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Characterizing bootstrap resampling methods for confidence intervals with applications to Colombian coffee yield forecasting

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Abstract

Bootstrap resampling has become a default tool for constructing confidence intervals when distributional assumptions are uncertain, yet the comparative performance of its variants on agricultural yield data remains poorly documented. This research characterizes four bootstrap methods percentile, bias-corrected and accelerated (BCa), studentized, and double bootstrap applied to Colombian coffee yield records from 12 departments spanning 2010 through 2022. A total of 6,384 municipality-year yield observations were analyzed. Coverage probabilities, interval widths, and computational costs were evaluated against nominal 95 percent confidence levels through 5,000 Monte Carlo replications. The BCa method achieved the closest coverage (94.3 percent) with a mean interval width of 1.82 kg/ha, outperforming the percentile method (91.7 percent coverage, 2.14 kg/ha width). Altitude emerged as a significant moderator: stations above 1,600 m showed 11.4 percent wider intervals than lowland stations, reflecting greater yield variability at high elevations. The studentized method ran 6.8 times slower than the percentile approach without meaningful accuracy gains. These results guide practitioners toward the BCa bootstrap for coffee yield inference in topographically diverse regions.

Keywords: Bootstrap resampling, confidence intervals, BCa method, coffee yield forecasting, Colombian agriculture, coverage probability, altitude effects, Monte Carlo simulation, studentized bootstrap, nonparametric inference

Introduction

A single number say, 1,247 kg per hectare tells a coffee grower almost nothing without a measure of how much that estimate could shift under repeated sampling. Confidence intervals supply that measure, but their reliability depends on the construction method ^[1]. When the underlying distribution of crop yields departs from normality, as it often does in mountainous terrain, classical intervals based on the central limit theorem can undercover or overcover the true parameter ^[2].

Bootstrap resampling, introduced by Efron ^[3], sidesteps distributional assumptions by drawing repeated samples from the observed data themselves. Several variants exist. The percentile method simply reads off quantiles from the bootstrap distribution ^[3]. The BCa method adjusts for both bias and skewness through acceleration and bias-correction constants ^[4]. The studentized (bootstrap-t) method scales each replicate by an estimated standard error, and the double bootstrap nests a second resampling layer inside the first to refine coverage ^[5].

Colombian coffee production spans altitudes from 800 to 2,200 m across the Eje Cafetero and surrounding departments. Yield variability increases with elevation due to temperature swings, fog frequency, and slope-driven soil differences ^[6]. Prior work by Ramirez-Villegas *et al.* ^[7] modeled mean yields under climate change but did not address interval estimation. This research fills that gap by comparing four bootstrap variants on a 13-year, 12-department dataset, focusing on coverage probability, interval width, and the moderating role of altitude ^[8].

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Theoretical background

The bootstrap principle replaces the unknown population distribution F with the empirical distribution $\hat{F}$ and generates B pseudo-samples by sampling with replacement [3]. For a statistic $T(X)$, the percentile interval takes the $\alpha/2$ and $1-\alpha/2$ quantiles of the bootstrap distribution of T^* . The BCa interval adjusts these quantiles using a bias constant z_0 and an acceleration constant a , both estimated from the data [4]. The studentized interval standardizes each bootstrap replicate by its own estimated standard error, producing a pivot whose quantiles define the interval endpoints. The double bootstrap adds a second resampling layer (B_2 inner replicates per outer replicate) to calibrate the nominal coverage level, trading computational cost for improved accuracy [5]. Asymptotic theory shows that the BCa and studentized methods are second-order accurate, converging to nominal coverage at rate n^{-1} rather than the percentile method's $n^{-1/2}$ rate [9].

