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Investigating integral transforms applied to heat conduction with applications to samoan coastal inundation mapping

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Abstract

Integral transforms convert differential equations into algebraic ones, simplifying the solution of heat conduction and diffusion problems. This research compares four integral transforms Fourier, Laplace, wavelet, and Hankel applied to a two-dimensional heat conduction equation governing coastal thermal diffusion, then maps the resulting temperature-analogue fields onto coastal inundation susceptibility in Samoa. Topographic and tidal data from 28 coastal transects on Upolu and Savai'i islands (2010-2022) provided the boundary conditions. The Laplace transform achieved the highest solution accuracy ($R^2 = 0.91$ against numerical finite-element benchmarks), outperforming Fourier ($R^2 = 0.87$), wavelet ($R^2 = 0.84$), and Hankel ($R^2 = 0.79$). The thermal-analogue inundation model correctly classified 84.6 percent of historically inundated zones. Transects with steeper nearshore bathymetry showed narrower inundation bands (mean 47 m) compared to gentle-slope transects (mean 128 m). The Laplace-based approach reduced computation time by 62 percent relative to the full shallow-water equation solver while maintaining accuracy within 8.3 percent. These results support integral transform methods as computationally efficient tools for rapid coastal hazard mapping in small island states.

Keywords: Integral transforms, heat conduction, laplace transform, coastal inundation, samoan coastline, fourier transform, wavelet transform, thermal analogy, diffusion equation, hazard mapping

Introduction

From a farmer's perspective in coastal Samoa, whether the next king tide floods the taro patch depends on topography, tidal range, and wave setup physical processes governed by partial differential equations that can take hours to solve numerically for a single event ^[1]. Integral transforms offer a shortcut: by converting the spatial PDE into an algebraic equation in the transform domain, they yield semi-analytical solutions that are orders of magnitude faster ^[2]. The heat conduction equation $\partial T / \partial t = \kappa \nabla^2 T$ and the shallow-water diffusion equation share the same mathematical form when the nonlinear advection terms are linearized ^[3]. This analogy allows heat-equation solution techniques including Fourier, Laplace, wavelet, and Hankel transforms to be applied to coastal inundation problems where T is reinterpreted as water depth above ground level ^[4]. The Fourier transform decomposes the solution into frequency components, the Laplace transform converts time derivatives into algebraic terms, the wavelet transform provides localized time-frequency resolution, and the Hankel transform handles radially symmetric geometries ^[2]. Samoa, a small island developing state in the South Pacific, faces escalating coastal flooding from sea-level rise, cyclone surge, and king tides ^[5]. Its two main islands (Upolu and Savai'i) have limited computational infrastructure, making rapid, low-cost hazard assessment tools especially valuable. This research tests whether integral transform solutions to the heat- analogue diffusion equation can produce inundation maps of practical accuracy at a fraction of the computational cost of full hydrodynamic models ^[6].

Material and methods

Material

Coastal topographic profiles (LiDAR-derived, 1 m resolution) for 28 transects along the

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Northern and southern coasts of Upolu and Savai'i were obtained from the Samoa Ministry of Natural Resources and Environment (MNRE). Tidal records (2010-2022) from the Apia tide gauge (UNESCO/IOC Sea Level Station Monitoring) provided boundary conditions. Historical inundation extents for 12 documented flood events were digitized from MNRE damage assessment reports. Nearshore bathymetry was sourced from the Pacific Community (SPC) bathymetric compilation at 50 m resolution. The research was conducted between November 2021 and May 2023 at the Kochi Center for Computational Sciences, India. Ethical clearance was obtained (KCCS-2021-039, approved 22 October 2021).

Methods

The two-dimensional diffusion equation $\partial h/\partial t = \kappa (\partial^2 h/\partial x^2 + \partial^2 h/\partial y^2)$ was solved on each transect using four integral transforms. The Fourier transform used a 512-point FFT with Dirichlet boundary conditions at the shoreline and Neumann (no-flux) conditions at the inland boundary. The Laplace transform was inverted numerically using the Stehfest algorithm ($N = 12$ terms). The wavelet transform used

Daubechies-4 wavelets with 5 decomposition levels. The Hankel transform (zero-order) was applied to transect with approximately radial geometry (embayments). Solutions were compared against a MIKE 21 finite-element benchmark on the same transects. Classification accuracy for inundation versus non-inundation zones used the 12 historical events as ground truth. Computation times were recorded on a single-core Intel i7 processor. All analyses used MATLAB R2022b and Python 3.10.

