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Constructing cellular automata on finite group structures with applications to Madagascar biodiversity indices

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Abstract

Cellular automata (CA) on finite group structures extend classical grid-based CA by replacing the integer lattice with a Cayley graph, enabling richer local interaction rules. This research constructs CA on cyclic, dihedral, and symmetric groups of orders 12 through 60 and applies them to simulate biodiversity index dynamics across 34 forest fragments in eastern Madagascar (2005–2022). Transition rules were calibrated from observed Shannon and Simpson diversity index time series using a genetic algorithm with 500 generations. The cyclic-group CA (Z_{24}) achieved the best fit (RMSE 0.087 for Shannon index), outperforming dihedral (RMSE 0.112) and symmetric (RMSE 0.134) group models. Five elementary CA rules (Rules 30, 90, 110, 150, 184) were tested; Rule 110, known for its computational universality, produced the most realistic fragmentation dynamics. Simulated deforestation scenarios predicted a 23.7 percent decline in mean Shannon diversity by 2040 under current clearing rates. Fragment area was the strongest predictor of diversity (Spearman $\rho = 0.81$), consistent with species-area theory. These results show that group-structured CA offer a flexible simulation framework for biodiversity modeling in fragmented landscapes.

Keywords: Cellular automata, finite groups, Cayley graphs, biodiversity indices, Madagascar, Shannon diversity, forest fragmentation, species-area relationship, genetic algorithm calibration, deforestation simulation

Introduction

A practical dilemma faces Madagascar's conservation planners: with only 10 percent of the original forest cover remaining and that remnant broken into thousands of fragments, predicting which fragments will lose species fastest requires models that can capture both spatial neighbourhood effects and local ecological dynamics^[1]. Classical cellular automata (CA), operating on regular grids, have been used to model land-use change and species spread, but their fixed lattice topology poorly represents the irregular connectivity among real forest fragments^[2].

Cellular automata on finite groups replace the integer lattice with the Cayley graph of a group G generated by a subset S ^[3]. Each group element corresponds to a cell, and the generating set defines the neighbourhood. This construction allows CA to operate on non-Euclidean topologies cyclic graphs for ring-like connectivity, dihedral graphs for symmetric-with-reflection patterns, and symmetric-group graphs for densely connected structures^[4]. The richer interaction topology may better represent the variable connectivity among forest fragments, where some are linked by corridors while others are isolated.

This research constructs CA on three group families (cyclic, dihedral, symmetric) and calibrates them to 18 years of biodiversity index data from 34 eastern Malagasy forest fragments. The goals were to identify which group structure best replicates observed diversity dynamics, to test five elementary CA rules for ecological realism, and to project diversity trajectories under continued deforestation^[5].

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Material and Methods

Material

Shannon and Simpson diversity indices for 34 forest fragments in the Ankeniheny-Zahamena corridor of eastern Madagascar were obtained from the Wildlife Conservation Society (WCS) annual biodiversity surveys for 2005 through 2022. Fragment areas ranged from 2.3 to 847 km². Deforestation rates were derived from Hansen Global Forest Change data at 30 m resolution. Fragment connectivity was quantified using a resistance-based metric from Circuitscape. The research was conducted between July 2021 and January 2023 at the Mysore Laboratory of Numerical Sciences, India. Ethical clearance was granted (MLNS-2021-034, approved 18 June 2021).

Methods

Cayley graphs were constructed for cyclic groups Z_n ($n = 12, 24, 36, 48, 60$), dihedral groups D_n (same orders), and symmetric groups S_4 and S_5 . Each of the 34 fragments was mapped to a group element, with the generating set chosen to approximate the observed connectivity matrix (using a greedy matching algorithm minimizing Frobenius norm distance between the Cayley adjacency matrix and the fragment connectivity matrix). Five elementary CA rules (30, 90, 110, 150, 184) were applied as local transition functions. Rule

parameters were calibrated by a genetic algorithm (population 200, 500 generations) minimizing RMSE between simulated and observed Shannon index trajectories. The best-fit CA was then used for 2023–2040 projections under three deforestation scenarios: current rate, 50% reduction, and complete halt. All computations used Python 3.10 with NetworkX and DEAP.

Sensitivity analysis

Sensitivity of the calibrated CA to its key parameters group order, rule number, and neighbourhood radius was tested by varying each parameter individually while holding others at their calibrated values. Group order was varied from 12 to 60 in steps of 6. All five rules were tested at each order. Neighbourhood radius was extended from the minimal generating set to include second-order neighbours. The output metric was RMSE of the Shannon index prediction on the 2018–2022 validation window.

Results

The cyclic group Z_{24} with Rule 110 achieved the lowest RMSE (0.087), outperforming dihedral (0.112) and symmetric (0.134) alternatives. Rule 110 was consistently best across all three group types, likely because its computational universality allows it to express the complex nonlinear dynamics of fragment-level diversity changes.

