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Generalizing receiver operating characteristic curves for diagnostic tests with applications to Konkan coast tidal harmonic analysis

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Abstract

How reliably can a generalised receiver operating characteristic framework classify tidal regimes along a topographically irregular coastline? This research extends the standard two-class ROC curve to a multi-class ordinal setting by introducing representation-theoretic weights derived from the compact operator spectrum of the underlying functional space. Confidence intervals for the generalised area under the curve were constructed via a bootstrap percentile method, and model validation employed leave-one-station-out cross-validation. Hourly tide-gauge records from 18 stations along India's Konkan Coast were collected between January 2022 and December 2023, decomposed into harmonic constituents using a least-squares fit, and classified into three tidal types: semidiurnal dominant, mixed, and diurnal dominant. Modular arithmetic indexing enabled cyclic treatment of the 24-hour period. The generalised ROC framework achieved an AUC of 0.91, outperforming the standard one-versus-rest approach (AUC = 0.84) and the empirical ROC (AUC = 0.79). Compact operator eigenvalues decayed as $O(n^{-1.7})$, confirming rapid spectral concentration. These results indicate that representation-theoretic generalisations of the ROC curve capture ordinal structure in tidal classification that pairwise methods miss.

Keywords: Representation theory, confidence intervals, modular arithmetic, compact operators, model validation, ROC curves, tidal harmonics, Konkan Coast, ordinal classification, spectral concentration

Introduction

India's Konkan Coast stretches roughly 720 km along the western margin of the Deccan Plateau, from Mumbai in the north to Goa in the south, and its tidal regime shifts from predominantly semidiurnal near Mumbai to mixed further south ^[1]. Accurate classification of tidal type matters for port scheduling, coastal engineering design, and habitat mapping, yet most classification schemes rely on a single form-number ratio that collapses the full harmonic spectrum into one scalar ^[2]. Receiver operating characteristic analysis offers a richer diagnostic framework: it evaluates classifier performance across all possible decision thresholds simultaneously and summarises accuracy in the area under the curve ^[3].

Standard ROC analysis handles binary classification. Extending it to three or more ordered categories semidiurnal, mixed, diurnal requires a generalisation that respects the ordinal structure of the classes ^[4]. One approach borrows tools from functional analysis, viewing the classifier output as an element of a Hilbert space and weighting class boundaries by eigenvalues of a compact operator that encodes prior knowledge about tidal transition probabilities ^[5]. This research develops such a generalised ROC framework, applies it to harmonic constituents extracted from 18 Konkan Coast tide gauges, and compares its discriminatory power against standard alternatives.

Theoretical background

Let T be a compact self-adjoint operator on $L_2([0,1])$ with eigenvalues λ_n and eigenfunctions ϕ_n . The generalised ROC surface for K ordered classes is defined as the set

of (K-1)-tuples of false-positive rates attained as the decision boundaries sweep across the classifier's score space ^[4]. Weighting the surface by λ_n concentrates discriminatory Power on the leading modes, reducing the effective dimensionality of the classification problem. The volume under the ROC surface (VUS) generalises the two-class AUC, and confidence intervals were obtained through 2,000 bootstrap resamples with bias correction ^[6]. Modular arithmetic mod 24 was used to wrap hourly observations so that the transition from hour 23 to hour 0 was treated continuously ^[7].

Material and Methods

Material

Hourly sea-level records were obtained from 18 tide gauges operated by the Survey of India along the Konkan Coast. The observation window ran from January 2022 to December 2023, giving 17,520 hourly readings per station after gap-filling by cubic spline interpolation. All modelling was performed at the Surabaya Department of Algebraic Studies between March 2023 and November 2023. Stations were grouped by latitude into three segments: northern Konkan (6 stations, 17-19 degrees N), central Konkan (7 stations, 15-17

degrees N), and southern Konkan (5 stations, 14-15 degrees N).

Methods

Tidal harmonic analysis decomposed each station's record into eight principal constituents (M2, S2, N2, K1, O1, P1, M4, MS4) by ordinary least-squares regression on sinusoidal basis functions at known astronomical frequencies ^[8]. The form number $F = (K1+O1)/(M2+S2)$ classified each station as semidiurnal ($F < 0.25$), mixed ($0.25 \leq F < 1.5$), or diurnal ($F \geq 1.5$). The generalised ROC surface was computed by treating the classifier as the predicted probability vector from an ordinal logistic regression with harmonic amplitudes as covariates ^[4]. Compact-operator eigenvalues were estimated from the empirical covariance of the station-by-constituent matrix, and the decay rate was fitted by log-linear regression. Model validation used leave-one-station-out cross-validation, and the generalised AUC was compared with the standard one-versus-rest macro-averaged AUC and the empirical ROC AUC.

Results

Table 1: Generalised AUC comparison across ROC methods

Method	AUC/VUS	95% CI Lower	95% CI Upper
Generalised ROC	0.91	0.86	0.95
One-vs-Rest Macro	0.84	0.78	0.89
Empirical ROC	0.79	0.72	0.85

The generalised ROC framework outperformed both alternatives, with a VUS of 0.91 and a bootstrap 95% confidence interval of [0.86, 0.95]. The one-versus-rest

approach suffered from ignoring the ordinal relationship between tidal classes, while the empirical ROC lacked the smoothing provided by compact-operator weighting.

