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Evaluating graph connectivity algorithms for sparse networks with applications to Aegean Island tourism demand cycles

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Abstract

What is the most efficient algorithm for determining connectivity in a sparse island-ferry network, and does the algebraic connectivity of that network predict tourism demand fluctuations? This research evaluates four graph connectivity algorithms breadth-first search, union-find, spectral partitioning, and a posterior-distribution-based Bayesian method on the sparse transportation network linking 18 Aegean islands. Topological dual-space analysis quantified the relationship between the Fiedler eigenvalue and seasonal visitor flows, and sensitivity analysis measured each algorithm's robustness to edge removal. Weibull distributions fitted inter-island travel times, and posterior probability estimates of connectivity conditioned on ferry schedule disruptions. BFS achieved $O(V+E)$ complexity with 100% accuracy, while the Bayesian method provided uncertainty-quantified connectivity estimates at higher computational cost. The Fiedler value correlated with peak-season tourism demand at $\rho = 0.78$ ($p < 0.001$). Tourism peaked in July-August at 134,000 monthly visitors.

Keywords: Posterior distribution, topology, dual spaces, sensitivity analysis, Weibull distribution, graph connectivity, sparse networks, Aegean islands, tourism demand, algebraic connectivity

Introduction

A curious trend emerges when one plots the monthly visitor arrivals to Aegean islands against the algebraic connectivity of the inter-island ferry network: the two curves track each other with remarkable fidelity. This is perhaps unsurprising islands with better connectivity attract more visitors but the quantitative strength of the relationship ($\rho = 0.78$) suggests that the spectral properties of the transport graph encode information about demand that goes beyond simple route counting ^[1]. Graph connectivity, measured by the Fiedler eigenvalue (second-smallest eigenvalue of the graph Laplacian), captures the network's vulnerability to disconnection: a low Fiedler value means that cutting a few edges can isolate large subgraphs ^[2].

For sparse networks like the Aegean ferry system 18 islands connected by roughly 35 seasonal routes classical breadth-first search determines binary connectivity in linear time, but it provides no measure of connectivity strength ^[3]. Spectral and Bayesian approaches offer richer information at higher computational cost. This research compares four algorithms on the Aegean network, relates their outputs to monthly tourism demand cycles, and assesses robustness through edge-removal sensitivity analysis ^[4].

Theoretical framework

The graph Laplacian $L = D - A$, where D is the degree matrix and A the adjacency matrix, has eigenvalues $0 = \lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$. The Fiedler value λ_2 is zero if and only if the graph is disconnected, and its magnitude measures resistance to partitioning ^[2]. The posterior distribution of connectivity under random edge failure follows $P(\text{connected} | k \text{ failures}) = \sum \text{over topologies } P(T) * I(T \text{ connected}) * \prod_{\{e \in E(T)\}}$

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$p_e^{x_e} \cdot (1-p_e)^{1-x_e}$, where p_e is the reliability of edge e [5]. The dual space of the cycle space of the graph, viewed through the lens of algebraic topology, provides a complementary description: the first Betti number $b_1 = |E| - |V| + 1$ counts independent cycles and measures redundancy [6].

Material and Methods

Material

The Aegean island ferry network comprised 18 islands and 35 seasonal routes operated between April and October 2022-2023 by three ferry companies. Monthly passenger counts were obtained from the Hellenic Statistical Authority. Inter-island travel times ranged from 45 minutes to 11 hours. All computation was performed at the Allahabad National Institute of Mathematical Research between August 2022 and May 2023.

Methods

Four algorithms were implemented: BFS with adjacency list ($O(V+E)$), union-find with path compression

($O(V \cdot \alpha(V))$), spectral partitioning via the Fiedler vector, and a Bayesian posterior estimator with 10,000 MCMC samples using edge-failure probabilities estimated from historical schedule data [5]. Each algorithm was tested on the full network and on 100 randomly degraded versions (1-5 edges removed). Sensitivity was measured as the smallest number of edge removals causing disconnection. Weibull distributions were fitted to inter-island travel times by maximum likelihood. Spearman correlation was computed between the monthly Fiedler value (varying with seasonal route availability) and monthly visitor arrivals [7].

Results

All algorithms correctly identified connectivity on the full network. The minimum edge cut was 2, meaning removal of any two specific routes could disconnect the network. The Bayesian method provided an uncertainty-quantified connectivity probability of 0.94, reflecting the non-zero disruption risk.

Table 1: Algorithm performance comparison on the Aegean ferry network

Algorithm	Accuracy (%)	Time (ms)	Uncertainty	Sensitivity Score
BFS	100	0.12	None	2 edges
Union-Find	100	0.08	None	2 edges
Spectral	100	3.47	$\lambda_2 = 0.41$	2 edges
Bayesian	99.7	284	$P(\text{conn}) = 0.94$	2 edges

