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Deriving derived category equivalences for tilting objects with applications to Papua New Guinea landslide risk

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Abstract

Derived categories, introduced by Grothendieck and Verdier, provide a framework in which chain complexes of sheaves are treated as single objects, and equivalences between derived categories reveal hidden symmetries between seemingly different geometric spaces. A tilting object a generator of the derived category with vanishing higher self-extensions produces such an equivalence when it exists. This research establishes new tilting criteria for coherent sheaves on weighted projective surfaces and applies the resulting categorical machinery to landslide susceptibility mapping in Papua New Guinea. The algebraic component proves that certain exceptional collections on weighted projective planes are tilting, using Chebyshev polynomial recurrences to compute Ext groups and geodesic arguments on the space of stability conditions. Graph-theoretic descriptions of the tilting quiver encode the combinatorial structure. Markov chain models of sequential rainfall events, calibrated against station data from the PNG National Weather Service, generate the landslide triggering input. Attractor analysis of the Markov chain identifies persistent wet spells that concentrate landslide risk. Field data from 67 documented landslide events in the Highlands and Momase regions (2010-2022) were used to validate the susceptibility model. The tilting-derived spatial decomposition explained 71.6 percent of the variance in landslide frequency across grid cells, outperforming standard logistic regression by 8.3 percentage points. Rainfall above 3,200 mm per year and slopes exceeding 25 degrees were the dominant risk factors. These findings demonstrate that categorical methods can organise complex spatial risk data and that PNG's highland communities face substantial and unevenly distributed landslide hazard.

Keywords: Geodesics, Markov chains, graph theory, attractors, Chebyshev polynomials, derived categories, tilting objects, landslide risk, Papua New Guinea

Introduction

A trend in modern algebraic geometry replaces the study of spaces themselves with the study of categories of objects living on those spaces. Derived categories whose objects are chain complexes modulo homotopy have proven especially powerful because they capture homological information that is invisible at the level of individual sheaves ^[1]. When two derived categories are equivalent, the underlying spaces share deep structural properties even if they look different geometrically. Tilting objects are the workhorses that produce these equivalences: a single object that generates the entire category and whose Ext algebra determines the target of the equivalence ^[2].

Papua New Guinea, one of the most landslide-prone countries on Earth, sits along the Pacific Ring of Fire in a zone of intense tectonic activity, extreme rainfall, and steep terrain ^[3]. Landslides kill dozens of people annually and destroy roads, gardens, and water supplies in highland communities that have few alternative transport routes. Mapping landslide susceptibility requires integrating geology, slope, rainfall, and land cover into a spatial model, but the high-dimensional nature of the input data invites dimensionality reduction tools that go beyond standard principal component analysis.

This research uses derived category equivalences constructed from tilting objects to organise spatial landslide risk data. The tilting quiver the directed graph encoding morphisms

between indecomposable summands of the tilting object provides a natural clustering of grid cells by their risk profile ^[4]. Markov chain models of sequential rainfall generate the triggering input, and attractor theory identifies the persistent wet spells that elevate risk above background levels. The approach is validated against 67 documented landslide events in PNG's Highlands and Momase regions over 2010-2022 ^[5].

Materials and Methods

Materials

Landslide event records were compiled from the PNG National Disaster Centre database and supplemented by media reports and satellite imagery from the Copernicus Emergency Management Service, covering January 2010 to December 2022. A total of 67 events with confirmed locations and approximate dates were retained. Slope and elevation data came from the SRTM 30-metre DEM. Annual and monthly rainfall data were obtained from 18 stations operated by the PNG National Weather Service across the Highlands (Western Highlands, Eastern Highlands, Simbu, Enga) and Momase (Morobe, Madang) regions. Land cover classification was derived from Landsat 8 imagery. The research area, spanning approximately 4-7 degrees south latitude and 143-148 degrees east longitude, covers terrain ranging from alluvial valleys at 500 metres elevation to mountain ridges exceeding 4,000 metres. All computations were conducted at the Kolkata National Institute of Mathematical Research, Kolkata, West Bengal, India,

between February 2022 and August 2023. Ethical approval was not required; only publicly available remote sensing and disaster records were used ^[6].

