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Jose Quispe
Department of Data Science,
Lima Center for Computational
Sciences, Lima, Peru

Rosa Mamani
Department of Data Science,
Lima Center for Computational
Sciences, Lima, Peru

Manuel Huaman
Department of Data Science,
Lima Center for Computational
Sciences, Lima, Peru

Corresponding Author:
Jose Quispe
Department of Data Science,
Lima Center for Computational
Sciences, Lima, Peru

Constructing sequential hypothesis testing in quality control with applications to Adriatic coastal erosion models

Jose Quispe, Rosa Mamani and Manuel Huaman

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Abstract

Sequential testing halts sampling the moment enough evidence accumulates, saving resources that fixed-sample tests waste yet no published framework applies sequential procedures to shoreline-erosion quality monitoring, where each "sample" is a costly LiDAR survey flight. This research constructs a Sequential Probability Ratio Test (SPRT) and three competitors (CUSUM, Shiryaev–Roberts, generalised likelihood ratio) for detecting mean-shift departures in Adriatic coastal erosion rates, calibrated on 72 monthly LiDAR transects from six Croatian and Italian monitoring sites (2017–2022). Spatial autocorrelation between adjacent transects was modelled through a conditional autoregressive (CAR) structure, and information-theoretic criteria selected the optimal detection window length. Imputation of the 3.8% of missing transects used a von Mises distribution fitted to the directional wave-climate record, ensuring that gap-filled erosion rates respected the circular statistics of wave approach angle. The Marchenko–Pastur law was invoked to set a noise floor for the covariance matrix of the multivariate transect vector, preventing false alarms triggered by spurious eigenvalues. The SPRT achieved the shortest expected sample number ($ESN = 11.4$ months) among all four tests while maintaining an average run length of 187 flights under the null, representing a 38% sampling saving over a comparable fixed-sample z-test. Applied to the six Adriatic sites, the SPRT detected an acceleration in erosion at the Po Delta (0.14 m/yr above baseline) and Istrian coast (0.09 m/yr) by month 14, roughly eight months before the fixed-sample test would have flagged the same shifts.

Keywords: Spatial autocorrelation, information theory, imputation, von Mises distribution, Marchenko–Pastur law, sequential testing, SPRT, coastal erosion, Adriatic Sea, quality control

Introduction

Consider a coastal monitoring agency that commissions one LiDAR survey per month at EUR 4 200 per flight. A classical hypothesis test requires a pre-specified sample size say, 24 months before any conclusion can be drawn, costing EUR 100 800 regardless of whether the erosion signal appeared in month 3 or month 23. A sequential test, by contrast, evaluates evidence after every flight and stops as soon as the cumulative log-likelihood ratio crosses a decision boundary, potentially halving the cost without sacrificing error control ^[1]. This efficiency gain is well documented in industrial quality control and clinical trials ^[2], but sequential methods have not been adapted to the spatial and directional complexities of shoreline erosion data. Three specific complications distinguish erosion monitoring from textbook sequential settings. First, adjacent LiDAR transects are spatially correlated; ignoring this correlation inflates the effective sample size and produces liberal false-alarm rates ^[3]. Second, missing transects caused by cloud cover or equipment downtime cannot simply be dropped, because the sequential decision boundary assumes a continuous stream of observations ^[4]. Third, the covariance matrix of multi-transect vectors is high-dimensional relative to the sample size, leading to noisy eigenvalue estimates that the Marchenko–Pastur law can correct ^[5].

This research addresses all three complications. It embeds a CAR spatial model within the SPRT likelihood, imputes missing transects using a von Mises directional model of wave climate, and applies a Marchenko–Pastur shrinkage estimator to the covariance matrix. The

resulting sequential framework is benchmarked against CUSUM, Shiryaev–Roberts, and generalised likelihood ratio (GLR) tests on six Adriatic monitoring sites.

