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Comparative analysis between symmetric and asymmetric dynamic conditional correlation multivariate GARCH modeling of macro-financial variables in Nigeria

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Abstract

The interconnected dynamics of oil prices, exchange rates, stock market performance and inflation are central to Nigeria's economic stability. This study examined the dynamic interrelationships among these macro-financial variables to enhance understanding of risk transmission and policy effectiveness in Africa's largest economy. Specifically, the research models time-varying volatility spillovers and conditional correlations, testing for asymmetric effects whereby negative shocks may exert a stronger influence on market co-movements than positive ones. Using both symmetric and asymmetric Dynamic Conditional Correlation Multivariate GARCH (DCC-GARCH and ADCC-GARCH) models under multivariate normal, Student's t, and Laplace distributions, the study analyzes monthly data from 2003 to 2024 to capture complex volatility dynamics. Model selection criteria indicated the superiority of the symmetric DCC specification with a student t distribution, as evidenced by the lowest Akaike (AIC) and Bayesian (BIC) information criteria. Results show significant volatility persistence and heavy-tailed distributions across all variables, yet dynamic correlations among them are generally weak and symmetric, with no statistically significant evidence of asymmetric leverage effects in correlation dynamics. The strongest positive correlation was observed between oil prices and stock returns, whereas inflation exhibited minimal connectivity with other variables. The study concludes that the parsimonious symmetric DCC model adequately captures the interdependence structure, offering valuable insights for policymakers and investors by highlighting the importance of dynamic frameworks for risk assessment and portfolio management in Nigeria's oil-dependent economy.

Keywords: Asymmetry, dynamic conditional correlation, multivariate GARCH, macro-financial variables, symmetry

1. Introduction

The dynamic interrelationship between macroeconomic indicators and financial market variables constitutes a fundamental area of inquiry in financial econometrics, particularly within emerging economies characterized by inherent volatility and structural vulnerabilities. Nigeria, as Africa's largest economy, presents a compelling case study due to its oil-dependent fiscal structure, managed exchange rate regime, and evolving capital market dynamics. The persistence of volatility clustering and cross-market dependencies among key variables exchange rates, oil prices, stock market indices, and inflation necessitates advanced modeling approaches that capture both time-varying correlations and asymmetric responses to shocks. The methodological evolution from univariate GARCH specifications to multivariate frameworks, particularly the Dynamic Conditional Correlation (DCC-GARCH) model and its asymmetric extension (ADCC-GARCH), provides robust tools for examining these complex interactions while addressing the leverage effect phenomenon whereby negative innovations exert a disproportionate impact on volatility compared to positive shocks of equal magnitude.

Empirical investigations into Nigeria's macro-financial nexus have employed diverse methodological approaches, yielded substantive insights while revealed persistent limitations. Domestic studies by Adeosun *et al.* (2023)^[2] and Eze *et al.* (2022)^[11] utilizing symmetric DCC-GARCH frameworks demonstrate significant volatility transmission between oil prices, exchange rates, and equity returns, confirming oil price shocks as a primary transmitter of financial volatility. Udegbonam and Eriki (2021)^[22] advanced this literature through asymmetric GJR-GARCH specifications, documenting stronger volatility spillovers from negative shocks across financial variables. Complementary research by Obi and Uddin (2023) incorporates inflation within time-frequency domain analyses, while Ogbonna *et al.* (2021)^[18] expands the variable set to include external reserves. Internationally, methodological innovations by Kumar *et al.* (2024)^[12] and Yarovaya *et al.* (2020)^[23] demonstrate the superior performance of ADCC models in capturing asymmetric connectedness in emerging markets, while theoretical foundations by Engle (2021)^[9] and Caporin and McAleer (2022)^[5] provide rigorous justification for multivariate GARCH implementations. Pan-African studies by Lawal *et al.* (2024)^[13] further establish the significance of global financial shocks and commodity price movements in driving domestic financial volatility.

