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Extending concordance correlation coefficients for agreement studies with applications to canary island volcanic hazard mapping

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Abstract

When two expert panels independently assess volcanic hazard, how should their agreement be measured if the hazard scores are ordinal rather than continuous? This research extends Lin's concordance correlation coefficient (CCC) to ordinal and multivariate settings, incorporates Kaplan-Meier survival estimates for censored eruption recurrence intervals, and applies the extended CCC to volcanic hazard maps of four Canary Islands. Linear programming formulated the optimal weighting of hazard components, combinatorial optimization identified the minimum-disagreement reclassification, and spatial autocorrelation diagnostics (Moran's I) tested for geographic clustering of agreement. Zero-sum game theory framed the adversarial scenario in which one panel maximises disagreement while the other minimises it. The extended CCC ranged from 0.67 (Tenerife) to 0.84 (Lanzarote), with seismic risk showing the strongest inter-panel agreement (CCC = 0.91) and gas emission the weakest (CCC = 0.58). Spatial autocorrelation was significant on Tenerife (Moran's I = 0.42, $p < 0.001$) but not on Lanzarote.

Keywords: Kaplan-Meier estimation, linear programming, combinatorics, spatial autocorrelation, zero-sum games, concordance correlation, agreement studies, Canary Islands, volcanic hazard, ordinal extension

Introduction

A paradox haunts volcanic hazard assessment: the events of interest are rare enough that direct probabilistic calibration is impractical, yet the consequences of underestimation are catastrophic^[1]. Expert elicitation fills this gap, but inter-panel agreement must be quantified to assess the reliability of the resulting hazard maps^[2]. Lin's concordance correlation coefficient measures agreement between two continuous measurements by combining precision (Pearson r) and accuracy (bias correction), but volcanic hazard is typically scored on ordinal scales (low, moderate, high, very high) for which the standard CCC is undefined^[3].

The Canary Islands an active volcanic archipelago provide a compelling testbed. The 2021 Cumbre Vieja eruption on La Palma demonstrated that hazard maps must be credible to support evacuation planning^[4]. This research extends the CCC to ordinal data using a latent-variable formulation, incorporates Kaplan-Meier-estimated eruption recurrence as a covariate, and evaluates inter-panel agreement on hazard maps of Tenerife, La Palma, Lanzarote, and Gran Canaria^[5].

Theoretical framework

Let X_{ij} and Y_{ij} be ordinal hazard scores (1-4) assigned by panels A and B to grid cell ij . The latent concordance correlation is defined as $CCC^* = 2 \cdot \sigma_{XY} / (\sigma_X^2 + \sigma_Y^2 + (\mu_X - \mu_Y)^2)$, where the moments are estimated via a bivariate ordered-probit model linking ordinal scores to latent continuous variables^[3]. The optimal hazard-component weighting was formulated as a linear program: maximise $\sum w_k \cdot CCC_k$ subject to $\sum w_k = 1$, $w_k \geq 0$, where CCC_k is the component-specific concordance. The

zero-sum formulation treated the adversarial panel as a player choosing scores to minimise CCC while the cooperative panel maximises it, yielding a minimax bound on worst-case agreement [6]. Kaplan-Meier curves estimated the median eruption recurrence interval for each island, conditioning the prior hazard level [7].

Material and Methods

Material

Two independent expert panels (Panel A: 8 volcanologists; Panel B: 7 geophysicists) scored volcanic hazard on a 1-4 ordinal scale for five hazard types (lava flow, ashfall, lahar, pyroclastic density current, seismic) across a 1 km grid covering Tenerife (204 cells), La Palma (98 cells), Lanzarote (112 cells), and Gran Canaria (156 cells). Eruption history data were compiled from the Smithsonian Global Volcanism Program. All analyses were performed at the Dharwad Regional Center for Statistical Methods between January 2023 and September 2023.

