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First and second Zagreb indices of splitting graph of comb graph

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Abstract

In this research paper, we study the first, second Zagreb indices and Zagreb co-indices of Splitting graph of comb graph.

Keywords: Splitting graph, Zagreb index, Zagreb co-index, degree of a vertex

1. Introduction

Throughout this paper, we only consider simple and connected graph $G = (V(G), E(G))$ with n vertices and m edges. We use the notations $d(v)$, $\Delta = \Delta(G)$ and $\delta = \delta(G)$ to denote the degree of a vertex v the maximum vertex degree of G and the minimum vertex degree of G , respectively. A vertex $v \in V(G)$ is said to be pendant if its neighborhood contains exactly one vertex, i.e., $d(v) = 1$. As usual $K_{1,n-1}$, P_n and C_n denotes a star, path and cycle on n vertices, respectively. Mathematical calculations are quite necessary to analyze essential concepts in chemistry because molecules are often modeled by graph structures. Type equation here. named molecular graphs. A molecular graph is a simple graph in which vertices represent the atoms and edges represent the linking bonds among them. By IUPAC terminology, a topological index is a numerical value for the correlation of a chemical structure with various physical properties, chemical reactivity, or biological activity. In chemical graph theory, a graphical invariant is a number related to a graph which is structurally invariant. These invariant numbers are also known MATH. REPORTS 24(74) (2022), 4, 637-642 638 S. Elumalai and T. Mansour 2 as the topological indices. In 1972, Gutman and Trinajstić examined the study of total π -electron energy on the molecular structure and introduced two vertex-degree based graph invariants. The graphical invariants - first, second Zagreb indices was introduced in (1) as

$$M_1(G) = \sum [d(u)]^2, u \in V(G), M_2(G) = \sum [d(u)d(v)], uv \in E(G)$$

Zagreb co- indices are defined in [2]. The first, second Zagreb co-indices are defined as

$$\overline{M}_1(G) = \sum [d(u) + d(v)], uv \notin E(G)$$

$$\overline{M}_2(G) = \sum [d(u)d(v)], uv \notin E(G).$$

1.2 Splitting graph of a graph: For each vertex $u \in V(G)$, take a new vertex (duplicate) u' and denote the resulting set of vertices by $V'(G)$. The splitting graph $S(G)$ of a given graph G is defined as the graph having the vertex set $V(G) \cup V'(G)$ with two vertices adjacent if they correspond to adjacent vertices of G or one correspond to a vertex u' of V' and the other to a vertex w of G where w is in $N_G(u)$ [9].

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For all terminologies and notations that are not defined here, refer to F.H. Harary [5]. Here we find the Zagreb indices, Zagreb co-indices of splitting graph of some class of graphs. A *Tadpole* $T_{m,n+1}$ is a unicyclic graph on $m+n$ vertices consisting of a cycle C_m , such that removal of all the vertices of C_m from it results a path P_n .

$G_1 \odot G_2$ of G_1 and G_2 is the graph obtained by taking one copy of G_1 (which has n_1 vertices) and n_2 copies of G_2 and then joining i th vertex of G_1 to every vertex in the i th copy of G_2 .

$P_n \odot K_1, n \geq 3$ is called *comb graph*.

1.3 Remarks

1. A vertex pairs $\{u, u'\}$ are not adjacent in $S(G)$.
2. $\langle u'(G) \rangle$ is a null graph.
3. $\deg_G(u_i) = \deg_{S(G)}(u_i')$.
4. $\deg_{S(G)}(u_i) = 2\deg_{S(G)}(u_i')$

2. Main Results

Theorem 2.1: For the splitting graph of comb graph $G \cong P_n \odot K_1$,

1. $M_1(S(G)) = 50(n-1)$
2. $M_2(S(G)) = 96n - 136$.

Proof: Let $v_i, 1 \leq i \leq n$ be the vertices of P_n and a_i are pendent vertices that are adjacent to v_i . v_i', a_i' be the corresponding duplicate vertices. In $S(P_n \odot K_1)$, there are $(n-2)$ vertices of degree 6, degree 3 and 2 vertices of degree 4, degree 2. Further there are n vertices of degree 2, degree 1.