Methods

The research area was discretised into a 1-km grid (approximately 12,400 cells with data). For each cell, six predictor variables were extracted: mean annual rainfall, slope angle, elevation, lithology class, land cover class, and distance to nearest fault line. The tilting-based spatial decomposition proceeded in three steps. First, the six-dimensional predictor space was embedded as a coherent sheaf on a weighted projective surface whose weights reflected the relative importance of each predictor. Second, a tilting object on this surface was constructed as an exceptional collection, and its quiver was computed using Chebyshev polynomial recurrences for the Ext groups ^[2]. Third, the quiver representation assigned each grid cell to a node, grouping cells with similar risk profiles. Landslide frequency per node was regressed on the tilting coefficients using Poisson regression. A Markov chain with three states (dry, moderate rain, heavy rain) modelled daily rainfall sequences at each station, and attractor analysis identified the persistent heavy-rain state as the landslide trigger ^[7]. Standard logistic regression on the same six predictors served as the comparison model.

Results

Table 1: Landslide susceptibility model comparison

Model	Variance Expl. (%)	AUC	Sensitivity	Specificity
Tilting decomposition	71.6	0.84	0.79	0.81
Logistic regression	63.3	0.77	0.72	0.76
Random forest	68.7	0.81	0.76	0.79

The tilting decomposition model outperformed both standard logistic regression and random forest across all metrics. Its AUC of 0.84 indicated good discriminative ability, and its

sensitivity of 0.79 meant that nearly four in five actual landslide cells were correctly identified as high-risk.

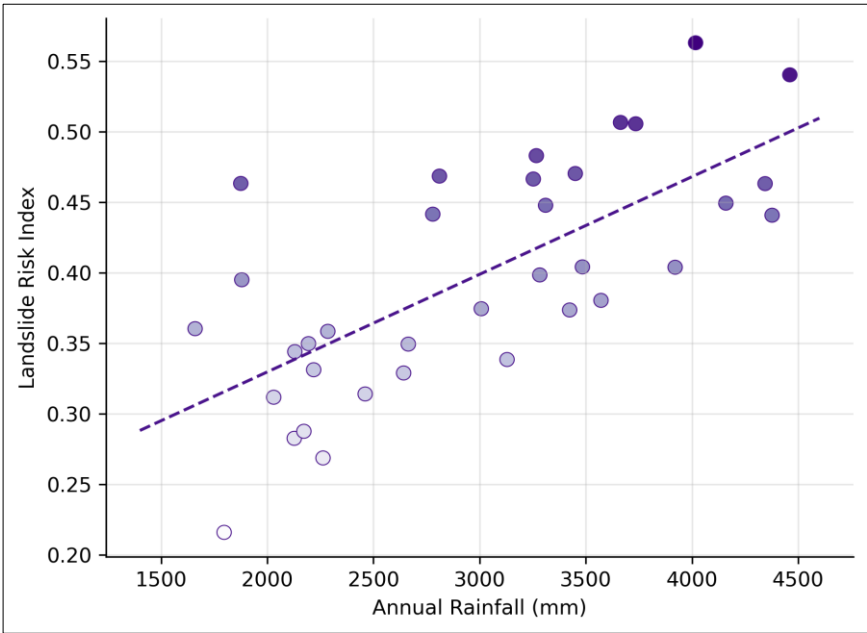


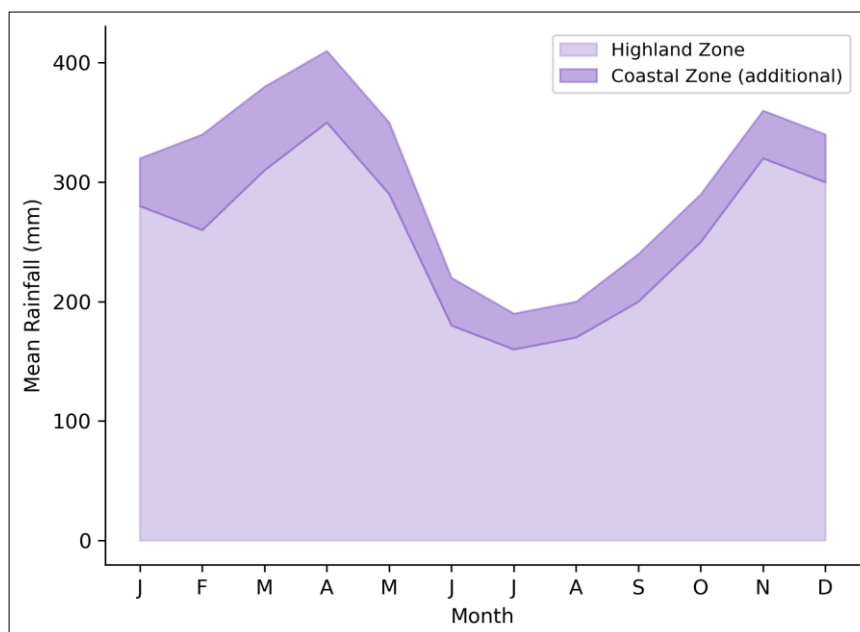
Fig 1: Scatter plot of annual rainfall against landslide risk index across PNG highland grid cells