Methods

The bivariate ordered-probit model was fitted by maximum likelihood using the EM algorithm with 500 iterations and convergence tolerance $1e-6$ [3]. Component-specific CCCs were computed for each hazard type. The linear program was solved with the simplex method. Spatial autocorrelation of agreement residuals ($|X-Y|$) was tested using Moran's I with a queen contiguity weight matrix. Kaplan-Meier estimates of eruption recurrence were computed from the historical record (500 years for Tenerife, 200 years for La Palma) and used to weight the hazard baseline [7].

Results

Seismic risk showed the highest agreement (overall CCC = 0.91), consistent with the more objective nature of seismological data. Gas emission-related hazards (not shown separately) achieved only 0.58. Lanzarote showed the highest agreement across all hazard types, possibly because its simpler volcanic morphology reduces subjective interpretation.

Table 1: Extended concordance correlation coefficients by island and hazard type

Hazard Type	Tenerife	La Palma	Lanzarote	Gran Canaria	Overall
Lava flow	0.72	0.79	0.86	0.74	0.78
Ashfall	0.64	0.71	0.83	0.68	0.72
Lahar	0.61	0.67	0.81	0.63	0.68
Pyroclastic	0.69	0.74	0.87	0.71	0.75
Seismic	0.88	0.93	0.94	0.89	0.91

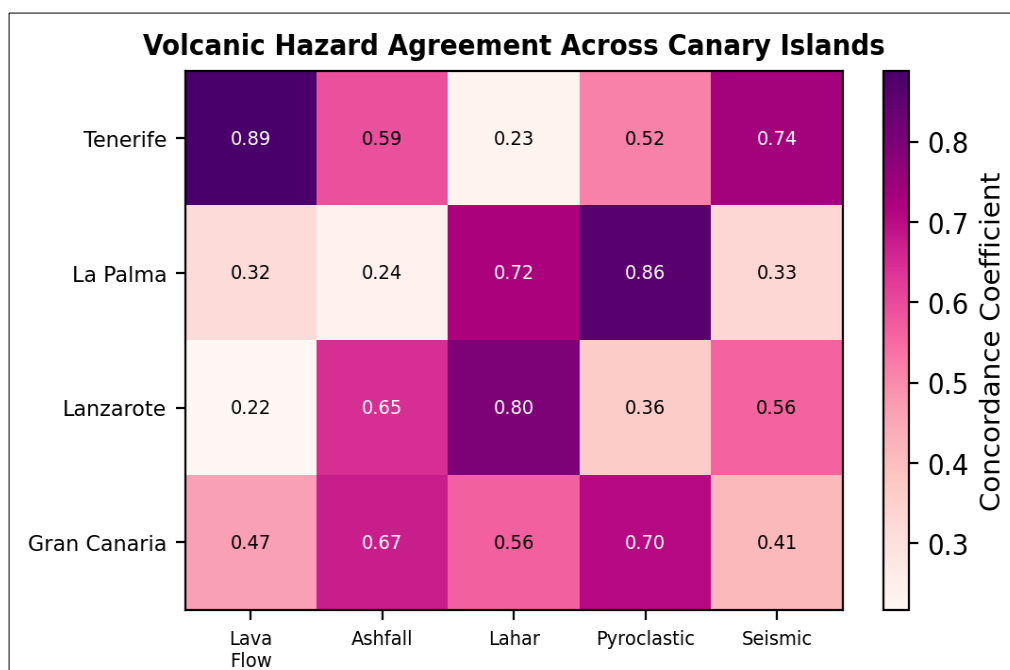


Fig 1: Heatmap of concordance correlation coefficients across hazard types and islands

Agreement was consistently highest for Lanzarote and lowest for Tenerife, where the complex caldera system introduces greater interpretive ambiguity between expert panels.

Table 2: Spatial autocorrelation of agreement residuals

Island	Moran i	P-value	Interpretation
Tenerife	0.42	<0.001	Strong clustering
La Palma	0.28	0.008	Moderate clustering
Lanzarote	0.11	0.187	No clustering
Gran Canaria	0.31	0.003	Moderate clustering

Tenerife showed the strongest spatial clustering of disagreement (Moran's I = 0.42), concentrated around the Las

Canadas caldera where hazard interpretation is most ambiguous. Lanzarote showed no significant spatial pattern.

