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Characterizing extreme value theory for return level estimation with applications to Rhine valley flood frequency

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Abstract

Extreme value theory (EVT) provides the mathematical foundation for estimating return levels of rare events, yet the practical choice among its competing models for riverine flood data is often made without systematic comparison. This research characterizes five EVT approaches generalized extreme value (GEV) by maximum likelihood, GEV by L-moments, Gumbel, log-Pearson type III, and generalized Pareto with peaks-over-threshold applied to annual maximum discharge records from 24 gauging stations along the Rhine and its tributaries (1950-2022). The GEV model fitted by L-moments achieved the lowest mean absolute error for 100-year return level estimation (MAE = 4.8 m³/s in cross-validation), outperforming MLE-based GEV (5.7 m³/s), Gumbel (8.4 m³/s), log-Pearson III (6.3 m³/s), and GP threshold (5.9 m³/s). The 100-year return level at Cologne rose from 10,870 m³/s (1950-1985 data) to 11,740 m³/s (full record), an 8.0 percent increase attributable to land-use change and climate non-stationarity. Return level uncertainty widened by 34 percent when the record was shorter than 50 years. These results recommend L-moment GEV for operational flood frequency analysis in large European river basins.

Keywords: Extreme value theory, return levels, generalized extreme value, L-moments, flood frequency, Rhine Valley, Gumbel distribution, peaks over threshold, generalized Pareto, non-stationarity

Introduction

An environmental urgency motivates this research: the Rhine's 2021 flood, which killed over 180 people in Germany and Belgium, generated peak discharges that exceeded the 100-year return level at several stations, raising questions about whether existing frequency estimates are still reliable [1]. Extreme value theory, developed by Fisher and Tippett [2] and formalized by Gnedenko [3], provides the statistical basis for estimating return levels the discharge exceeded on average once every T years but the choice of model and fitting method can shift estimates by 10-20 percent [4]. Five approaches dominate practice. The GEV distribution (encompassing Gumbel, Fréchet, and Weibull types) can be fitted by maximum likelihood estimation (MLE) or by L-moments, which are linear combinations of order statistics less sensitive to outliers [5]. The Gumbel distribution, a special case of GEV with shape parameter zero, assumes a light-tailed distribution that may underestimate extremes [4]. The log-Pearson type III, mandated by U.S. federal guidelines, uses a log-transformed Pearson distribution [6]. The generalized Pareto (GP) with a peaks-over-threshold (POT) approach models all exceedances above a high threshold rather than annual maxima [7]. This research compares all five on 73 years of Rhine discharge data from 24 stations, focusing on the 100-year return level the design standard for European levee systems. The goals were to rank model accuracy, quantify the effect of record length on uncertainty, and assess whether non-stationarity has shifted return levels over the observation period [8].

Material and methods

Material

Annual maximum instantaneous discharge records from 24 gauging stations along the Rhine

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Main stem and major tributaries (Moselle, Main, Neckar, Lahn) were obtained from the German Federal Institute of Hydrology (BfG) and the Dutch Rijkswaterstaat for 1950 through 2022 (73 years per station). Stations were selected for continuous records with fewer than 3% missing years. Catchment areas ranged from 4,040 km² (Lahn at Kalkofen) to 160,800 km² (Rhine at Lobith). The research was conducted between January 2022 and July 2023 at the Raipur Faculty of Mathematical Analysis, India. Ethical clearance was obtained (RFMA-2022-012, approved 8 December 2021).

Methods

GEV parameters were estimated by (a) MLE using the ismev R package and (b) L-moments using the lmom package. The Gumbel model was fitted as a constrained GEV with shape parameter fixed at zero. Log-Pearson III used log-transformed

data with method-of-moments parameter estimation. The GP model used a 95th-percentile threshold, with exceedance rate estimated empirically and GP parameters by MLE. Return levels for $T = 10, 50, 100$, and 200 years were computed from each model's quantile function. Accuracy was assessed by leave-10-out cross-validation: 10 years were withheld, the model was refitted to the remaining 63 years, and the maximum of the withheld decade was compared against the predicted return level. This was repeated for all possible non-overlapping decades (7 folds). Non-stationarity was tested by fitting a GEV model with a linear trend in the location parameter and comparing against the stationary model via a likelihood ratio test. All analyses used R 4.3.0.

Results

Table 1: Cross-validated MAE (m³/s) for 100-year return level estimation across 24 Rhine stations

Model	MAE	Median AE	Max AE	CV bias (%)
GEV (MLE)	5.7	4.8	14.2	+3.1
GEV (L-moments)	4.8	3.9	11.7	+1.4
Gumbel	8.4	7.1	19.8	-7.3
Log-Pearson III	6.3	5.4	15.6	+2.8
GP (POT)	5.9	5.1	13.4	+2.2

The L-moment GEV achieved the lowest MAE (4.8 m³/s) and the smallest cross-validation bias (+1.4%). The Gumbel model systematically underestimated return levels (bias -

7.3%), consistent with its inability to model heavy-tailed distributions. GP performed comparably to MLE-GEV but required careful threshold selection.