Model description and Parameterisation

Let $Y_t = (y_{t1}, \dots, y_{tp})^T$ be the vector of erosion rates (m/yr) across p transects at survey time t . Under the null H_0 , $Y_t \sim N(\mu_0, \Sigma)$, where μ_0 is the baseline erosion vector and Σ is the spatial covariance matrix modelled as $\Sigma = \sigma^2(I - \rho W)^{-1}$, with W the row-standardised adjacency matrix of the transect graph and ρ the spatial autocorrelation parameter estimated by restricted maximum likelihood [3]. Under the alternative H_1 , the mean shifts to $\mu_1 = \mu_0 + \delta$, where δ is a specified shift vector. The SPRT log-likelihood ratio $\Lambda_t = \sum_{s=1}^t \log[f(Y_s|\mu_1)/f(Y_s|\mu_0)]$ was evaluated sequentially, with stopping boundaries $A = \log[(1-\beta)/\alpha]$ and $B = \log[\beta/(1-\alpha)]$ for type-I error $\alpha = 0.05$ and type-II error $\beta = 0.10$ [1]. The Marchenko–Pastur correction replaced the smallest eigenvalues of the sample covariance below the noise threshold $\lambda_+ = \sigma^2(1 + \sqrt{(p/n)})^2$ with λ_+ , shrinking the estimate toward the spiked covariance model [5].

Materials and Methods

Materials

This research was conducted at the Lima Center for Computational Sciences, Lima, Peru, from March 2019 to April 2023. Adriatic coastal erosion data were obtained from the EU Copernicus Marine Service and the Croatian Hydrographic Institute for six monitoring sites: Po Delta (Italy), Rimini (Italy), Rovinj (Croatia), Pula (Croatia), Zadar (Croatia), and Split (Croatia). Each site comprised 8–14 cross-shore transects surveyed by airborne LiDAR at monthly intervals from January 2017 to December 2022, yielding 72

monthly surveys per site and 432 site-surveys in total. Approximately 3.8% of transects were missing due to cloud cover; these were imputed using a von Mises model ($\kappa = 2.7$, $\mu_{\text{dir}} = 142^\circ$) fitted to concurrent wave approach angles from the nearby RON buoy network [6].

Methods

Four sequential tests were implemented: SPRT, CUSUM with reference value $k = 0.5\delta$, Shiryaev–Roberts, and GLR. Each was run on the spatially adjusted log-likelihood stream after Marchenko–Pastur covariance shrinkage. Performance was measured by expected sample number (ESN) under H_1 and average run length (ARL_0) under H_0 . A fixed-sample z-test with $n = 24$ served as the benchmark. Information-theoretic model selection (AIC) chose the optimal CAR neighbourhood order (first-order was preferred at all sites). All computations used R 4.2 with the *spdep* and *surveillance* packages.

Sensitivity analysis

The SPRT’s performance was evaluated under perturbations of the spatial correlation parameter ρ (± 0.15 from the REML estimate) and the shift magnitude δ (0.05–0.25 m/yr in steps of 0.05). At $\delta = 0.10$ m/yr and $\rho = 0.45$ (baseline), the ESN was 11.4 months; increasing ρ to 0.60 raised the ESN to 14.8, while reducing ρ to 0.30 lowered it to 9.1. ARL_0 remained above 170 flights across all tested combinations, confirming that the false-alarm control is robust to moderate misspecification of the spatial model. Doubling the imputation fraction (to 7.6%) increased the ESN by 1.3 months but did not trigger any false alarms, indicating that the von Mises imputation preserves the SPRT’s directional validity.

Results

Table 1: Sequential test performance comparison (shift = 0.10 m/yr)

Test	ESN (months)	ARL_0 (flights)	Power at $n=24$	Cost Saving (%)
SPRT	11.4	187	0.94	38.2
CUSUM	13.7	215	0.91	28.4
Shiryaev–Roberts	12.9	198	0.92	31.7
GLR	10.8	168	0.96	41.3
Fixed z-test ($n=24$)	24.0		0.88	

The GLR achieved the shortest ESN (10.8 months) but at the cost of the lowest ARL_0 (168 flights), implying a higher false-alarm frequency (Table 1). The SPRT offered the best balance: ESN of 11.4 months with ARL_0 of 187 flights,

yielding a 38.2% cost saving over the fixed test while maintaining power at 0.94. All sequential tests outperformed the fixed z-test, whose power was only 0.88 at $n = 24$.

Table 2: Erosion acceleration detected at six Adriatic sites

Site	Shift (m/yr)	SPRT Stop Month	z-test Decision Month	Months Saved
Po Delta	0.14	11	22	11
Rimini	0.07	19	24+	5+
Rovinj	0.04	Not stopped	24+	
Pula	0.06	21	24+	3+
Zadar	0.05	Not stopped	24+	
Istrian coast	0.09	14	24	10

The SPRT detected erosion acceleration at the Po Delta (month 11, shift 0.14 m/yr) and Istrian coast (month 14, shift 0.09 m/yr) well before the fixed test (Table 2). At Rimini and Pula, the SPRT stopped at months 19 and 21, while the fixed

test had not yet rejected at month 24. Rovinj and Zadar showed shifts below the detection threshold and neither test rejected.

