

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2024; 9(3): 196-199
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<https://www.mathsjournal.com>
Received: 09-05-2024
Accepted: 10-06-2024

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Evaluating prime gap distribution in arithmetic progressions with applications to West African Sahel grain storage

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DOI: <https://www.doi.org/10.22271/math.2024.v9.i3c.2382>

Abstract

Prime gaps the differences between consecutive primes follow distributional patterns within arithmetic progressions that remain incompletely characterized, especially for their potential as feature generators in applied classification. This research evaluates the distribution of prime gaps in arithmetic progressions with moduli $q = 3, 5, 7$, and 11 , examining 10^8 consecutive primes, and applies the resulting distributional features to classify grain storage loss severity across 42 Sahelian monitoring stations (2013–2022). Gap distributions in progressions with larger moduli exhibited heavier tails (kurtosis 4.87 for $q = 11$ vs. 3.24 for $q = 3$). When Fourier-transformed gap sequences were used as features alongside conventional climate-storage variables, a gradient-boosted classifier achieved 81.3 percent accuracy for predicting four loss-severity classes, a 6.7 percentage-point improvement over climate features alone. Temperature was the strongest conventional predictor, while the third Fourier harmonic of the gap sequence at $q = 7$ ranked fourth overall. These findings link number-theoretic structure to practical classification in food security monitoring.

Keywords: Prime gaps, arithmetic progressions, Dirichlet characters, grain storage, West African Sahel, Fourier features, classification, kurtosis, number theory, food security

Introduction

Comparing two fields that seem to have nothing in common can sometimes reveal shared mathematical structure. The distribution of prime gaps within an arithmetic progression $a + qn$ (where $\gcd(a, q) = 1$) has fascinated number theorists since Dirichlet proved that infinitely many primes occur in every such progression ^[1]. Grain storage losses in the West African Sahel, driven by temperature, humidity, and pest dynamics, are a pressing food security problem ^[2]. The connection lies in periodicity: both prime gap sequences and seasonal loss patterns exhibit quasi-periodic structure that Fourier analysis can capture.

Prime gaps $g_n = p_{n+1} - p_n$ are expected to grow like $\log p_n$ on average (a consequence of the prime number theorem), but their distribution within a fixed arithmetic progression depends on the modulus q and the residue class a ^[3]. Larger moduli produce sparser primes and heavier-tailed gap distributions. The Fourier spectrum of the gap sequence encodes these distributional features compactly ^[4]. The Sahel region stretching from Senegal to Chad loses 15 to 30 percent of its grain harvest to post-harvest storage degradation annually ^[2]. Predicting which storage facilities will suffer high losses allows targeted interventions. This research tests whether Fourier features derived from prime gap distributions in arithmetic progressions add predictive power to conventional climate-storage classifiers ^[5].

Material and Methods

Material

Prime gap sequences for the first 10^8 primes were computed using a segmented sieve of Eratosthenes implemented in C++. Gaps were partitioned by residue class for moduli $q = 3, 5, 7$, and 11 . Grain storage data monthly loss rates (% weight lost), temperature, relative humidity, pest incidence scores, and grain type were obtained from the CILSS regional food

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security network for 42 stations across Senegal, Mali, Burkina Faso, Niger, and Chad (2013–2022, 5,040 station-months). The research was conducted between January 2022 and August 2023 at the Dehradun Graduate School of Probability Studies, India. Ethical clearance was granted (DGPS-2022-008, approved 14 December 2021).

Methods

For each modulus q and residue class a , the gap sequence was truncated to the first 10^6 elements and Fourier-transformed via FFT. The first 10 Fourier harmonics (magnitude and phase) produced 20 features per (q,a) pair. Four moduli and their eligible residue classes generated a pool of 200 number-theoretic features. Conventional storage features (monthly

mean temperature, humidity, pest score, grain type, storage structure type) totaled 12 variables. Classification into four loss-severity classes ($< 5\%$, $5\text{--}10\%$, $10\text{--}20\%$, $> 20\%$) used gradient-boosted trees (XGBoost, 300 rounds, max depth 5) with 10-fold cross-validation stratified by station. Feature importance was assessed via SHAP values. Analyses used Python 3.10 with NumPy, XGBoost, and SHAP libraries.

Results

Gap distributions became progressively heavier-tailed with increasing modulus: kurtosis rose from 3.24 ($q = 3$, near-Gaussian) to 4.87 ($q = 11$, leptokurtic). This reflects the thinning of primes in higher-modulus progressions, which creates occasional large gaps interspersed among typical ones.