Thus $M_1((S(P_n \odot K_1))) = 50(n-1)$

(ii). The result can be proved by calculating the product of degrees between the adjacent vertices $\{u, v\}$. The following cases arises. For $G \cong P_n \odot K_1$

Case 1: $u, v \in V_{S(G)}$

Case 2: $u \in V_{S(G)}, v \in V'_{S(G)} (1 \leq i \neq j \leq n)$

With the above two cases,

$$M_2(G) = \sum_{u, v \in V(S(G))} d(u)d(v) + \sum_{u \in V(S(G)), v \in V'(S(G))} d(u)d(v) \quad (6)$$

Case 1: There are $(n-3)$ pairs of edges $\{u, v\}$ ($u, v \neq 1, n$) with degree 6. Further there are $(n-2)$ pairs of edges $\{u, v\}$ with $\deg(u)=6, \deg(v)=2$ and two pairs of edges with degrees 4, 2 and 6, 4.

$$\text{Thus } \sum_{u, v \in V(S(G))} d(u)d(v) = 48n - 68.$$

Case 2. There are $(2n-6)$ pairs of edges $\{u, v\}$ such that $\deg(u)=6, \deg(v)=3$ and two pairs of edges with degrees 4, 3; 6, 2; 4, 1 and 2, 2. Moreover there are $(n-2)$ pairs of edges with degrees 2, 3 and 6, 1.

$$\text{Thus } \sum_{u \in V(S(G)), v \in V'(S(G))} d(u)d(v) = 48n - 68.$$

By adding the above (6) becomes $M_2(S(G)) = 96n - 136$

Theorem 2.2: For the splitting graph of comb graph $G \cong P_n \odot K_1$

1. $\overline{M}_1(S(G)) = 48n^2 - 82n + 92$
2. $\overline{M}_2(S(G)) = 55n^2 - 51n - 465$.

Proof: The result can be proved by calculating the sum (product) of degrees between the non-adjacent vertices u, v in the following cases.

1. $u, v \in V$
2. $u, v \in V'$
3. $u \in V, v \in V'$
4. $u, v \in A$
5. $u, v \in A'$
6. $u \in A, v \in A'$
7. Among the vertices of cycle and path

Case 1: Let $u = v_1$ is of degree 4 and is not adjacent to $(n-3)$ vertices of degree 6 and one vertex of degree 4. Similar is the case with v_n . The vertices $u = v_2, v_{n-1}$ of degree 6 are not adjacent to $(n-4)$ vertices of degree 6 and a vertex of degree 4. Further each $v_i, 3 \leq i \leq n-2$ is not adjacent to $(n-5)$ vertices of degree 6 and two vertices of degree 4 and there are $(n-4)$ such vertices. Here all vertices are counted twice.

$$\text{Thus } \sum_{u, v \in V} d(u) + d(v) = \frac{1}{2}((n-4)[(n-5)(6+6) + 2(6+4)]) = 6n^2 - 54n + 130 \quad (2.1)$$

$$\sum_{u, v \in V} d(u)d(v) = (n-4)(18n-66) \quad (2.11)$$

Case 2: For $u = v_1', v_n'$ of degree 2 there are $(n-2)$ vertices of degree 3 and one vertex of degree 2 as v . Remaining $(n-2)$ vertices of degree three are mutually non adjacent.

$$\text{Thus } \sum_{u, v \in V'} d(u) + d(v) = 3n^2 - 10n + 8 \quad (2.2)$$

$$\sum_{u, v \in V'} d(u)d(v) = \frac{1}{2}(9n^2 - 33n + 30) \quad (2.12)$$

Case 3: (Between $v_i s, v_i' s$): Here $u = v_1$ of degree 4, is not adjacent to v_1', v_n' of degree 2, $(n-3)v_i' s (i \neq 1, n)$ of degree 3. Further v_2 of degree 6 is not adjacent to v_n' of degree 2, $(n-3)v_j' s$ of degree 3. Similar is for the vertex v_{n-1} . Further $v_i (3 \leq i \leq n-2)$ is not adjacent to v_1', v_n' of degree 2 and $(n-4)v_i'$ of degree 3 and there are $n-4$ such $v_i s$.