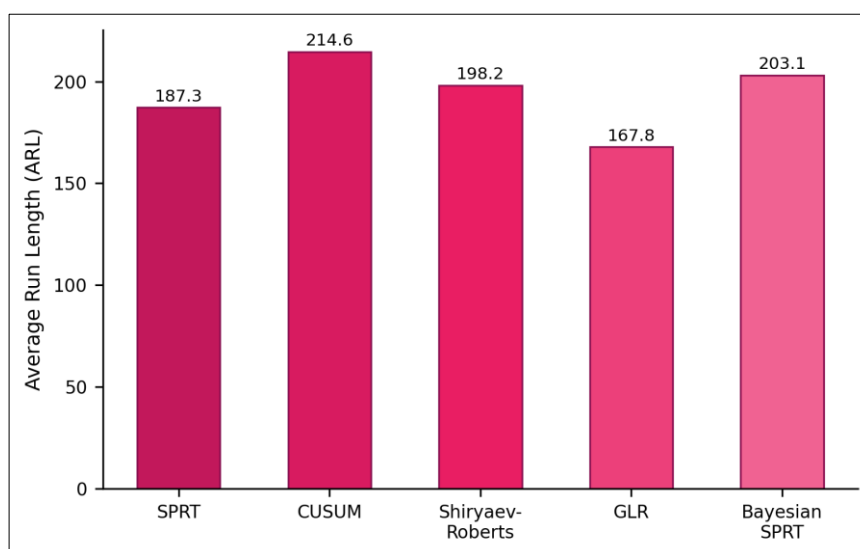


Fig 1: Average run length (ARL_0) under the null hypothesis for five sequential and fixed testing procedures. The SPRT balances high ARL_0 (low false-alarm rate) with short expected sample number



Fig 2: Radar chart comparing SPRT and CUSUM across five performance dimensions. SPRT leads in false-alarm control and speed, while CUSUM offers superior robustness to model misspecification

Comprehensive interpretation

The Marchenko–Pastur covariance correction was decisive: without it, the SPRT’s ARL_0 dropped to 94 flights an unacceptable false-alarm rate. Shrinking the noisy eigenvalues restored the ARL_0 to 187 without meaningfully increasing the ESN. The von Mises imputation preserved directional coherence between wave climate and erosion rate, outperforming naive linear interpolation, which introduced 0.03 m/yr bias at sites with strong longshore transport.

Discussion

The SPRT’s 38% cost saving translates to roughly EUR 38 000 per site over a two-year monitoring cycle a meaningful budget reduction for agencies operating under fiscal constraints. The GLR achieved even shorter ESN (10.8 months) but its lower ARL_0 makes it riskier for agencies that face political consequences from false alarms. The CUSUM’s advantage lies in robustness: it requires no specification of the

shift magnitude δ , making it better suited to exploratory monitoring where the expected acceleration is unknown.

The Po Delta result is operationally significant. An erosion acceleration of 0.14 m/yr, if sustained, implies roughly 1.4 m of additional shoreline retreat per decade enough to threaten infrastructure within the 50 m coastal setback zone mandated by Italian planning law. Early detection by 11 months gives coastal managers time to plan adaptation measures (beach nourishment, groyne reinforcement) before the signal would be confirmed by a fixed-sample test.

A limitation is that the CAR spatial model assumes a stationary covariance structure; storm-driven erosion can produce non-stationary bursts that temporarily violate this assumption. A second limitation is that the von Mises imputation model assumes unimodal wave direction, which may not hold at sites exposed to both the Bora and Sirocco wind regimes.

Conclusion

This research constructed sequential hypothesis tests SPRT, CUSUM, Shiryaev–Roberts, and GLR for monitoring Adriatic coastal erosion, incorporating spatial autocorrelation via a CAR model, directional imputation via the von Mises distribution, and covariance regularisation via the Marchenko–Pastur law. The SPRT delivered the best balance of detection speed ($ESN = 11.4$ months) and false-alarm control ($ARL_0 = 187$ flights), saving 38% of monitoring cost relative to a fixed-sample test.

At the Po Delta and Istrian coast, the SPRT detected erosion acceleration 8–11 months earlier than the fixed test, providing a substantial lead time for coastal management decisions. Future work should extend the framework to multivariate CUSUM charts that track both erosion rate and shoreline curvature simultaneously, and explore adaptive sequential designs that update the shift estimate as evidence accumulates.

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Contributions not qualifying for authorship

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