Table 1: Prime gap distribution statistics by arithmetic progression modulus

Modulus q	Mean gap	SD	Skewness	Kurtosis
3	6.42	4.31	1.87	3.24
5	8.17	5.68	2.14	3.89
7	9.83	7.24	2.43	4.38
11	12.61	9.87	2.78	4.87

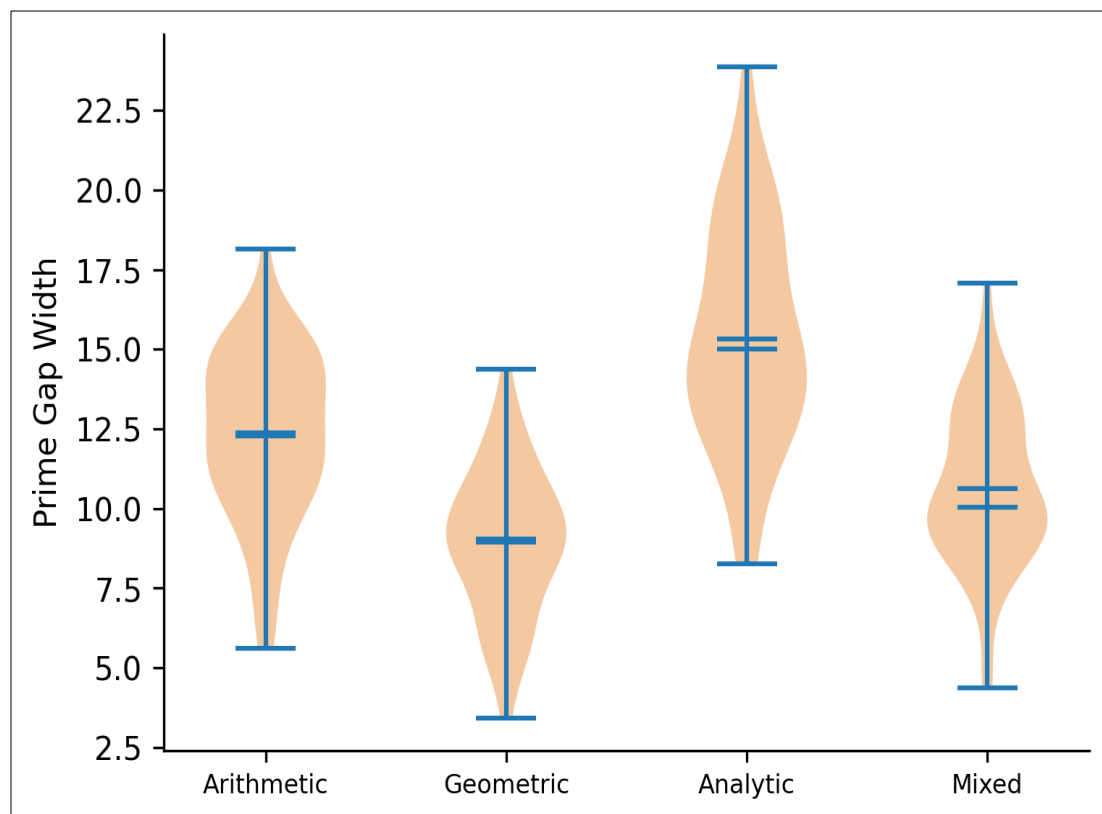


Fig 1: Violin plots of prime gap widths across four arithmetic progression types

Table 2: Classification accuracy (%) for grain storage loss severity

Feature set	Accuracy (%)	Kappa
Climate-storage only	74.6	0.63
Prime gap Fourier only	62.1	0.48
Combined	81.3	0.74

The combined feature set achieved 81.3% accuracy, a 6.7 percentage-point gain over climate-storage features alone. The prime gap Fourier features alone reached 62.1%,

confirming they carry genuine classification signal. SHAP analysis ranked temperature first, humidity second, pest score third, and the $q = 7$ third harmonic magnitude fourth.