Notwithstanding these contributions, significant methodological and contextual gaps persist within the extant literature. Current research on Nigeria remains fragmented through limited variable inclusion particularly the exclusion of inflation from multivariate volatility frameworks and predominant reliance on symmetric specifications that implicitly assume uniform responses to positive and negative innovations. This constitutes a substantive limitation given Nigeria's demonstrated susceptibility to asymmetric shocks from oil price collapses, currency depreciations, and inflationary pressures. Furthermore, while international studies provide robust evidence of asymmetric volatility transmission in emerging markets, their findings are not directly transferable to Nigeria's unique economic structure and policy environment. This study therefore addresses these gaps through the implementation of both symmetric (DCC) and asymmetric (ADCC) multivariate GARCH frameworks to model the dynamic conditional correlations and volatility spillovers among exchange rates, oil prices, stock market returns, and inflation in Nigeria. The specific research objectives are threefold: to investigate the presence and magnitude of asymmetric volatility responses across these variables; to quantify the time-varying conditional correlations and volatility spillovers using both DCC and ADCC methodologies; and to evaluate the statistical superiority and economic significance of asymmetric specifications for risk management and policy formulation in Nigeria's macro-financial system.

2. Methodology

2.1 Source of Data and Variables of the Study

This study used secondary data from two reliable sources: the World Bank's statistical database (<https://data.worldbank.org/indicator/FP.CPI.TOTL.ZG>) and the Central Bank of Nigeria's official website (<https://www.cbn.gov.ng/>). Four key macroeconomic indicators were analyzed in the study: crude oil prices (OP), inflation rates (INF), exchange rates (EXR), and stock market performance (SE). Strong temporal coverage for volatility analysis was ensured by the dataset's 21-year duration, which included monthly observations from January 2003 to December 2024. The R programming version 4.4.2 was used for all econometric calculations and statistical tests, allowing for a thorough analysis of the time-series characteristics and dynamic interactions between the variables.

2.2 Statistical technique for data analysis

2.2.1 Return Series

In order to estimate the variables' volatility, we first convert the monthly raw data into returns series. Equations 1 through 4 illustrate how to carry out this transition by dividing the current value by its previous value using the natural logarithm.

$$EXRR_t = \ln\left(\frac{EXR_t}{EXR_{t-1}}\right) \quad (1)$$

$$OPR_t = \ln\left(\frac{OP_t}{OP_{t-1}}\right) \quad (2)$$

$$SER_t = \ln\left(\frac{SE_t}{SE_{t-1}}\right) \quad (3)$$

$$INFR_t = \ln\left(\frac{INFLA_t}{INFLA_{t-1}}\right) \quad (4)$$

Where EXRR - exchange rate returns; OPR - oil price returns; SER - stock exchange returns; INFR - inflation returns.

2.2.2 Unit Root Test

An augmented Dickey Fuller (ADF) (Dickey & Fuller, 1981) was employed to test the stationarity of the data. The ADF model for the exchange rate (EXR), oil prices (OP), stock exchange (SE) and inflation (INF) are given as follows:

$$\Delta EXR_t = aEXR_{t-1} + x'_t\delta + B_1\Delta EXR_{t-1} + B_2\Delta EXR_{t-2} + \dots + B_p\Delta EXR_{t-p} + v_t \quad (5)$$

$$\Delta OP_t = aOP_{t-1} + x'_t\delta + B_1\Delta OP_{t-1} + B_2\Delta OP_{t-2} + \dots + B_p\Delta OP_{t-p} + v_t \quad (6)$$

$$\Delta SE_t = aSE_{t-1} + x'_t\delta + B_1\Delta SE_{t-1} + B_2\Delta EXR_{t-2} + \dots + B_p\Delta EXR_{t-p} + v_t \quad (7)$$

$$\Delta INFLA_t = aINFLA_{t-1} + x'_t\delta + B_1\Delta INFLA_{t-1} + B_2\Delta INFLA_{t-2} + \dots + B_p\Delta INFLA_{t-p} + v_t \quad (8)$$

Where x_t is an optional exogenous regression that may consist of a constant or a constant and a trend.

The ADF t-test of the null hypothesis states the following:

$H_0: \theta = 0$, implying that the data need to be different to make them stationary. In contrast to the alternative hypothesis of $H_1: \theta < 0$, the data tend to be stationary and need to be analysed by means of using a time trend in the regression model instead of differencing the data.

The test statistic is the conventional-to-t ratio for θ :

$$t_\theta = \frac{\hat{\theta}}{se(\hat{\theta})} \quad (9)$$

Where $\hat{\theta}$ is the estimate of θ and where $se(\hat{\theta})$ is the coefficient standard error.