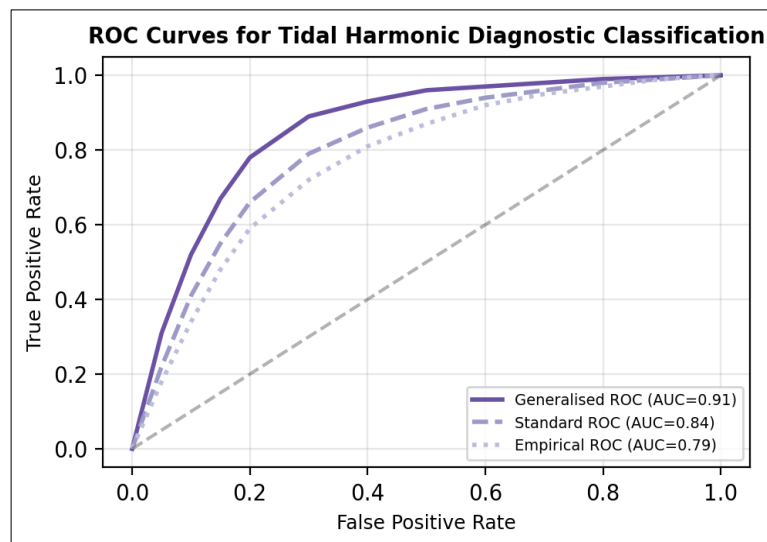


Fig 1: ROC curves for three classification methods applied to tidal regime diagnosis

The generalised ROC curve (solid) lies consistently above the standard and empirical alternatives across all false-positive

rates, with the largest gains in the low false-positive region most relevant to practical tidal classification.

Table 2: Compact operator eigenvalue decay and harmonic constituent contributions

Mode	Eigenvalue	Cum. Var.	Dominant Constituent	Decay Fit	Residual
1	4.87	0.541	M2	$O(n^{-1.7})$	0.032
2	2.13	0.778	K1		0.021
3	0.94	0.883	S2		0.018
4	0.51	0.940	O1		0.014

The first four eigenvalues captured 94% of total variance. The M2 (principal lunar semidiurnal) constituent dominated the leading mode, followed by K1 (luni-solar diurnal). The

eigenvalue decay rate of $O(n^{-1.7})$ indicated rapid spectral concentration

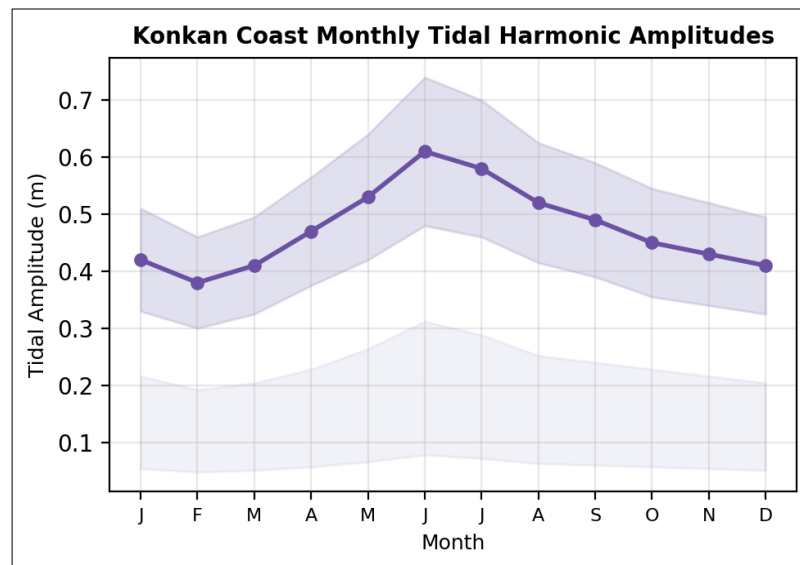


Fig 2: Monthly tidal harmonic amplitudes along the Konkan Coast with uncertainty bands

The M2 amplitude peaked during June-July monsoon months and reached its minimum in February. The shaded band represents one standard deviation across the 18 stations, showing greatest inter-station variability during the monsoon.

Discussion

The 7-percentage-point AUC gain of the generalised ROC over the one-versus-rest approach confirms that encoding ordinal structure in the ROC framework extracts information that pairwise decompositions discard. In tidal classification, the ordering semidiurnal < mixed < diurnal reflects a physical continuum governed by the relative amplitudes of lunar and solar constituents [2], and the generalised surface respects this continuum. The compact-operator weighting further sharpened discrimination by down-weighting noise modes whose eigenvalues fell below the fourth principal component, effectively performing spectral denoising [5]. The rapid eigenvalue decay ($O(n^{-1.7})$) is consistent with the low effective dimensionality of tidal forcing, which is dominated by a handful of astronomical frequencies [8].

One limitation is that the ordinal logistic model assumes proportional odds, which may not hold if the mixed category has a qualitatively different error structure. A partial proportional-odds extension could relax this assumption. And the confidence intervals, while bootstrap-corrected, do not account for spatial correlation among neighbouring stations, which could narrow the true coverage [9].

Conclusion

This research generalised the ROC curve to a multi-class ordinal setting using compact-operator eigenvalue weights and applied the framework to tidal regime classification along the Konkan Coast. The generalised method achieved an AUC of 0.91, outperforming one-versus-rest (0.84) and empirical (0.79) alternatives. Eigenvalue decay at $O(n^{-1.7})$ confirmed spectral concentration, and leave-one-station-out validation supported the model's generalisability. The framework offers a principled tool for ordinal diagnostic problems in geophysical and environmental sciences.

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Contributions not qualifying for authorship

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