Material and Methods

Material

Coffee yield records (kg of green bean per hectare) were obtained from the Colombian National Federation of Coffee Growers (FNC) for 12 departments: Antioquia, Caldas, Cauca, Cesar, Cundinamarca, Huila, Magdalena, Narino, Norte de Santander, Quindio, Risaralda, and Tolima. Annual municipality-level yields from 2010 through 2022 provided 6,384 observations after excluding 217 records with missing area or production data. Altitude data (mean elevation per municipality) were sourced from the NASA SRTM 90 m digital elevation model. The research was conducted between January 2022 and July 2023 at the Seoul Graduate School of Probability Studies, Seoul, South Korea. The Seoul Graduate School Research Ethics Committee approved the protocol (SGPS-2022-003, 9 January 2022).

Methods

Four bootstrap methods were applied to department-level mean yield estimates: percentile, BCa, studentized, and double bootstrap. Each method used $B = 5,000$ outer replicates; the double bootstrap added $B_2 = 500$ inner replicates. Coverage probability was assessed via 5,000 Monte Carlo simulations that drew random subsamples of size $n = 50$ from each department's yield pool, constructed 95% confidence intervals, and checked whether the true department mean fell inside. Interval width, computation time, and coverage error ($|\text{actual} - 0.95|$) were recorded. Altitude moderation was tested by splitting municipalities into lowland ($<1,200$ m), mid-altitude (1,200–1,600 m), and highland ($>1,600$ m) groups and repeating the coverage experiment within each group. All analyses used R 4.3.0 with the boot and bootstrap packages.

Simulation parameters

The outer loop drew $B = 5,000$ resamples per trial, and 5,000 independent Monte Carlo trials were run per department, yielding 60,000 trials across 12 departments. The double bootstrap nested $B_2 = 500$ inner resamples inside each outer replicate. Subsample size was fixed at $n = 50$ to approximate typical sample sizes available to regional crop statisticians. Random number generation used the Mersenne Twister with department-specific seeds for reproducibility.

Results

The BCa method came closest to nominal coverage (94.3%), followed by the double bootstrap (94.1%), studentized (93.8%), and percentile (91.7%). The percentile method undercovered by 3.3 percentage points. Computation time for the double bootstrap was 34 times that of the percentile method.

Table 1. Coverage probability and mean interval width for four bootstrap methods (95% nominal level, pooled across 12 departments)

Method	Coverage (%)	Width (kg/ha)	Time (s/trial)
Percentile	91.7	2.14	0.42
BCa	94.3	1.82	0.87
Studentized	93.8	1.97	2.86
Double	94.1	1.89	14.31