2.2.3 Univariate GARCH Model

To model time series data exhibiting volatility clustering, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is widely used (Somarajan *et al.*, 2019)^[20]. An extension of the ARCH model, the univariate GARCH model specifies that the conditional variance at a given time is a function of both past squared error terms and its own lagged conditional variances. Given a time series y_t the univariate GARCH model is given by:

$$y_t = \mu_t + \varepsilon_t, \varepsilon_t = \sigma_t Z_t, Z_t \sim iidN(0,1) \quad (10)$$

Where y_t is the observed financial data, μ_t is the expected value or mean of the observed returns or financial data, and σ_t is the conditional variance representing the volatility process specified by:

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \quad (11)$$

Where $\omega > 0$ is a constant term, $\alpha_i \geq 0$ represents the impact of past squared errors (ARCH term with order of p), $\beta_j \geq 0$ represents the impact of past conditional variances (GARCH term with order of q) and ε_t is the error term at time t .

2.2.4 Multivariate GARCH Models

Univariate GARCH models capture the volatility of a single time series but fail to account for interdependencies between assets. Multivariate GARCH (MGARCH) models addressed this by simultaneously modeling the volatility of each series and their dynamic correlations, enabling better risk assessment. Major MGARCH specifications include the conditional covariance models VECM (Lütkepohl, 2004)^[15] and BEKK (Engle & Kroner, 1995)^[10], and the conditional correlation models CCC (Bollerslev, 1990)^[4] and DCC (Engle, 2002)^[8] (Stelzer, 2008; Nelson, 1991)^[21, 16]. This study employed the the symmetric and asymmetric DCC model in modeling exchange rate, oil price, stock exchange index and inflation return.

The symmetric Dynamic Conditional Correlation MGARCH Model

The dynamic conditional correlation MGARCH model preserves the simplicity of the univariate GARCH model for each return series while permitting time-varying correlations across multiple time series. The modeling of correlations and conditional variances is carried out separately in the DCC-MGARCH model. It uses univariate GARCH models to represent the conditional variances initially, and a different procedure to explain the dynamic correlations (Likassa *et al.*, 2025)^[14].

Let y_t be an $k \times 1$ vector of return series, where $k = 1, 2, \dots, K$. The DCC- GARCH model can be expressed in two steps: The first step is to model the conditional variances for the i^{th} time series, which is governed by a univariate GARCH model:

$$\sigma_{i,t}^2 = \omega_i + \sum_{j=1}^p \alpha_{ij} \varepsilon_{i,t-j}^2 + \sum_{k=1}^q \beta_{ik} \sigma_{i,t-k}^2 \quad (12)$$

Where $\sigma_{i,t}^2$ is the conditional variance of the i^{th} time series, where p is denoting ARCH effects and q is denoting the GARCH parameters.

Then the second step is to model the dynamic correlations, where the conditional covariance matrix is defined as follows:

$$H_t = D_t M_t D_t \quad (13)$$

Where $D_t = \text{diag}(\sigma_{1,t}, \sigma_{2,t}, \dots, \sigma_{n,t})$ is the diagonal matrix of conditional standard deviations, and M_t is the time-varying correlation matrix. The correlation matrix M_t is updated as:

$$M_t = (Q_t^*)^{-1} Q_t (Q_t^*)^{-1} \quad (14)$$

$$Q_t = (1 - a - b) \bar{Q} + a(z_{t-1} z_{t-1}') + b Q_{t-1} \quad (15)$$

Where $\bar{Q} = \text{cov}(z_t z_t') = E(z_t z_t')$ is a $T \times n$ unconditional correlation matrix of standardized error z_t where $z_t = D_t^{-1} \varepsilon_t$, $Q_t^* = \text{diag}(\sqrt{\sigma_{1,t}}, \sqrt{\sigma_{2,t}}, \dots, \sqrt{\sigma_{n,t}})$ and a and b are non-negative parameters that determine the impact of past shocks and correlations on current correlations. As in Engle (2002)^[8], Q_t can be estimated by:

$$\bar{Q}_t = \frac{1}{T} \sum_{t=1}^T z_t z_t'. \quad (16)$$

In general, as in Engle (2002)^[8], the DCC-GARCH model is given by:

$$Q_t = (1 - \sum_{m=1}^M a_m - \sum_{n=1}^N b_n) \frac{1}{T} \sum_{t=1}^T z_t z_t' + \sum_{m=1}^M a_m z_{t-1} z_{t-1}' + \sum_{n=1}^N b_n Q_{t-1} \quad (17)$$