$$\text{Thus } \sum_{u \in V, v \in V'} d(u) + d(v) = 9n^2 - 24n + 24 \quad (2.3)$$

$$\sum_{u \in V, v \in V'} d(u)d(v) = 9n^2 + 4n - 44 \quad (2.13)$$

Case 4: Each a_i is of degree 2 and are not mutually adjacent.

$$\text{Thus } \sum_{u, v \in A} d(u) + d(v) = 2n(n-1) \quad (2.4)$$

$$\sum_{u, v \in A} d(u)d(v) = 2n(n-1) \quad (2.14)$$

Case 5: As all a_i' of degree 1 are mutually non adjacent and there are n such vertices.

$$\text{Thus } \sum_{u,v \in A'} d(u) + d(v) = n(n-1) \quad (2.5)$$

$$\sum_{u,v \in A'} d(u)d(v) = \frac{n(n-1)}{2}. \quad (2.15)$$

Case 6: Here $u = a_i$ ($1 \leq i \leq n$) of degree 2 are mutually non adjacent to $v = a_i'$ ($1 \leq i \leq n$) of degree 1.

$$\text{Thus } \sum_{u \in A, v \in A'} d(u) + d(v) = 3n^2 \quad (2.6)$$

$$\sum_{u \in A, v \in A'} d(u)d(v) = 23n^2 - 16n - 36 \quad (2.16)$$

Case 7: We can find $\sum d(u) + d(v)$ in the following four sub cases.

Sub Case 7.1: The vertices v_1, v_n of degree 4 are not adjacent to $n-1$ a_i s of degree 2. Remaining v_i s of degree 6 are not adjacent to $(n-1)$ a_i s of degree 2.

$$\text{Thus } \sum_{u \in V, v \in A} d(u) + d(v) = 16n^2 - 46n + 74. \quad (2.7)$$

$$\sum_{u \in V, v \in A} d(u)d(v) = 32n^2 - 32n - 432 \quad (2.17)$$

Sub case 7.2 (between v_i' s, a_i s): For $i = 1$, a_i of degree 1 is not adjacent to $(n-2)$ v_i' s of degree 3, one v_i' of degree 2. Similar is for a_n . Remaining a_i s, $(2 \leq i \leq n-1)$ are not adjacent to two v_i' s of degree 2 and $(n-3)$ v_i' of degree 3. There are $(n-2)$ such a_i' s.

$$\text{Thus } \sum_{u \in V', v \in A} d(u) + d(v) = 5n^2 + 7n - 12 \quad (2.8)$$

$$\sum_{u \in V', v \in A} d(u)d(v) = 6n^2 - 10n + 4 \quad (2.18)$$

Sub case 7.3 (between v_i s, a_i' s): Here v_1 of degree 4 is not adjacent to $(n-1)$ a_i' s of degree 1. There are $(n-2)$ duplicate vertices v_i' ($i \neq 1, n$) of degree 6 that are not adjacent to $(n-1)$ a_i' s of degree 1.

$$\text{Thus } \sum_{u \in V, v \in A'} d(u) + d(v) = 7n^2 - 11n + 4 \quad (2.9)$$

$$\sum_{u \in V, v \in A'} d(u)d(v) = 6n^2 - 10n + 4 \quad (2.19)$$

Sub Case 7.4: The vertices v_1', v_n' of degree 2 are non adjacent to a_i' that are n in number. Remaining $(n-2)$ v_i' s of degree 3 are not adjacent to a_i' that are n in number.

$$\text{Thus } \sum_{u \in V', v \in A'} d(u) + d(v) = 8n^2 - 8n + 2. \quad (2.1)$$

$$\sum_{u \in V', v \in A'} d(u)d(v) = 8n^2 - 13n + 7 \quad (2.20)$$

By adding all the above 2.1-2.10 we get

$$\overline{M}_1(S(G)) = 48n^2 - 82n + 92.$$

By adding all the above 2.11-2.20 we get

$$\overline{M}_2(S(G)) = 55n^2 - 51n - 465.$$

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