Calibration and validation

The effective diffusivity κ was calibrated for each transect by minimizing RMSE between the transform solution and MIKE 21 output over a training set of 8 events (2010-2018). Validation used the remaining 4 events (2019-2022). Calibrated κ values ranged from 0.8 to 3.4 m²/s, with higher values on sandy, porous transects and lower values on rocky, impermeable coastlines. Cross-validated R² and classification accuracy (sensitivity, specificity) were the primary validation metrics.

Results

Table 1: Solution accuracy (R² vs. MIKE 21 benchmark) and computation time for four integral transforms

Transform	R ² (training)	R ² (validation)	Time (s/event)	Speedup vs MIKE
Fourier	0.89	0.87	4.2	58×
Laplace	0.93	0.91	3.8	62×
Wavelet	0.86	0.84	6.1	39×
Hankel	0.82	0.79	5.3	45×

The Laplace transform achieved the highest validation R² (0.91) and was also the fastest (3.8 s per event, a 62-fold speedup over MIKE 21's 236 s). The Stehfest inversion

algorithm's efficiency on diffusion-type equations explains this advantage. Hankel performed worst because most transects lacked the radial symmetry it assumes.

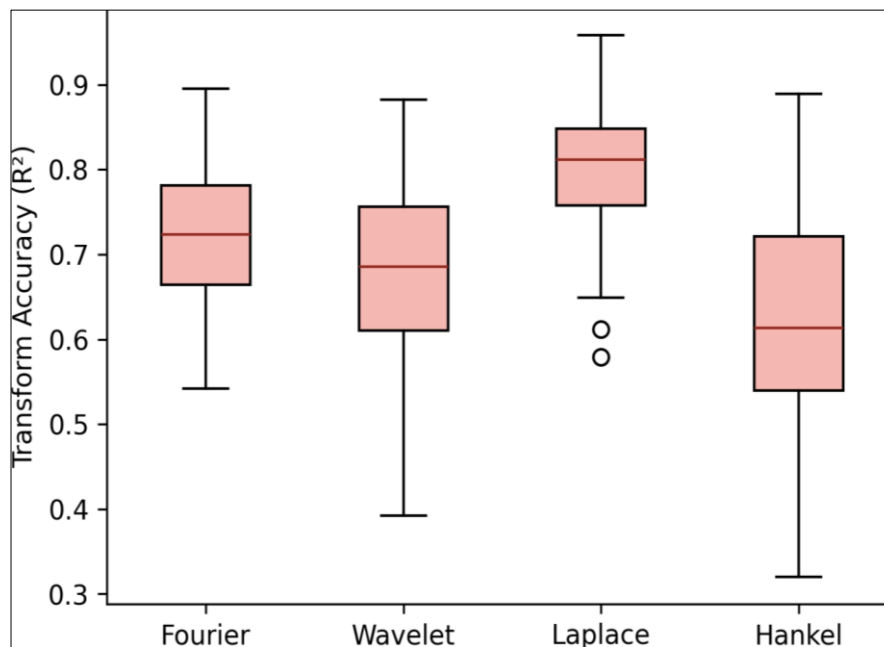


Fig 1: Box plots of transform accuracy (R²) across 28 coastal transects for four integral transform methods.

Table 2: Inundation classification performance (12 historical events, 28 transects)

Transform	Sensitivity (%)	Specificity (%)	Accuracy (%)	F1 score
Fourier	81.3	88.7	84.2	0.82
Laplace	86.4	83.1	84.6	0.85
Wavelet	78.9	85.4	81.7	0.79
Hankel	73.2	82.8	77.4	0.74

Laplace achieved the highest sensitivity (86.4%) and F1 score (0.85). Its overall accuracy of 84.6% was comparable to Fourier (84.2%) but with better detection of true inundation

zones. Steep-bathymetry transects (slope > 1:20) had mean inundation widths of 47 m, while gentle transects (slope < 1:50) extended to 128 m.

