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Fixed value theorem and its implications

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Abstract

Fixed value theorem is a central area in modern arithmetic with significant applications in investigation, differential equations, optimization, and applied sciences. Yet rigorous overview of fixed-point theorem, its, mathematical formulation, and practical applications. Several illustrative examples are provided to demonstrate the effectiveness of iterative methods in approximating fixed points.

Keywords: Fixed-point theorem, fixed point iteration, iterative methods, differential equations, optimization, mathematical analysis, convergence analysis, applied mathematics

1. Introduction

The fixed point theorem studies conditions under which a mapping $f: X \rightarrow X$ has a point $x \in X$ such that $f(x) = x$, known as a fixed point. These points represent equilibrium states in many systems. A key result in this field is the Banach Fixed Point Theorem, which guarantees the existence and uniqueness of a fixed point in complete metric spaces.

2. Basic Definitions

Definition 1 (Measured Space)

Ametria interplanetary (X, d) is a set X with a coldness purpose d such that

$$d(x, y) \geq 0, d(x, y) = 0 \leftrightarrow x = y$$

$$d(x, y) = d(y, x) \quad d(x, z) \leq d(x, y) + d(y, z)$$

Definition 2 (Fixed Point)

X is a fixed point if $f(x)=x$.

Definition 3 (Contraction Mapping)

Contractive $(f: X \rightarrow X)$ is defined as:

$$d(f(x), f(y)) \leq k \cdot d(x, y), 0 < k < 1$$

1. Banach's Constant Pointer Theorem. Theorem

Let (X, d) be complete and $f: X \rightarrow X$ contraction. Then:

1. Exists an exclusive fixwd point $x^* \in X$
2. he arrangement: $x_{n+1} = f(x_n)$ congregates to (x^*)
3. Illustrative Example

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Example 1: Iterative Method

Solve:

$$x = \cos(x)$$

Start with $x_0=1$

$$x_1 = \cos(1) = 0.5403$$

$$x_2 = \cos(0.5403) = 0.8575$$

$$x_3 = \cos(0.8575) = 0.6543$$

$$x_4 = \cos(0.6543) = 0.7935$$

$$x_n \rightarrow 0.739$$

Conclusion

The arrangement touches to the static opinion.

Example 2 (Contraction Mapping)

$$\text{Let } f(x) = \frac{x+2}{3}.$$

Solve $x = f(x)$

$$x = \frac{x+2}{3} \rightarrow 3x - x = 2 \rightarrow x = 1$$

Check contraction:

$$|f(x)-f(y)| = |(x+2)/3 - (y+2)/3| = |x-y|/3$$

$$\text{So } k = 1/3 < 1 \rightarrow \text{contraction}$$

Conclusion: Unique fixed point exists and equals $x=1$ **Example 3 (Graphical Interpretation)**Consider $f(x) = x^2$

solve:

$$x = x^2 \rightarrow x(x-1) = 0 \quad x = 0, x = 1$$

Fixed point occur at intersections of: $y = f(x)$ and $y = x$ **2. Additional Example (Advanced)****Example4: Nonlinear Equation**Solve: $x = e^{-x}$

Using iteration:

$$x_{n+1} = e^{-x_n} \text{ starting with } [x_0 = 0, x_1 = 1, x_2 = 0.367, x_3 = 0.692, x_4 = 0.500 \dots x \approx 0.567]$$

Solve: $x = e^{-x}$

Using iteration:

$$x_{n+1} = e^{-x_n} \text{ starting with } [x_0 = 0, x_1 = 1, x_2 = 0.367, x_3 = 0.692, x_4 = 0.500 \dots x \approx 0.567]$$

2. Application**2.1 Differential Equations**

Ensure existence and uniqueness.

2.2 Optimization

Iterative methods for solving nonlinear problems.

2.3 Economics

Equilibrium models (Nash equilibrium).

2.4 Computer Science

Recursive algorithms and machine learning.

2.5 Image Processing

Iterative image enhancement and filtering.

3. Conclusion

Secure opinion philosophy is a powerful mathematical framework that provides both theoretical guarantees and practical methods for solving complex problems. Through contraction mappings and iterative schemes, it enables efficient approximation of solutions across various disciplines.

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