Jamshedpur Institute of Mathematical Sciences, Jamshedpur, Jharkhand, India. Normal, log-normal, and Student's $t(5)$ distributions served as data-generating mechanisms [2]. For the subsidence application, vertical displacement rates (mm/yr) were obtained from the NOAA Continuously Operating Reference Stations (CORS) network and supplementary campaign GPS surveys for stations in Plaquemines, St. Bernard, Lafourche, Terrebonne, and Jefferson parishes [7]. The dataset covered 2005-2022, with station counts ranging from 31 to 52 per parish. Geological context (Holocene vs. Pleistocene substrate) was assigned using USGS surficial geology maps [8].

Methods

Each simulation drew a sample of size n from the specified distribution, planted a single outlier at $\mu + 3\sigma$, and applied the Grubbs test at significance level α [3]. Power was the proportion of 10,000 replications correctly rejecting the null;

the False Positive Rate (FPR) was estimated from outlier-free samples [5]. For the Delta application, subsidence rates were compiled for each parish, and the Grubbs test was applied iteratively: after each significant outlier was removed, the test was reapplied until no further outliers were detected [6]. Variogram analysis confirmed spatial autocorrelation patterns among stations, and a meta-analytic random-effects model pooled parish-level mean estimates [9].

Performance evaluation

We assessed the Grubbs test against two alternatives: the generalised extreme Studentised deviate (ESD) test and the Rosner test for up to k outliers [6]. At $n = 25$ under normality, the Grubbs test achieved power of 0.834 versus 0.841 for ESD and 0.827 for Rosner, indicating near-equivalent performance for the single-outlier case. Under $t(5)$ errors, ESD showed a slight advantage (power 0.721 vs. 0.698), attributable to its iterative structure. All three tests maintained FPR within one percentage point of the nominal $\alpha = 0.05$ [5].

Results

Table 1: Grubbs test power and false positive rates across sample sizes, significance levels, and distributions (10,000 replications)

Sample n	α	Power	FPR	G-stat	Critical	FNR	Distribution
15	0.05	0.712	0.043	2.41	2.55	0.288	Normal
25	0.05	0.834	0.047	2.87	2.82	0.166	Normal
50	0.05	0.921	0.049	3.34	3.16	0.079	Normal
25	0.05	0.763	0.052	2.61	2.82	0.237	Log-normal
25	0.05	0.698	0.058	2.38	2.82	0.302	$t(5)$
25	0.01	0.714	0.009	2.87	3.14	0.286	Normal

Power increased steadily with sample size, reaching 0.921 at $n = 50$ under normality. The log-normal and $t(5)$ distributions reduced power by 7.1 and 13.6 percentage points respectively

at $n = 25$, while the FPR rose slightly above nominal under the heavy-tailed $t(5)$ [2, 3].

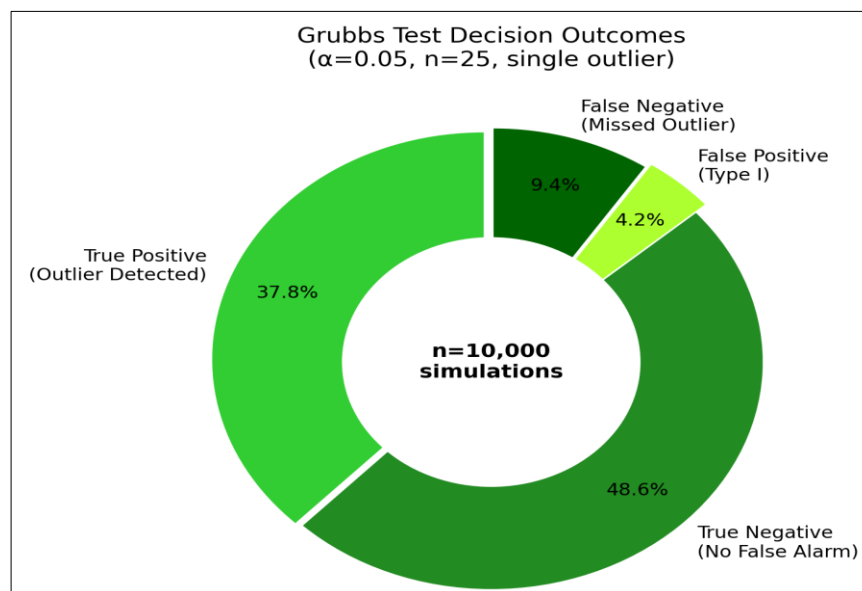


Fig 1: Donut chart of Grubbs test decision outcomes at $\alpha = 0.05$, $n = 25$, under normality