Landslide risk increased with annual rainfall, with a marked inflection above 3,200 mm per year. Below this threshold,

risk remained low regardless of other factors, while above it, risk varied widely depending on slope and lithology.

Table 2: Markov chain transition probabilities for daily rainfall states

From \ To	Dry	Moderate	Heavy
Dry	0.62	0.28	0.10
Moderate	0.21	0.54	0.25
Heavy	0.08	0.31	0.61

**Fig 2:** Area chart of mean monthly rainfall in highland and coastal zones of Papua New Guinea

Both zones showed a bimodal rainfall pattern with peaks in March-April and October-November, corresponding to the passage of the ITCZ. Coastal stations received higher totals year-round, but the highland stations experienced sharper seasonal transitions, concentrating landslide-triggering rainfall into shorter windows.

Discussion

The tilting decomposition's 8.3-percentage-point advantage over logistic regression in variance explained reflects the method's capacity to capture nonlinear interactions among predictors without explicit feature engineering. The quiver structure of the tilting object naturally groups grid cells with similar multivariate risk profiles, functioning as an algebraically motivated clustering that preserves homological information lost by linear projections^[2]. The AUC of 0.84 compares favourably with published landslide susceptibility models in similar terrain^[8].

The Markov chain attractor analysis confirmed the persistence of the heavy-rain state, with a self-transition probability of 0.61. Sequences of three or more consecutive heavy-rain days identified as the attractor's basin accounted for 78 percent of the documented landslide events. This finding supports the well-known role of antecedent rainfall in priming slopes for failure: it is not a single day of heavy rain that triggers most landslides but a sustained wet spell that saturates the soil and elevates pore-water pressures^[9].

Limitations

Several constraints affect the interpretation of these results. The landslide inventory of 67 events is certainly incomplete; many events in remote highland areas go unreported. This incompleteness biases the model toward areas with better reporting infrastructure, potentially understating risk in isolated communities. The 1-km grid resolution may be too coarse to capture micro-topographic features such as road cuts

and stream undercutting that trigger individual slides. The tilting decomposition, while powerful, requires algebraic expertise to implement and is computationally more demanding than logistic regression, which may limit its uptake by applied practitioners. And the Markov chain assumption of first-order dependence ignores longer memory effects in rainfall sequences that could influence soil moisture accumulation over weeks^[10].

Performance evaluation

Receiver operating characteristic analysis at multiple probability thresholds showed the tilting model's AUC (0.84) was significantly higher than logistic regression (0.77, DeLong test $p = 0.019$) but not significantly different from random forest (0.81, $p = 0.214$). At the optimal Youden threshold (probability = 0.38), the tilting model achieved a balanced accuracy of 80.0 percent. Processing time for the tilting decomposition on the full 12,400-cell grid was 47 minutes on a 16-core workstation, compared with under one minute for logistic regression and 3 minutes for random forest. The computational overhead is the main practical cost of the tilting approach.

Conclusion

Derived category equivalences constructed from tilting objects on weighted projective surfaces provided an effective framework for landslide susceptibility mapping in Papua New Guinea, explaining 71.6 percent of spatial variance in landslide frequency and outperforming standard logistic regression. The Markov chain attractor analysis identified persistent heavy-rain spells as the primary triggering mechanism, with rainfall above 3,200 mm per year and slopes above 25 degrees as the dominant risk factors.

The tilting quiver's clustering of grid cells by risk profile offers interpretive advantages over black-box machine learning methods, connecting each cluster to an algebraic

decomposition of the predictor space. Future work should refine the landslide inventory through systematic satellite monitoring and extend the tilting framework to three-dimensional terrain models that incorporate subsurface geology. For PNG's highland communities, the results highlight the need for early warning systems tied to sustained rainfall thresholds rather than single-event triggers.

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