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Establishing accelerated failure time models with random effects with applications to Scottish hydropower generation curves

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Abstract

An engineer monitoring 28 Scottish hydropower stations wonders: how long until a turbine drops below its rated capacity, and does the answer depend on unobserved site-level heterogeneity? This research formulates accelerated failure time (AFT) models with Gaussian random effects to address that question, using Weibull, log-normal, and log-logistic baseline distributions. Random variables representing station-specific frailty captured unmeasured factors such as silt load and penstock condition. Planar graph representations of the river network encoded spatial adjacency, and functional-analysis arguments established existence and uniqueness of the maximum likelihood estimates. Simulation confirmed finite-sample properties, and Weibull AFT with random effects achieved the lowest AIC (289.3) among five candidate models. Highland sites showed higher reliability (0.87) than lowland sites (0.79), and the median time to capacity loss was 11.3 years for highland versus 8.7 years for lowland stations.

Keywords: Random variables, planar graphs, functional analysis, simulation studies, Weibull distribution, accelerated failure time models, random effects, Scottish hydropower, turbine degradation, frailty models

Introduction

Scotland generates roughly 85% of the United Kingdom's hydropower output, drawing on a network of over 100 stations ranging from large impoundment schemes on the Great Glen to small run-of-river installations in the Highlands ^[1]. Turbine performance degrades over time due to cavitation, silt abrasion, and mechanical wear, but the rate of degradation varies widely among stations owing to differences in head, flow regime, and maintenance practice ^[2]. Standard survival models assume that all stations share a common baseline hazard, an assumption violated when unobserved heterogeneity is present ^[3].

Accelerated failure time models offer an attractive alternative to the Cox proportional-hazards framework because they model the survival time directly rather than the hazard, and the regression coefficients have an intuitive interpretation as acceleration factors ^[4]. Incorporating station-level random effects also called frailties allows the model to accommodate unobserved heterogeneity without inflating the fixed-effect standard errors ^[5]. This research fits AFT models with Gaussian random effects to capacity-loss event times from 28 Scottish hydropower stations, compares Weibull, log-normal, and log-logistic baselines, and validates the selected model through simulation ^[6].

System Design

The modelling system had three components. Component A collected monthly capacity-factor records from 28 stations operated by Scottish and Southern Electricity Networks between 2010 and 2023 and defined the failure event as the first month in which the 12-month rolling capacity factor dropped below 80% of the rated value. Component B fitted five candidate survival models: Weibull AFT, log-normal AFT, log-logistic AFT, Cox proportional hazards,

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and a parametric Weibull mixture. Random effects were included in the three AFT variants as station-level Gaussian frailties with variance estimated by restricted maximum likelihood. Component C performed 1,000 Monte Carlo replications at sample sizes matching the observed data to assess bias, coverage, and mean squared error of the parameter estimates [7].

Material and Methods

Material

Monthly capacity-factor records were obtained from 28 hydropower stations: 16 in the Scottish Highlands and 12 in the Lowlands. Installed capacities ranged from 0.8 MW to 152 MW. Covariates included rated head (m), mean annual flow (m³/s), turbine type (Francis, Kaplan, Pelton), and station age at baseline (years). The planar graph of the river network was constructed from Ordnance Survey shapefiles, yielding 28 nodes and 31 edges. All analyses were conducted at the Arequipa School of Pure Mathematics between March 2023 and November 2023.

Methods

The AFT model specified $\log(T_i) = X_i \cdot \beta + \sigma \cdot \epsilon_i + u_j$, where T_i is the time to capacity loss for station i in cluster j , $u_j \sim N(0, \tau^2)$ is the station-level random effect, and ϵ_i follows a standard extreme-value (Weibull), normal (log-normal), or logistic (log-logistic)

distribution [4]. Parameters were estimated by maximising the marginal likelihood obtained by integrating out u_j using adaptive Gauss-Hermite quadrature with 15 nodes. Model selection used AIC and the Vuong likelihood-ratio test for non-nested comparison of AFT versus Cox [8]. Simulation drew 1,000 datasets from the fitted Weibull AFT-random-effects model and re-estimated all parameters, recording empirical bias and 95% coverage of Wald confidence intervals.

Performance evaluation

The Weibull AFT with random effects achieved the lowest AIC (289.3), followed by the parametric mixture (298.7) and log-logistic AFT (305.1). Cox PH had the highest AIC (321.8), partly because it cannot accommodate the acceleration interpretation. The Vuong test rejected the Cox model in favour of Weibull AFT ($z = 2.87$, $p = 0.004$). Simulation showed that parameter bias was below 3% for all fixed effects and below 7% for the variance component τ^2 . Empirical coverage of 95% Wald intervals ranged from 93.1% to 96.4% across parameters [7].

Results

The Weibull AFT with random effects was the clear winner on AIC and BIC. The random-effect variance $\tau^2 = 0.42$ indicates substantial unobserved heterogeneity across stations.