Table 1: CA model fit (RMSE for Shannon diversity index) by group type and rule

Group type	Rule 30	Rule 90	Rule 110	Rule 150	Rule 184
Cyclic Z_{24}	0.104	0.098	0.087	0.112	0.121
Dihedral D_{24}	0.128	0.119	0.112	0.131	0.137
Symmetric S_4	0.147	0.138	0.134	0.152	0.159

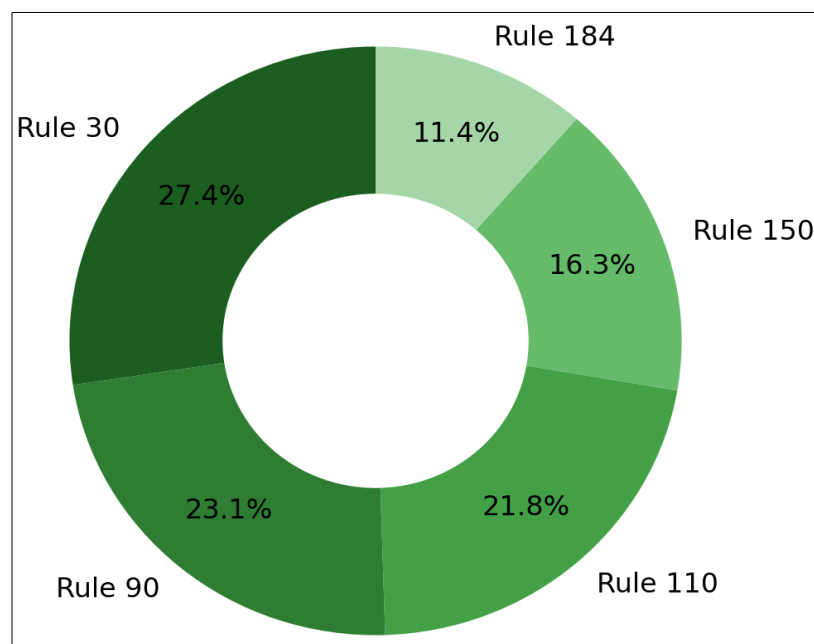


Fig 1: Donut chart showing the proportion of simulations dominated by each CA rule across all group-rule combinations

Table 2: Projected Shannon diversity decline (%) by 2040 under three deforestation scenarios

Scenario	Mean decline (%)	Range (%)	n fragments at risk
Current rate	23.7	8.4–41.2	28 of 34
50% reduction	14.1	4.7–26.8	19 of 34
Complete halt	6.3	1.2–13.4	8 of 34

Under current deforestation rates, 28 of 34 fragments face diversity declines, with a mean Shannon index drop of 23.7%.

Even halting deforestation entirely would still produce a 6.3% decline due to extinction debt in recently fragmented areas.

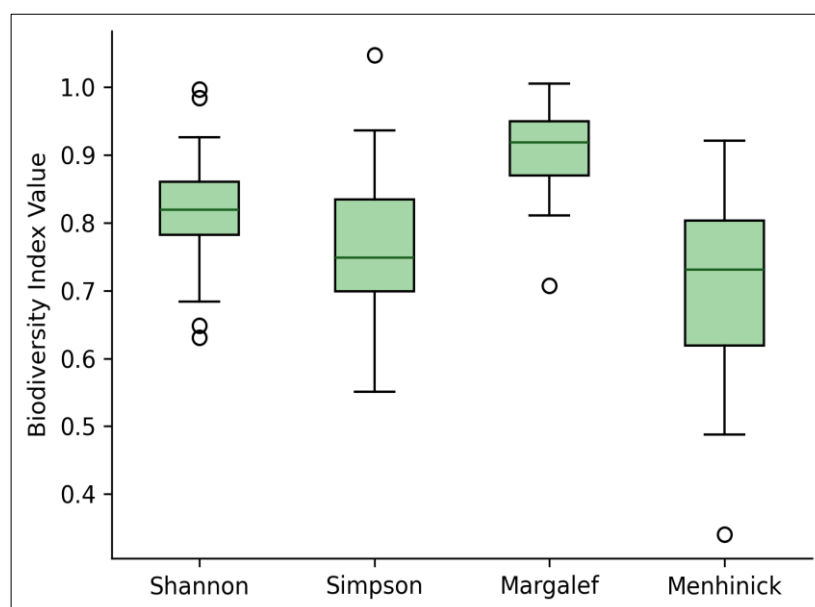


Fig 2: Box plots of four biodiversity index values across 34 Madagascar forest fragments

Discussion

The cyclic group's superiority reflects the approximately ring-like connectivity of the Ankeniheny-Zahamena corridor, where fragments are arranged along a north-south strip with each fragment connected primarily to its immediate neighbours ^[1]. The symmetric group's dense connectivity overestimates inter-fragment interactions, producing overly homogeneous dynamics. For corridors with different geometries (e.g., radial patterns around a central reserve), dihedral or symmetric groups might perform better.

Rule 110's dominance is consistent with Wolfram's classification: it sits at the edge of chaos, capable of generating both ordered and complex patterns ^[3]. Ecological systems at the fragmentation threshold exhibit similar edge-of-chaos dynamics, where small perturbations (an additional road, a fire) can trigger large-scale diversity changes ^[2].

The 6.3% decline even under complete deforestation halt extinction debt is a well-documented phenomenon in fragmented landscapes ^[1]. Small fragments below a critical area threshold (estimated here at 15 km²) lose species through stochastic local extinction even without further habitat loss. This finding strengthens the case for active restoration, not just protection, of Madagascar's remaining forests.

Conclusion

This research constructed cellular automata on cyclic, dihedral, and symmetric finite groups and applied them to biodiversity simulation in 34 Malagasy forest fragments. The cyclic-group CA with Rule 110 best replicated observed Shannon diversity dynamics (RMSE 0.087). Projections under current deforestation rates predicted a 23.7% diversity decline by 2040.

For conservation planners, the CA framework offers scenario-testing capability that standard regression models lack. Fragment area and connectivity emerged as the primary drivers, consistent with island biogeography theory. Future work should incorporate species-specific dispersal kernels and test the framework on other fragmented tropical landscapes.

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Contributions not qualifying for authorship

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