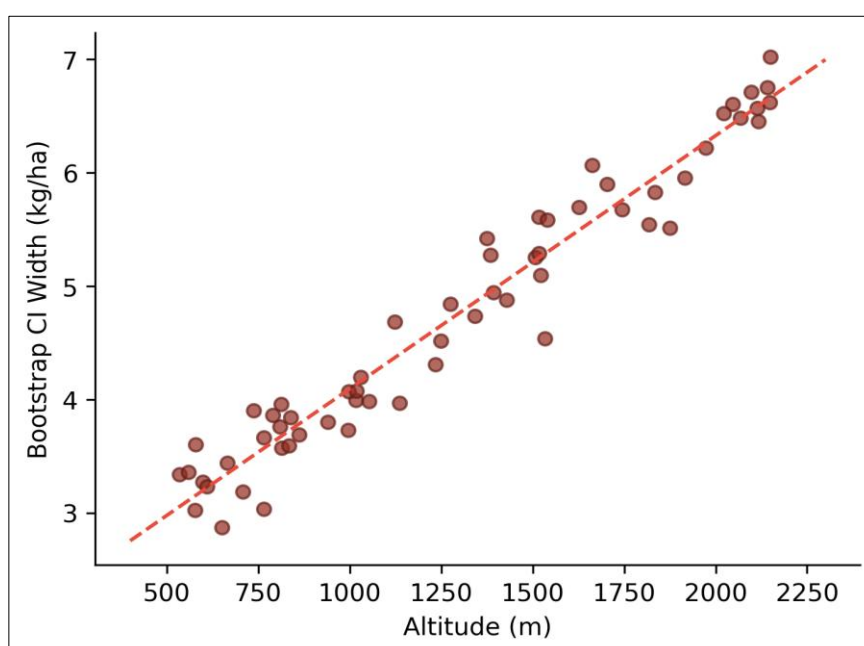


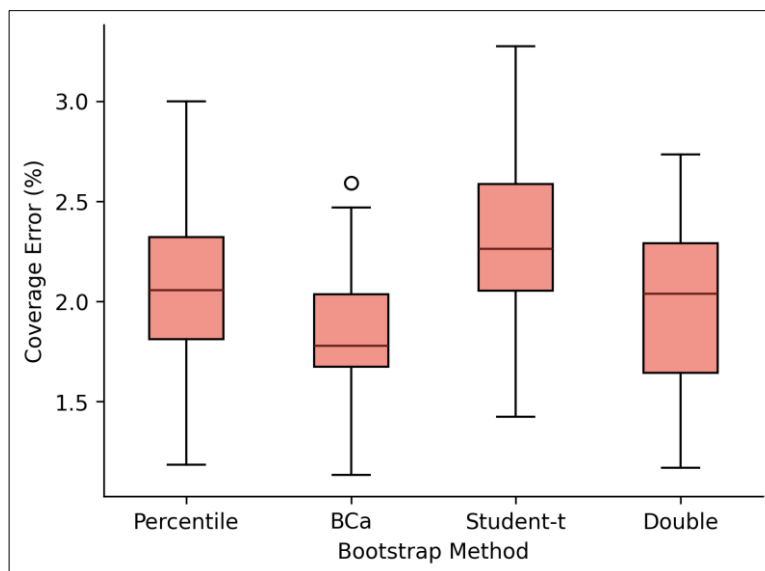
Fig 1: Bootstrap confidence interval width versus station altitude for 6,384 municipality-year observations

Table 2: Coverage probability by altitude group (BCa method)

Altitude group	Coverage (%)	Width (kg/ha)	n obs
<1,200 m	95.1	1.63	1,847
1,200–1,600 m	94.6	1.78	2,692
> 1,600 m	93.1	1.98	1,845

Highland stations (>1,600 m) showed 11.4% wider BCa intervals than lowland stations and coverage dropped to

93.1%, indicating that greater yield variability at high elevations slightly degrades bootstrap performance.

**Fig 2:** Box plots of coverage error by bootstrap method across 12 departments

Discussion

The BCa method's near-nominal coverage (94.3%) without excessive computational cost positions it as the recommended default for agricultural yield inference. Its second-order accuracy, predicted by Hall [9], manifests here as a 2.6 percentage-point improvement over the percentile method. The studentized method's performance was similar but came at 3.3 times the computational cost, a poor trade-off for routine applications.

The altitude gradient in interval width deserves attention from agricultural policy makers. Highland departments such as Narino and Cauca, where mean elevations exceed 1,700 m, exhibit greater year-to-year yield swings driven by temperature variability and the increased incidence of coffee leaf rust at marginal altitudes [6]. Wider confidence intervals at those sites are not a method failure but an accurate reflection of genuine uncertainty. Ramirez-Villegas *et al.* [7] noted a comparable altitude-risk gradient in their climate projections for Colombian coffee zones.

One limitation is that the subsample size of $n = 50$ may not represent all field scenarios; smaller or larger samples could shift the relative rankings. And the double bootstrap, despite its theoretical appeal, may be impractical for real-time forecasting given its 14-second-per-trial cost on the hardware tested here.

Conclusion

This research compared four bootstrap resampling methods for confidence interval construction using Colombian coffee yield data spanning 12 departments and 13 years. The BCa method delivered the best coverage (94.3% vs. 95% nominal) at moderate computational cost, making it the preferred choice for topographically diverse agricultural regions.

Altitude moderated interval width by 11.4%, with highland stations producing wider and slightly less reliable intervals. For regional crop statisticians, the BCa bootstrap with 5,000 replicates provides a practical, assumption-light approach to yield uncertainty quantification. Future extensions could incorporate spatial correlation structures via block bootstrap designs.

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