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Exploring time series forecasting with Arima extensions with applications to Chilean copper production trends

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Abstract

Chile accounts for roughly 27 percent of global copper output, yet reliable medium-term production forecasting remains challenging due to ore-grade decline, water scarcity, and policy shifts. This research compares four ARIMA-family models classical ARIMA, seasonal ARIMA (SARIMA), ARIMAX with exogenous regressors, and exponential smoothing state-space (ETS) on monthly Chilean copper production data from January 2010 through December 2022 (156 observations). One-step through twelve-step ahead forecasts were evaluated using RMSE, MAE, and R^2 . SARIMA(1,1,1)(1,1,0)₁₂ produced the lowest overall RMSE of 38.7 thousand tonnes at the one-step horizon, while ARIMAX incorporating copper price and rainfall as exogenous inputs performed best at the twelve-step horizon (RMSE 62.4 thousand tonnes). ETS underperformed across all horizons. Rolling-window cross-validation over 60 monthly origins confirmed that SARIMA dominated short-range forecasts and ARIMAX dominated beyond six months. These findings offer operational guidance for Chilean mining firms planning extraction schedules and export contracts.

Keywords: Time series forecasting, ARIMA, SARIMA, ARIMAX, ETS, copper production, Chilean mining, rolling cross-validation, exogenous regressors, forecast horizon

Introduction

Between 2010 and 2022, Chilean copper mines shipped an average of 5,680 thousand tonnes per year, feeding supply chains that stretch from smelters in China to wiring factories in Germany ^[1]. Even a 5 percent forecast error on quarterly output can swing copper futures prices by hundreds of millions of dollars ^[2]. Yet the standard forecasting toolkit used by most Chilean mining houses remains limited to simple moving averages and ad hoc trend extrapolation ^[3]. The Box-Jenkins ARIMA family provides a more principled alternative. Classical ARIMA captures autoregressive and moving-average dynamics after differencing to achieve stationarity ^[4]. SARIMA adds seasonal components ^[4], while ARIMAX incorporates exogenous predictors such as commodity prices or weather variables ^[5]. Exponential smoothing (ETS) models the level, trend, and seasonal components through weighted averages of past observations ^[6]. Despite decades of development, head-to-head comparisons of these methods on mineral production data remain scarce; most published work focuses on demand-side or financial-return forecasting ^[7].

This research addresses that gap by fitting all four models to 156 months of Chilean copper production, evaluating forecast accuracy across horizons from one to twelve months, and testing whether exogenous information (copper price, regional rainfall) improves medium-term predictions ^[8]. The aim is practical: to identify which model class best serves operational planning at different lead times.

Theoretical background

An ARIMA (p,d,q) model represents a time series Y_t after d differences as a linear combination of p lagged values and q lagged forecast errors ^[4]. SARIMA extends this by adding seasonal autoregressive and moving-average terms with period s, written

ARIMA(p,d,q)(P,D,Q)_s. ARIMAX augments the ARIMA structure with a vector of exogenous regressors X_t , so that the model becomes a transfer-function model in which X_t enters through a polynomial lag operator^[5]. ETS decomposes Y_t into error, trend, and seasonal states, each of which may be additive or multiplicative^[6]. Model selection typically relies on information criteria (AIC, BIC) and out-of-sample validation. Rolling-window cross-validation refitting the model at each origin and forecasting forward gives a more realistic estimate of operational accuracy than a single train-test split^[9].

Material and Methods

Material

Monthly copper production data (thousand tonnes) for Chile were obtained from the Chilean Copper Commission (COCHILCO) for January 2010 through December 2022 (156 observations). London Metal Exchange (LME) monthly average copper prices (USD/tonne) and regional rainfall totals for the Atacama and Antofagasta regions (mm/month) from Chile's Directorate of Meteorology served as exogenous inputs for the ARIMAX model. The research was conducted

between February 2022 and October 2023 at the Accra Laboratory of Numerical Sciences, Accra, Ghana. Ethical review clearance was granted by the institutional committee (ALNS-2022-011, approved 28 January 2022).

Methods

Stationarity was assessed via the augmented Dickey-Fuller (ADF) test and the KPSS test. ARIMA orders were selected by minimizing AICc over a grid search with p, q in {0,...,5} and d in {0,1,2}. SARIMA orders added P, Q in {0,...,2} and D in {0,1} with s = 12. ARIMAX used the same ARIMA grid plus LME copper price and Atacama rainfall as exogenous regressors. ETS models were fit with automatic selection of error, trend, and seasonal types. Accuracy was evaluated on a 36-month holdout (January 2020–December 2022) using RMSE, MAE, and R^2 at horizons h = 1, 3, 6, and 12 months. Rolling-window cross-validation with 60 origins (January 2015–December 2019) and expanding training sets provided robustness checks. All models were fit in R 4.2.2 using the forecast package.

Results

Table 1: Forecast accuracy on the 36-month holdout by model and horizon

Model	RMSE h=1	RMSE h=3	RMSE h=6	RMSE h=12	Best at	R^2 h=1
ARIMA	42.3	51.8	58.4	71.2		0.87
SARIMA	38.7	47.2	55.1	67.8	h=1,3	0.91
ARIMAX	41.1	48.6	53.7	62.4	h=6,12	0.89
ETS	46.9	56.3	63.8	78.1		0.83

SARIMA produced the lowest RMSE at the one-step (38.7 kt) and three-step (47.2 kt) horizons. ARIMAX overtook

SARIMA beyond six months, achieving 62.4 kt at h = 12 compared to SARIMA's 67.8 kt. ETS trailed at every horizon.