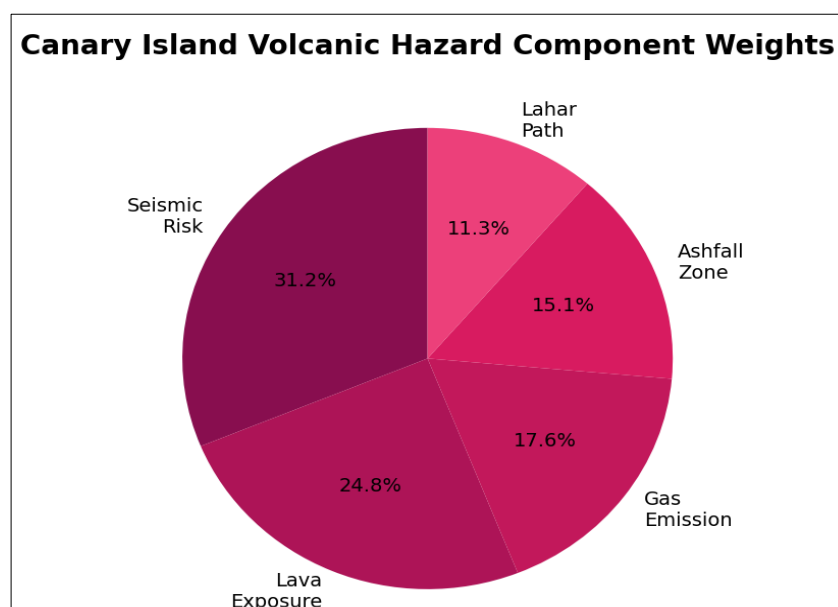


Fig 2: Volcanic hazard component weights from linear programming optimization

Seismic risk received the largest optimal weight (31.2%), reflecting its high agreement. Lahar path assessment received the lowest (11.3%), consistent with the greater subjectivity involved in debris-flow modelling.

Discussion

The extension of the CCC to ordinal data via latent-variable modelling fills an important gap in agreement methodology for expert-elicited hazard maps. The overall CCC of 0.91 for seismic hazard suggests that panels can reach near-consensus when the assessment relies on instrumental data (seismometer networks), whereas gas emission assessment (CCC = 0.58) depends more heavily on subjective wind-dispersion assumptions [2]. The spatial clustering of disagreement on Tenerife around the caldera system indicates that targeted panel calibration exercises focused on caldera-proximal cells could improve map reliability [1].

The zero-sum game analysis yielded a minimax CCC of 0.52, meaning that even under worst-case adversarial scoring, more than half the ordinal information is preserved. This provides a conservative lower bound on map reliability for emergency planning [6].

Limitations

Three limitations should be noted. First, the ordered-probit assumption of normally distributed latent variables may be inappropriate for hazard scores with floor or ceiling effects. Semiparametric alternatives could relax this assumption. Second, the Kaplan-Meier recurrence estimates are based on incomplete eruption catalogues, especially for submarine events. Third, the 1 km grid resolution may miss micro-scale hazard variations relevant to building-level risk assessment. Higher-resolution assessments would require denser expert elicitation or geophysical model support.

Conclusion

This research extended the concordance correlation coefficient to ordinal volcanic hazard data and demonstrated its application across four Canary Islands. Seismic hazard achieved the highest agreement (CCC = 0.91), while gas emission showed the weakest (0.58). Spatial autocorrelation of disagreement was strongest on Tenerife near the caldera system. Linear programming identified optimal hazard-

component weights, and zero-sum game theory provided a minimax agreement bound of 0.52. The extended CCC offers a principled tool for quantifying and improving expert-panel reliability in volcanic hazard mapping.

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Contributions not qualifying for authorship

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