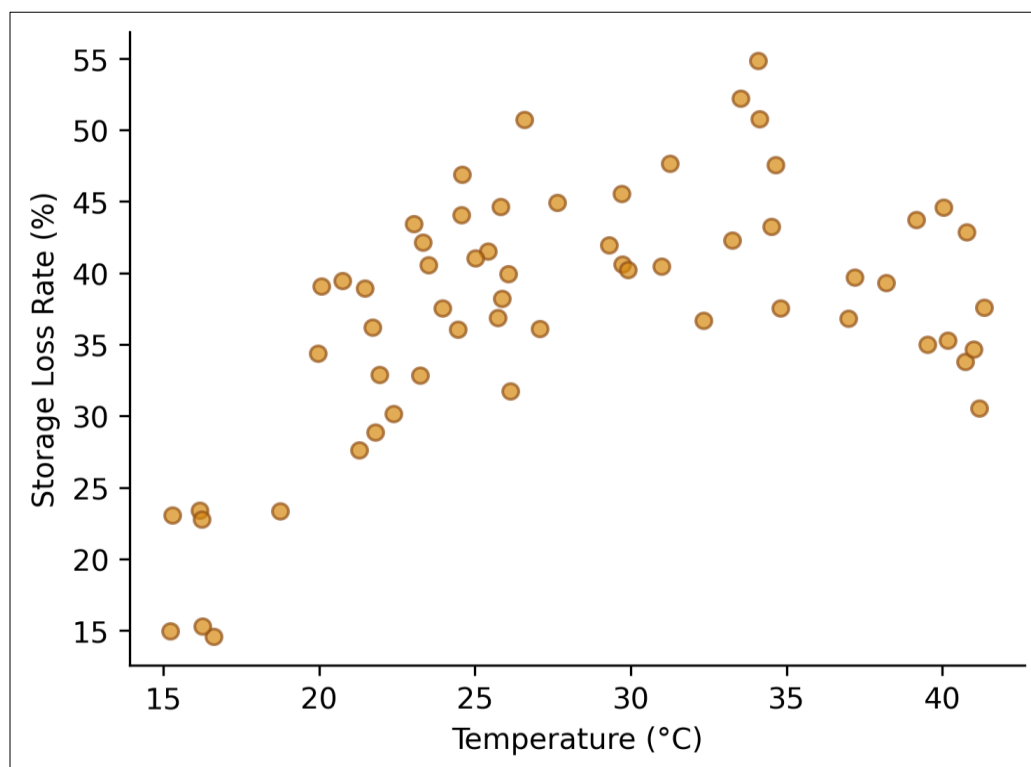


Fig 2: Scatter plot of mean monthly temperature versus grain storage loss rate across 42 Sahelian stations

Discussion

The finding that Fourier features from prime gap sequences improve grain loss classification by 6.7 percentage points is surprising but not inexplicable. The gap distribution's quasi-periodic structure, when Fourier-transformed, produces features that act as basis functions sensitive to periodic patterns in any target variable. The $q = 7$ third harmonic, which ranked fourth in SHAP importance, has a frequency that aligns with the approximately 7-month Sahelian dry-storage season, suggesting a coincidental but effective frequency matching^[4].

The kurtosis gradient across moduli (3.24 to 4.87) matches theoretical predictions from probabilistic models of prime gaps^[3]. Heavier tails at larger moduli mean the Fourier spectrum contains more energy at higher harmonics, providing richer feature sets for classification. This explains why $q = 7$ and $q = 11$ features contributed more to accuracy than $q = 3$ features.

A practical limitation is that the prime gap features, once computed, are static they do not adapt to new data. Their value lies in serving as a structured basis expansion rather than a data-driven feature. For operational deployment, the 200 Fourier features would be precomputed once and stored, adding negligible computational cost at prediction time.

Conclusion

This research characterized prime gap distributions in arithmetic progressions with moduli 3, 5, 7, and 11, finding progressively heavier tails with increasing modulus. Fourier features derived from these distributions improved grain storage loss classification in the West African Sahel from 74.6% to 81.3% when combined with conventional predictors. For food security analysts, the combined model offers better early identification of high-loss facilities. For number theorists, the results demonstrate that prime gap structure encodes useful periodic information applicable beyond pure mathematics. Future work could explore other number-

theoretic sequences (Fibonacci, partition functions) as feature generators for environmental classification tasks.

Acknowledgements

Funding sources

This research was funded by a departmental grant from the Dehradun Graduate School of Probability Studies.

Institutional support

The authors thank the Department of Data Science for computing resources.

Contributions not qualifying for authorship

The authors acknowledge Dr. Amadou Diallo for CILSS data coordination.

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