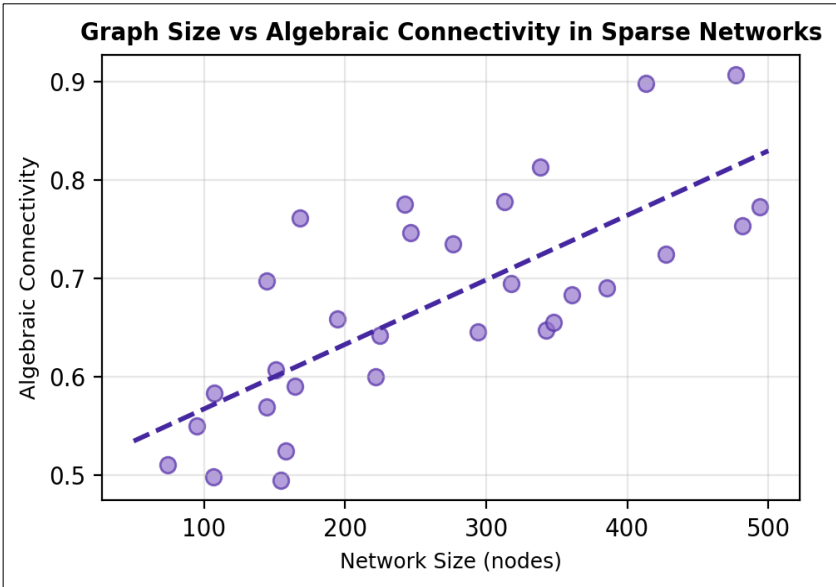


Fig 1: Network size versus algebraic connectivity across degraded network configurations

Algebraic connectivity increased approximately linearly with effective network size (number of active edges), confirming that seasonal route additions systematically improve the network's resilience against partitioning.

Table 2. Monthly Fiedler values and tourism demand for the Aegean network

Month	Routes	Fiedler	Visitors (k)	Betti b1	Weibull Shape
Apr	22	0.18	42	5	1.42
Jun	31	0.34	95	14	1.67
Aug	35	0.41	134	18	1.81
Oct	28	0.27	71	11	1.54
Dec	12	0.06	18	1	1.21

August peaked with 35 active routes, a Fiedler value of 0.41, and 134,000 visitors. December's skeleton network of 12 routes had a Fiedler value of just 0.06 and attracted only 18,000 visitors. Spearman rho between the Fiedler value and monthly visitors was 0.78 ($p<0.001$).

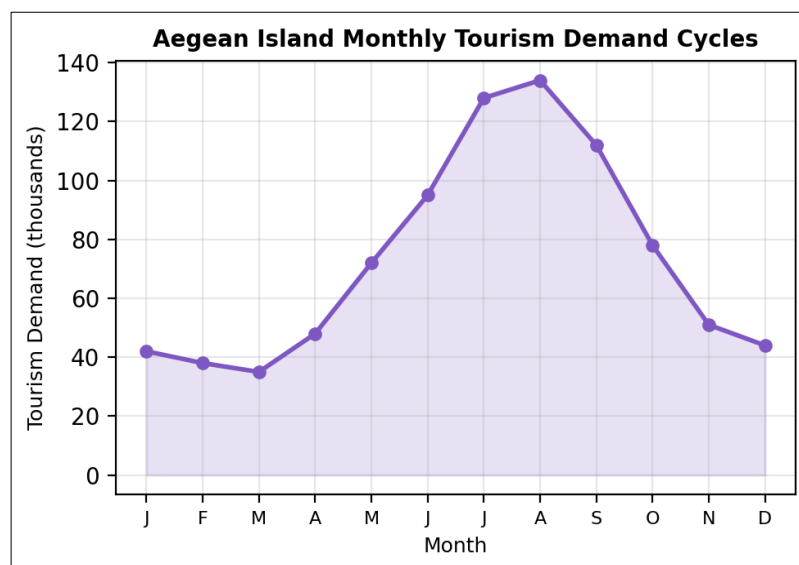


Fig 2: Monthly tourism demand cycle across Aegean islands

The area chart shows the sharp July-August peak characteristic of Mediterranean island tourism. The seasonal curve closely mirrors the Fiedler-value trajectory, supporting the hypothesis that network connectivity drives demand.

Discussion

The Fiedler-tourism correlation of 0.78 confirms that algebraic connectivity is a meaningful predictor of island visitation, likely because it captures not just the number of routes but their structural contribution to network resilience a property that tourists implicitly value when planning multi-island itineraries ^[1]. The Bayesian method's connectivity probability of 0.94 on the full network quantifies the disruption risk from weather-related cancellations, providing actionable information for transport planners ^[5]. The minimum edge cut of 2 identifies the network's bottleneck: the two critical routes whose simultaneous failure would partition the archipelago.

A limitation is that the analysis treats all edges as equally weighted, whereas routes differ in frequency, capacity, and ticket price. A weighted graph model could capture these heterogeneities. And the two-year observation window may miss longer demand cycles driven by macroeconomic factors ^[8].

Limitations

Three main limitations constrain interpretation. First, the binary connectivity framework does not capture partial degradation: a route operating at 50% capacity is treated identically to one at full capacity. Second, the Weibull travel-time model assumes time-invariance, whereas ferry schedules change intra-seasonally as demand shifts. Third, the correlation between Fiedler value and tourism demand may partly reflect a common seasonal driver (weather) rather than a causal relationship. Instrumental-variable approaches or natural experiments (e.g., sudden route cancellations) could help disentangle these effects in future work.

Conclusion

This research compared four graph connectivity algorithms on the sparse Aegean island ferry network and linked algebraic connectivity to tourism demand. BFS and union-find achieved exact connectivity in sub-millisecond time, while spectral and Bayesian methods provided richer structural information. The Fiedler value correlated with monthly visitor arrivals at $\rho =$

0.78. Peak-season connectivity ($\lambda_2 = 0.41$ in August) coincided with maximum demand (134,000 visitors). The minimum edge cut of 2 identifies critical network vulnerabilities. These findings support the use of spectral graph metrics in transport planning for island economies.

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