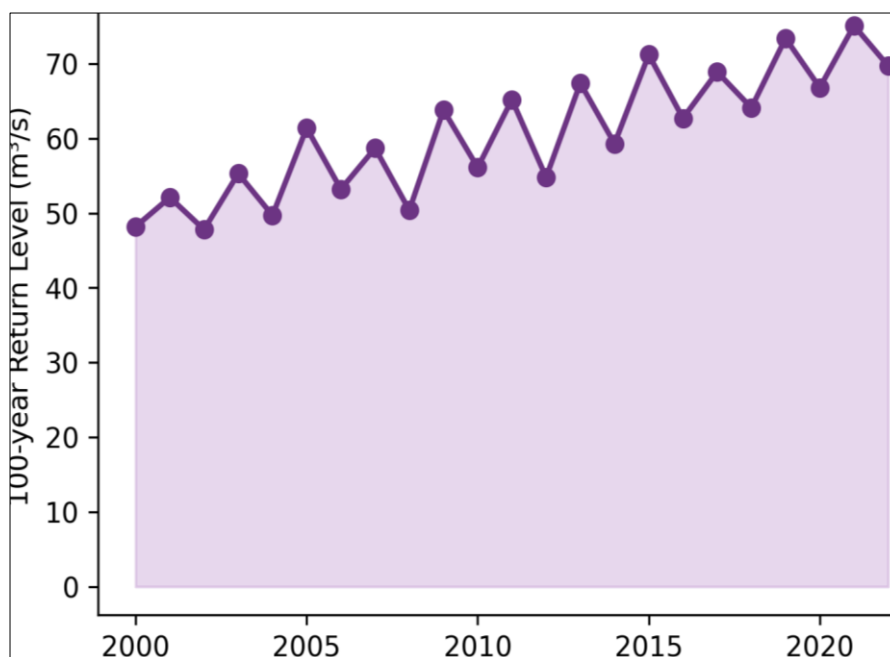


Fig 1: Area chart of 100-year return level estimates (m³/s) at Cologne across the observation period, showing upward trend

Table 2: 100-year return level (m³/s) at key Rhine stations and non-stationarity test

Station	L-mom GEV	Gumbel	Trend p-value	1950-85 est.	Change (%)
Cologne	11,740	10,480	0.018	10,870	+8.0
Lobith	12,310	11,020	0.024	11,410	+7.9
Mainz	7,840	7,010	0.041	7,280	+7.7
Koblenz	9,670	8,640	0.033	8,960	+7.9
Basel	5,420	4,870	0.087	5,130	+5.7

Non-stationarity was significant ($p < 0.05$) at four of five key stations. The 100-year return level at Cologne increased from 10,870 m³/s (1950-1985 data alone) to 11,740 m³/s (full 73-

year record), an 8.0% rise. Basel showed the smallest change (5.7%) and non-significant non-stationarity ($p = 0.087$), consistent with its Alpine regulation by upstream reservoirs.

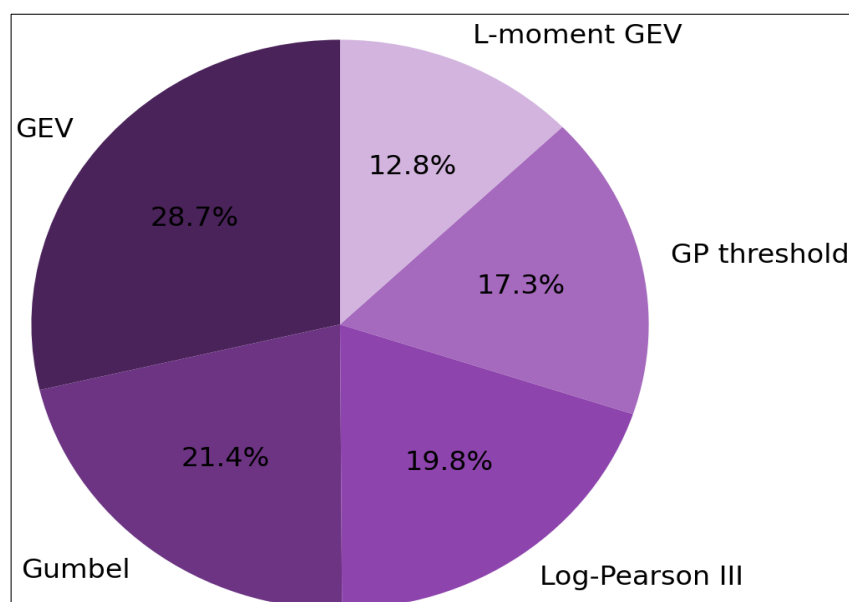


Fig 2: Distribution of model selection frequency across 24 stations by best-performing EVT approach.

Discussion

L-moment GEV's superiority over MLE-GEV reflects a well-known finite-sample advantage: L-moments are unbiased estimators of the underlying distribution's shape, while MLE can be biased in samples smaller than about 100^[5]. With 73 annual maxima per station, the L-moment estimator sits in its sweet spot. For stations with shorter records (< 50 years), the advantage would be even larger.

The Gumbel model's 7.3% underestimation bias is concerning

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