This model is estimated using a two-step approach: the first implies the estimation of univariate GARCH, and the second step is the correlation estimation. The number of the estimated parameters in the correlation process of DCC-GARCH is independent of the number of series correlated, hence allowing for a potentially large correlation matrices to be feasibly estimated. The limitation of this model is to hypothesis the same correlation dynamics of all the assets (Billio *et al.*, 2006; Otranto, 2010)^[3, 19].

The Asymmetric Dynamic Conditional Correlation Model

Cappiello *et al.* (2006)^[6] introduced an aDCC model to investigate whether conditional variances, covariances, and correlations of assets in a portfolio are sensitive to the sign of past innovations. Compared to the DCC model, this model further explores whether the positive and negative shocks are of the same magnitude or have different impacts. As in equation (13), the matrix H_t is decompose into:

$$H_t = D_t M_t D_t \quad (18)$$

The conditional variances, H_t is assumed to follow the GJR-GARCH models,

$$\sigma_{i,t}^2 = \omega_i + \sum_{i=1}^p (\alpha_i + \gamma_i I_{i,t-j}) \varepsilon_{i,t-j}^2 + \sum_{i=1}^q \beta_i \sigma_{i,t-i}^2 \quad (19)$$

Where $I_{i,t-j} = 1$ if $\varepsilon_{i,t-1}^2 < 0$ (a negative shock) and $I_{i,t-1} = 0$ otherwise. γ_i represents the asymmetric leverage effect or news impact parameters for asset i . In this model, equation (15) can be extended to consider the asymmetries, which can be written as

$$Q_t = (\bar{Q} - A' \bar{Q} A - B' \bar{Q} B - G' \bar{N} G) + A' z_{t-1} z_{t-1}' A + B' Q_{t-1} B + G' \eta_{t-1} \eta_{t-1}' G \quad (20)$$

Where $\bar{N} = E[z_t z_t']$ and $\eta_t = I_t \{z_{t-1} < 0\} \odot z_t$. The $\odot$ symbol is the element-by-element product of the residuals if shocks are negative, and $\eta_t = 0$, otherwise. The asymmetric term is denoted by a symbol G , which captures the period when both markets experience negative shocks, resulting $\eta_t \eta_t' = I_t$. The symbols A , B and G represent the diagonal parameter matrices, where $A, B, G > 0$ to ensure the positiveness and stationarity of Q_t . $\bar{Q} = E(z_t z_t')$ is a $T \times n$ unconditional matrix of the standardised errors z_t .

Basic Properties of MGARCH Family

Any MGARCH model should be unique and stationary satisfying:

$$\sum_{i=1}^p \theta_i + \sum_{j=1}^q \beta_j < 1 \quad (21)$$

As the necessary and sufficient condition to have a unique and stationary solution, where β_j denotes the coefficients for lagged conditional variances, determining the effect of past volatilities on current volatility and θ_i is denoting coefficients for lagged squared residuals, determining the effect of past shocks on current volatility.

2.2.5 Distribution forms and Estimation of GARCH Models

The estimation of GARCH models for this study was based on the assumption of normality and Students t distribution for the innovation series ε_t . The log-likelihood from the normal distribution is:

$$l_t = -\frac{1}{2} \left[n \log(2\pi) + \sum_{t=1}^n \frac{\varepsilon_t^2}{\sigma_t^2} + \sum_{t=1}^s \log \sigma_t^2 \right] \quad (22)$$

And $\varepsilon_t = \sigma_t z_t$ where $z_t = \varepsilon_t / \sigma_t$ is the GARCH time series innovation and N is the sample size of the time series. For the student t-distribution, we have:

$$l_t = -\frac{1}{2} \left\{ n \log \left(\frac{\pi(v-2) \Gamma(\frac{v}{2})^2}{\Gamma(\frac{v+1}{2})^2} \right) + \sum_{t=1}^n \log \sigma_t^2 + (v+1) \sum_{t=1}^n \log \left[1 + \frac{\varepsilon_t^2}{\sigma_t^2(v-1)} \right] \right\} \quad (23)$$

Where v is the degrees of freedom to be estimated and $\Gamma(\cdot)$ is the gamma function.