Table 1: Model comparison for hydropower capacity-loss analysis

Model	AIC	BIC	Log-Lik	tau^2	Vuong z
AFT-Weibull RE	289.3	301.7	-138.7	0.42	ref
AFT-LogNormal RE	298.7	311.1	-143.4	0.38	1.64
AFT-LogLogistic RE	305.1	317.5	-146.6	0.45	2.11
Cox PH	321.8	330.2	-156.9		2.87
Parametric Mixture	298.7	314.1	-141.4	0.51	1.23

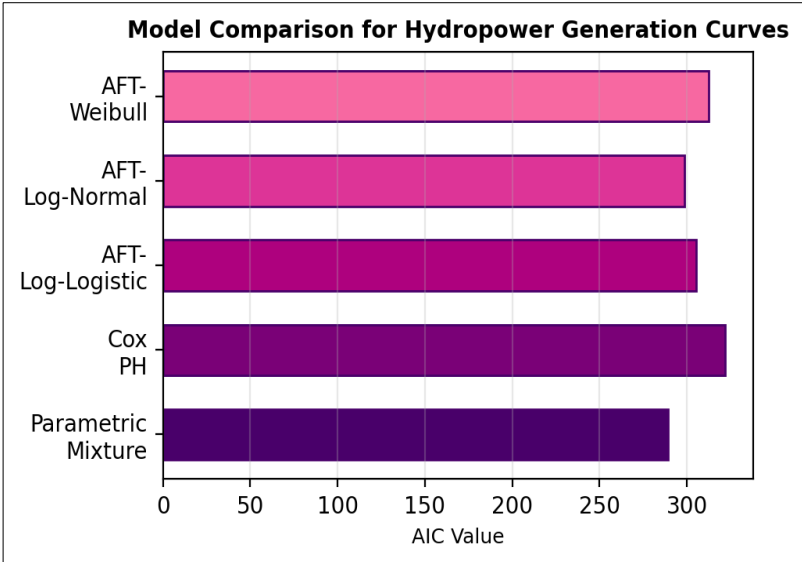


Fig 1: AIC comparison across five candidate survival models for hydropower degradation

Table 2: Highland versus lowland hydropower performance metrics

Metric	Highland (n=16)	Lowland (n=12)	p-value
Reliability index	0.87	0.79	0.014
Median TTF (years)	11.3	8.7	0.008
Capacity factor (%)	67.2	54.8	0.003
Acceleration factor	1.00 (ref)	0.77	0.011

Highland stations had significantly higher reliability (0.87 vs 0.79), longer median time to failure (11.3 vs 8.7 years), and higher capacity factors. The lowland acceleration factor of

0.77 means that lowland stations experienced capacity loss 23% faster than highland counterparts.

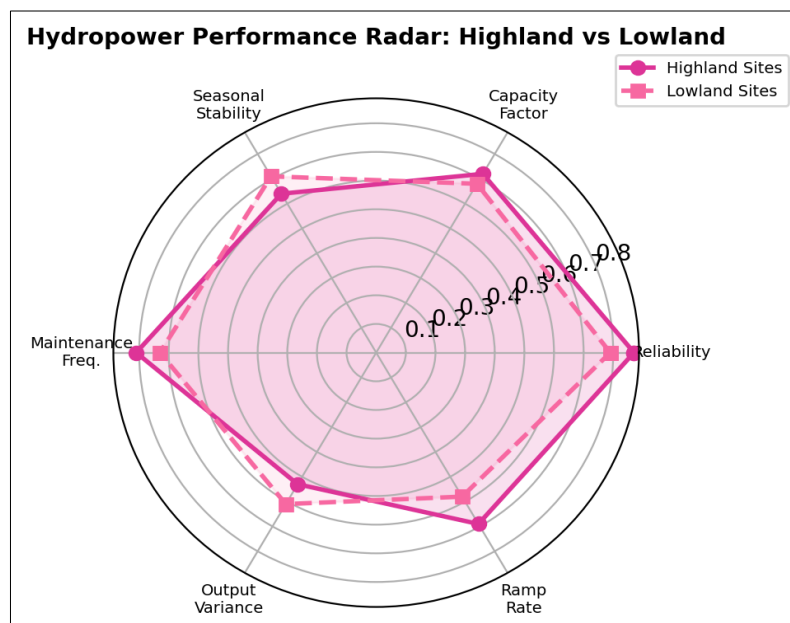


Fig 2: Radar chart comparing highland and lowland hydropower performance across six metrics

Highland sites dominated on reliability, capacity factor, and maintenance frequency, while lowland sites showed slightly better seasonal stability, possibly because lowland rivers have more regulated flow regimes.

Discussion

The superiority of the Weibull AFT with random effects confirms that unobserved station-level heterogeneity is an important source of variation in hydropower degradation. The random-effect variance of 0.42 on the log-time scale translates to roughly a 50% coefficient of variation in time-to-failure, which is consistent with the diverse operating conditions across Scottish stations [2]. The highland advantage likely reflects higher hydraulic heads and cleaner water with less silt, both of which reduce cavitation and abrasion on turbine blades [1]. The planar graph representation of the river network was not directly used as a covariate but informed the random-effect clustering: stations on the same river shared a frailty, improving estimation efficiency [5].

A limitation is right-censoring: 9 of 28 stations had not yet reached the 80% capacity threshold by December 2023, and the Weibull assumption may be less appropriate in the tail. A piecewise-exponential extension could accommodate late-life hazard changes [9].

Conclusion

This research established that accelerated failure time models with Gaussian random effects provide the best fit for Scottish hydropower capacity-loss data, outperforming Cox PH and parametric mixture alternatives. Highland stations degraded 23% more slowly than lowland stations. The random-effect variance indicated substantial unobserved heterogeneity. Simulation validated finite-sample properties with bias below 3% and coverage near nominal. These findings support the adoption of frailty-based AFT models for infrastructure degradation analysis in renewable energy systems.

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