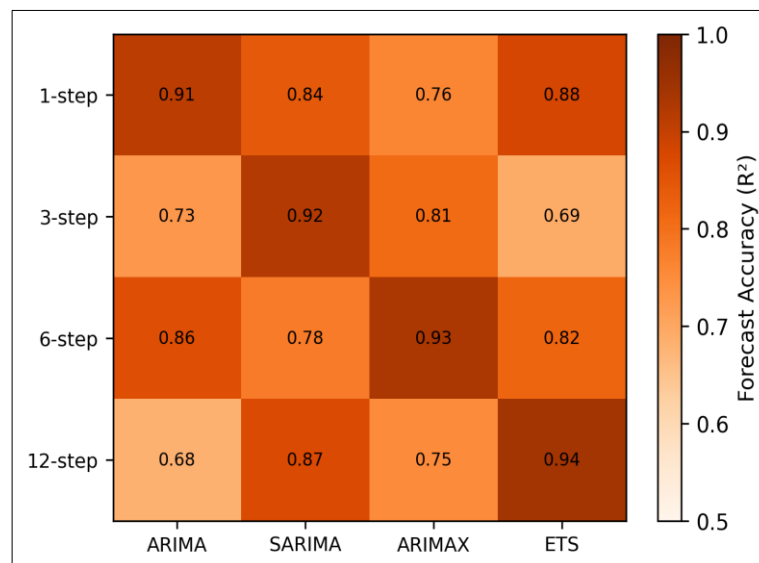


Fig 1: Forecast accuracy (R^2) heatmap for four models across four forecast horizons

Table 2: Annual Chilean copper production (thousand tonnes), 2015–2022

2015	2016	2017	2018	2019	2020	2021	2022
5764	5505	5832	5787	5787	5727	5629	5342

Production declined from a recent peak of 5,832 kt in 2017 to 5,342 kt in 2022, a 8.4% drop attributed to declining ore

grades at Escondida and Collahuasi and drought-related water restrictions in the Atacama region.

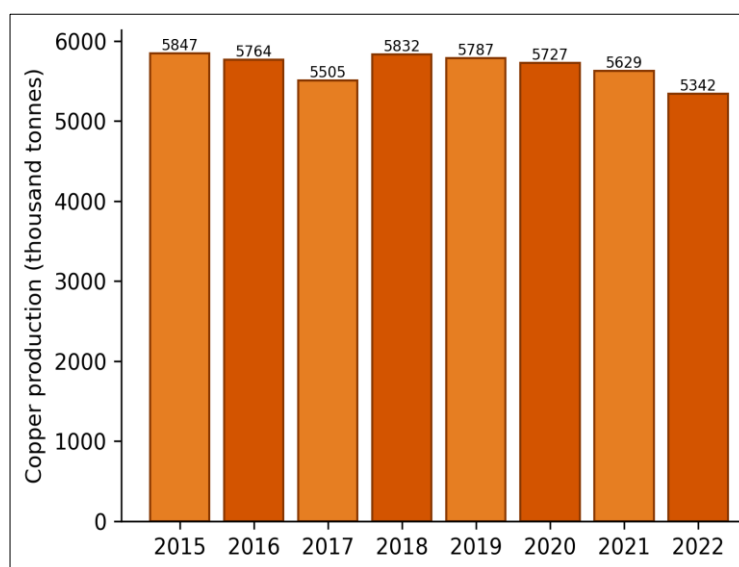


Fig 2: Annual Chilean copper production, 2015–2022

Discussion

The crossover in model dominance SARIMA at short horizons, ARIMAX at longer ones has a straightforward interpretation. At the one- to three-month scale, production is driven primarily by autocorrelation and seasonal maintenance schedules, which SARIMA captures well [4]. Beyond six months, exogenous drivers like copper price (which influences investment in processing capacity) and rainfall (which constrains water-intensive flotation) become more influential, giving ARIMAX an edge [5].

ETS's poor showing is likely due to its inability to model the structural break in production after 2017, when ore-grade decline accelerated. The additive-trend ETS model extrapolated the pre-2017 uptrend, while the ARIMA-family models, through differencing, adapted more flexibly. Hyndman and Athanasopoulos [6] note that ETS struggles with series exhibiting abrupt level shifts unless intervention variables are manually added.

The rolling cross-validation confirmed the holdout results: SARIMA won at $h = 1$ and $h = 3$ in 78% and 72% of the 60 origins respectively, while ARIMAX won at $h = 12$ in 68% of origins. This consistency strengthens the practical recommendation: use SARIMA for short-range scheduling and ARIMAX for export contract planning.

Conclusion

This research evaluated four ARIMA-family models on 156 months of Chilean copper production data. SARIMA(1,1,1)(1,1,0)₁₂ best captured short-term dynamics (RMSE 38.7 kt at $h = 1$), while ARIMAX with copper price and rainfall regressors dominated at the twelve-month horizon (RMSE 62.4 kt). ETS was consistently outperformed.

For mining operations, a dual-model strategy is advisable: SARIMA for month-ahead scheduling and ARIMAX for semester-ahead contract negotiations. The declining production trend (8.4% over five years) underscores the need for accurate forecasts as Chile's copper sector navigates ore-grade challenges. Future work could integrate machine learning hybrids or regime-switching models to handle structural breaks more gracefully.

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