2.2.6 Distribution forms and Estimation of MGARCH Models

The estimation of MGARCH models extends the univariate framework, requiring assumptions about the joint distribution of the innovation vector ε_t . For this study, the MGARCH model estimation was based on the assumption of multivariate normal, multivariate Student's t and multivariate Laplace distributions. Let ε_t be time vector of innovations at time t such that:

$$\varepsilon_t = H_t^{\frac{1}{2}} z_t \quad (24)$$

Where H_t is the conditional covariance matrix and z_t is the vector of i.i.d. innovations. The log-likelihood from the multivariate normal distribution is:

$$l(\theta) = -\frac{1}{2} \sum_{t=1}^T [k \log(2\pi) + \log(|H_t|) + \varepsilon_t' H_t^{-1} \varepsilon_t] \quad (25)$$

Where θ is the vector of all parameters to be estimated and $|H_t|$ is the determinant of the conditional covariance matrix. The log-likelihood from the multivariate student t- distribution is:

$$l(\theta) = \sum_{t=1}^T \left[\log \Gamma\left(\frac{v+k}{2}\right) - \log \Gamma\left(\frac{v}{2}\right) - \frac{k}{2} \log(\pi(v-2)) - \frac{1}{2} \log(|H_t|) - \left(\frac{v+k}{2}\right) \log\left(1 + \frac{\varepsilon_t' H_t^{-1} \varepsilon_t}{v-2}\right) \right] \quad (26)$$

Where v is the degrees of freedom and $\Gamma(\cdot)$ is the number of asset return series.

The log-likelihood from the multivariate Laplace distribution is:

$$l(\theta) = \sum_{t=1}^T \left[-\frac{1}{2} \log(|H_t|) + \log\left(\frac{(\varepsilon_t' H_t^{-1} \varepsilon_t)^{\frac{v}{2}}}{(2\pi)^{\frac{k}{2}}}\right) + \log K_v\left(\sqrt{2\varepsilon_t' H_t^{-1} \varepsilon_t}\right) \right] + C \quad (27)$$

The parameters θ of the MGARCH model are estimated by maximizing the log-likelihood function in (25 - 27) using numerical optimization techniques.

2.2.7 Model Selection Criterion

Information criteria Akaike Information Criterion (AIC), Bayesian Information Criterion (SBIC) and Hannan-Quinn Information Criterion (HQIC) evaluate GARCH model fit, with lower values indicating better performance. The preferred model among alternatives is the one that minimizes these metrics. Their formulations are:

$$AIC = -2\log(\hat{\sigma}^2) + 2(k) - 1 - \log(2\pi) \quad (28)$$

$$SBIC = -2\log(\hat{\sigma}^2) + (k) * \log(n) - 1 - \log(2\pi) \quad (29)$$

$$HQIC = -2\log(\hat{\sigma}^2) + 2(k) * \log(\log(n)) - 1 - \log(2\pi) \quad (30)$$

Where $\hat{\sigma}^2$ represents the model's fitted conditional variance, k is the number of estimated parameters in the GARCH model (including both GARCH and ARCH terms, and any additional parameters if applicable), n is the sample size (number of observations used in estimating the model) and π is the constant term.

4. Results and Discussion

The empirical findings of this study and their interpretation are presented in this section. The analysis begins by outlining the fundamental characteristics of the macro-financial variables through descriptive statistics and graphical representations. The stationarity of the data series is then established using the Augmented Dickey-Fuller (ADF) unit root test. The core empirical analysis proceeds with the application of the DCC and aDCC MGARCH models was also presented in this section.