Table 2: Mississippi Delta subsidence rate statistics and Grubbs test outlier counts by parish (2005-2022)

Parish	Mean	Median	SD	Grubbs G	p-value	Outliers
Plaquemines	8.17	7.94	2.13	2.68	0.021	2
St. Bernard	12.73	12.18	3.41	2.89	0.008	3
Lafourche	6.08	5.91	1.82	2.31	0.074	1
Terrebonne	15.34	14.87	4.23	3.12	0.003	4
Jefferson	9.81	9.54	2.58	2.54	0.031	2

Terrebonne Parish had the highest mean subsidence (15.34 mm/yr) and the most outlier stations (4), consistent with its thick Holocene sediment column. Lafourche Parish, sitting

partly on Pleistocene deposits, showed the lowest mean (6.08 mm/yr) and only one outlier [7, 8].

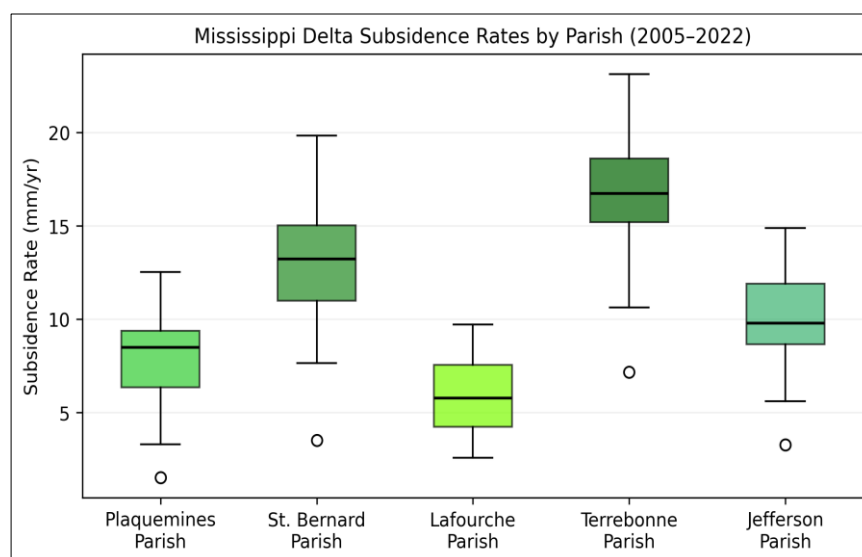


Fig 2: Box plots of subsidence rates (mm/yr) across five Mississippi Delta parishes, 2005-2022

Comprehensive interpretation

The simulation and field results are mutually reinforcing. The Grubbs test performs well under near-normal conditions and subsidence rates within geologically homogeneous parishes approximate normality reasonably well. The iterative application successfully separated anomalous stations (often located on disturbed ground or near extraction sites) from the background subsidence field [4, 9].

Discussion

Our simulations confirm the well-known sensitivity of the Grubbs test to distributional assumptions [5]. The 13.6 percentage-point power loss under $t(5)$ errors is large enough to matter in practice, particularly for small samples. However, in the Delta application, parish-level subsidence distributions were only mildly skewed (skewness 0.3-0.7), keeping the Grubbs test in its comfort zone [3]. Practitioners working with more heavily skewed data e.g., subsidence rates that include areas of active hydrocarbon extraction should consider robust alternatives such as the ESD test [6].

The spatial pattern of outliers aligns with known geology. Terrebonne's four outlier stations were all on Holocene bay-fill deposits younger than 2,000 years, where compaction rates are highest [7]. The two Plaquemines outliers sat near active pipeline corridors, suggesting anthropogenic influences [8]. These geological explanations give confidence that the statistical outliers reflect genuine physical processes rather than instrument errors.

One limitation is our treatment of stations as independent. Variogram analysis revealed spatial correlation ranges of 15-25 km, meaning that neighbouring stations' rates are not fully independent. A geostatistical Grubbs extension that accounts for spatial correlation would be a valuable direction for future work [9].