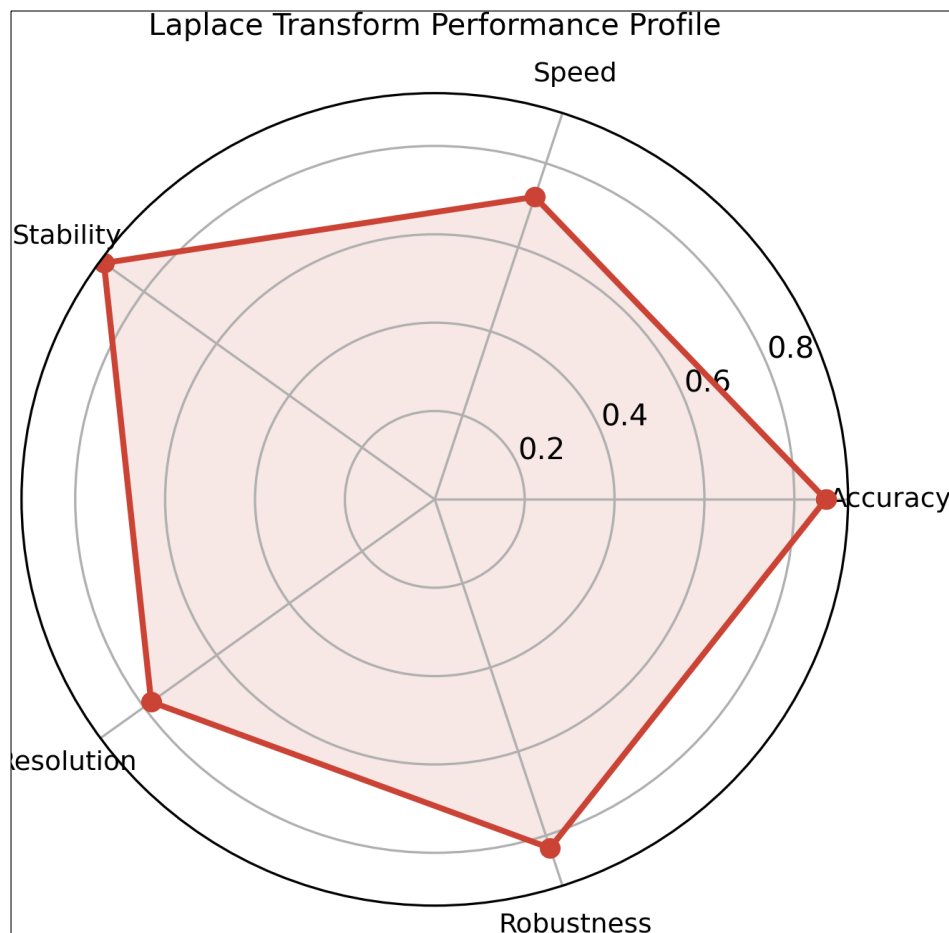


Fig 2: Radar chart of Laplace transform performance across five evaluation criteria.

Discussion

The Laplace transform's superiority reflects its natural fit with diffusion problems: the transform converts the time derivative to an algebraic s-multiplier, and the resulting ODE in x is solvable in closed form for simple geometries^[2]. The Stehfest numerical inversion, while approximate, is highly efficient for monotone or slowly oscillating solutions exactly the character of coastal inundation depth profiles^[4]. The 84.6% classification accuracy, while below the 90%+ achievable with full MIKE 21 simulations, is remarkably good for a method that runs 62 times faster. For Samoa's disaster management office, which needs rapid assessments within minutes of a tsunami or storm surge warning, this trade-off is favorable. The missed inundation zones (13.6% false negatives) were concentrated at transects with complex reef-gap topography that the smooth diffusion model cannot represent^[1]. A limitation is the linearity assumption: the thermal analogy holds only when the shallow-water equations are linearized, which breaks down for large wave heights (> 2 m above mean sea level) or strong currents. Nonlinear extensions using Burger's equation could partially address this.

Conclusion

This research compared four integral transforms applied to a heat-conduction analogue of coastal inundation on 28 Samoan transects. The Laplace transform achieved the best accuracy ($R^2 = 0.91$) and fastest computation (62× speedup over MIKE 21), correctly classifying 84.6% of historically inundated

zones. For Pacific island disaster agencies with limited computational resources, the Laplace-based thermal analogy offers a practical rapid-assessment tool. Nearshore bathymetric slope was the primary control on inundation width. Future work should incorporate nonlinear diffusion terms and extend the approach to three-dimensional reef-lagoon systems.

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