Table 1: Descriptive Statistic Results

	EXR	OP	INFLA	SE
Mean	288.9841	74.06689	14.01205	36128.12
Median	157.3200	7a2.72500	12.44000	30355.50
Maximum	1669.610	138.7400	34.60000	104562.1
Minimum	117.7200	14.28000	3.000000	13298.80
Std. Dev.	294.7075	26.67723	6.325296	18244.93
Skewness	3.236507	0.261720	1.285065	1.995953
Kurtosis	13.78375	2.229443	4.732540	7.294012
Jarque-Bera	1740.080	9.545217	105.6799	378.1124
P-value	0.000000	0.008458	0.000000	0.000000

Source: Researcher's Compilation

The descriptive statistics in Table 1 indicate that all variables exchange rate (EXR), oil price (OP), inflation (INFLA), and stock exchange (SE) exhibit non-normal distributions, as confirmed by Jarque-Bera tests. Significant right-skewness and high kurtosis values suggest asymmetric distributions with heavy tails and extreme outliers, particularly for EXR and SE. The substantial standard deviations relative to the means point to high volatility in the data series, underscoring the need for robust econometric techniques in subsequent analysis.

Table 2: Descriptive Statistic of Return Series

	EXRR	OPR	INFR	SER
Mean	0.953565	0.306157	0.143007	0.778076
Median	0.000000	1.609278	0.813380	0.329768
Maximum	43.25571	66.97667	53.22168	32.35158
Minimum	-20.98098	-81.58976	-104.1454	-36.58828
Std. Dev.	4.605322	11.66132	14.18968	7.182297
Skewness	4.889773	-1.243632	-1.876767	-0.150098
Kurtosis	41.92513	17.75789	18.55887	7.764259
Jarque-Bera	17651.74	2454.467	2807.167	249.7216
P-value	0.000000	0.000000	0.000000	0.000000

Source: Researcher's Compilation

The return series in Table 2 exhibit significant non-normality, as all Jarque-Bera p-values are 0.000, and are characterized by high volatility, evidenced by their substantial standard deviations. Most series show extreme skewness and very high kurtosis, indicating asymmetric return distributions with exceptionally heavy tails and a high likelihood of extreme values. The particularly negative skewness in OPR, INFR, and SER suggests a higher frequency of large negative returns compared to positive ones, while the extremely high positive skewness in EXRR implies the opposite.

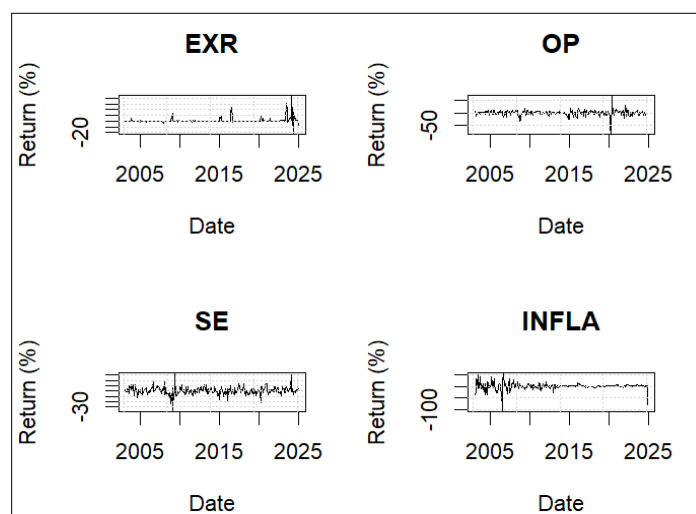


Fig 1: The Graph of monthly log Returns of Exchange Rate, oil price, stock exchange and inflation

The monthly log returns for all four variables (exchange rate, oil price, stock exchange index and inflation) in figure 1 exhibited clear evidence of volatility clustering, a hallmark of financial and macroeconomic time series. Periods of relative stability are interrupted by abrupt, large swings in returns, with particularly pronounced spikes and troughs around known periods of economic stress, such as the global financial crisis and the COVID-19 pandemic. The presence of extreme observations such as the severe negative return in oil prices and the sharp spike in inflation returns further underscores the leptokurtic, heavy-tailed nature of these series. These characteristics strongly justify the use of GARCH-type models to capture time-varying volatility and correlation dynamics in Nigeria's macro-financial environment.