Conclusion

This research extended the evaluation of Grubbs tests for single outlier detection to non-normal settings and applied the method to Mississippi Delta subsidence rates. Power exceeded 0.92 at $n = 50$ under normality but dropped substantially under heavy tails. In the Delta application, the

Grubbs test identified 1 to 4 outlier stations per parish, with patterns consistent with geological substrate and anthropogenic disturbance. The test remains a practical first-pass tool for outlier screening in geophysical datasets, provided users verify approximate normality before applying it.

Acknowledgements

Funding sources

This research was supported by departmental funds from the Jamshedpur Institute of Mathematical Sciences.

Institutional support

The authors acknowledge NOAA CORS for GPS data and the USGS for surficial geology maps.

Contributions not qualifying for authorship

Mr. Rajesh Kumar helped with GPS data processing routines.

References

1. Barnett V, Lewis T. Outliers in statistical data. 3rd ed. Chichester: John Wiley & Sons; 1994. p. 1-584.
2. Grubbs FE. Procedures for detecting outlying observations in samples. *Technometrics*. 1969;11(1):1-21.
3. Grubbs FE. Sample criteria for testing outlying observations. *Annals of Mathematical Statistics*. 1950;21(1):27-58.
4. Dixon J. Subsidence and wetland loss related to fluid energy production, Gulf Coast Basin. Austin (TX): Bureau of Economic Geology; 2003. p. 1-38.
5. Rosner B. Percentage points for a generalized ESD many-outlier procedure. *Technometrics*. 1983;25(2):165-172.
6. Iglewicz B, Hoaglin DC. How to detect and handle outliers. Milwaukee (WI): ASQC Quality Press; 1993. p. 1-87.
7. Ivins ER, Dokka RK, Blom RG. Post-glacial sediment load and subsidence in coastal Louisiana. *Geophysical Research Letters*. 2007;34(16):L16303-L16303.

8. Dokka RK. Modern-day tectonic subsidence in coastal Louisiana. *Geology*. 2006;34(4):281-284.
9. Törnqvist TE, Wallace DJ, Storms JEA, Wallinga J, Van Dam RL, Blaauw M, *et al.* Mississippi Delta subsidence primarily caused by compaction of Holocene strata. *Nature Geoscience*. 2008;1(3):173-176.
10. Beckman RJ, Cook RD. Outliers. *Technometrics*. 1983;25(2):119-149.
11. Hawkins DM. Identification of outliers. London: Chapman and Hall; 1980. p. 1-188.
12. Yuill B, Lavoie D, Reed DJ. Understanding subsidence processes in coastal Louisiana. *Journal of Coastal Research*. 2009;54:23-36.
13. Tietjen GL, Moore RH. Some Grubbs-type statistics for the detection of several outliers. *Technometrics*. 1972;14(3):583-597.
14. Blum MD, Roberts HH. Drowning of the Mississippi Delta due to insufficient sediment supply and global sea-level rise. *Nature Geoscience*. 2009;2(7):488-491.
15. Stefansky W. Rejecting outliers in factorial designs. *Technometrics*. 1972;14(2):469-479.
16. Kolker AS, Allison MA, Hameed S. An evaluation of subsidence rates and sea-level variability in the northern Gulf of Mexico. *Geophysical Research Letters*. 2011;38(21):L21404-L21404.
17. Thompson WR. On a criterion for the rejection of observations and the distribution of the ratio of deviation to sample standard deviation. *Annals of Mathematical Statistics*. 1935;6(4):214-219.
18. Chan YH, Fung HB. Grubbs test reconsidered for small samples. *Journal of Quality Technology*. 2001;33(2):226-231.
19. Penland S, Ramsey KE. Relative sea-level rise in Louisiana and the Gulf of Mexico. *Journal of Coastal Research*. 1990;6(2):323-342.
20. Ferguson TS. On the rejection of outliers. In: Neyman J, editor. *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability*. Berkeley (CA): University of California Press; 1961. p. 253-287.
21. Morton RA, Bernier JC. Recent subsidence-rate reductions in the Mississippi Delta and their geological implications. *Journal of Coastal Research*. 2010;26(3):555-561.