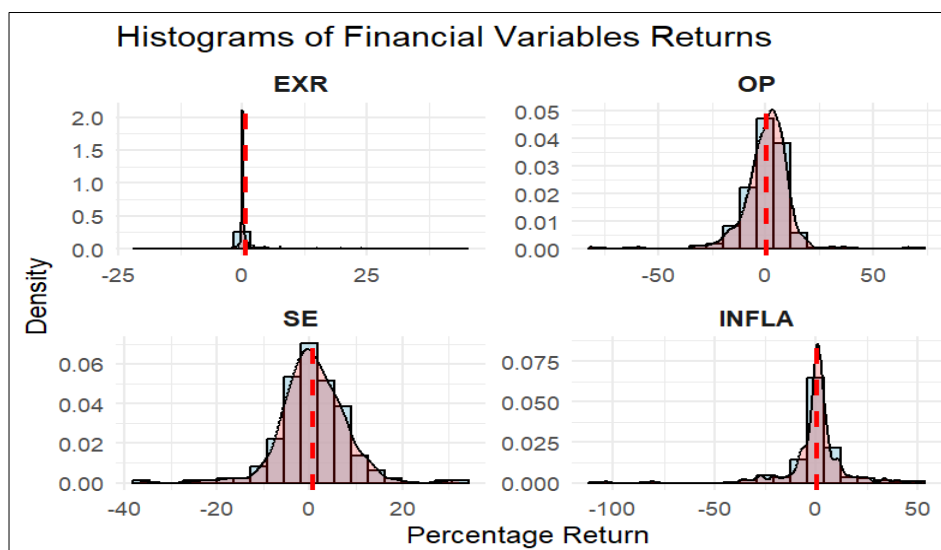


Fig 2: Density Plot of EXR, OP, SE and INFLA Returns

The histograms of return series across all variables EXR, OP, INFL, SEM, and SE in figure 2 exhibited strong departures from normality, characterized by pronounced peaks (leptokurtosis) and asymmetric tails. Most series show significant skewness, with INFLA and OP displaying particularly extreme ranges and negative outliers. The concentration of returns around zero with heavy tails suggests high kurtosis and volatility clustering, reinforcing the need for distributions that accommodate fat tails, such as the Student's t-distribution, in volatility modeling. These features are consistent with typical financial return data and justify the use of GARCH-type models to capture their time-varying variance and risk dynamics.

Table 3: ADF Unit Root Test Result of the Return Series

Variable	Critical Values			t-statistics	P-value
	1%	5%	10%		
EXRR	-3.9953	-3.4280	-3.1374	-4.0094	0.0096
OPR	-3.9936	-3.4271	-3.1369	-11.9977	0.0000
SER	-3.9940	-3.4273	-3.1370	-6.0917	0.0000
INFR	-3.9950	-3.4278	-3.1373	-6.6905	0.0000

Source: Researcher's Compilation

The ADF unit root test results in table 3 indicated that all return series (EXRR, OPR, SER, and INFR) are stationary at the 1% significance level, as their respective t-statistics are more negative than all the critical values (1%, 5% and 10%) and all p-values are below 0.01. This confirms the absence of a unit root in each series, validating their suitability for further econometric modeling without the need for differencing.

Table 4: Model Comparison

Model	AIC	BIC
Symmetric DCC-GARCH with Multivariate Normal Distribution	27.10	27.38
Symmetric DCC-GARCH with Multivariate Student-t Distribution	23.68	23.66
Symmetric DCC-GARCH with Multivariate Laplace Distribution	28.36	28.69
Asymmetric DCC-GARCH with Multivariate Normal Distribution	27.11	27.40
Asymmetric DCC-GARCH with Multivariate Student-t Distribution	23.69	24.04
Asymmetric DCC-GARCH with Multivariate Laplace Distribution	28.37	28.71

Source: Researcher's Compilation

Based on the AIC and BIC values in table 4, the Symmetric DCC-GARCH model using a Multivariate Student-t Distribution is the best-fitting model, as it has the lowest values for both criteria (AIC: 23.68, BIC: 23.66). This suggests that the data exhibit volatility clustering and heavy-tailed characteristics, which are adequately captured by the Student-t distribution. The symmetric variant outperforms its asymmetric counterparts, indicating that leverage effects may not be a dominant feature in this dataset. The normal and Laplace distributions yield higher AIC and BIC values, confirming their inferior fit relative to the Student-t specification.

Table 5: Estimation Results of the MGARCH-DCC and aDCC Model with Student-t Distributions

Variables	Parameter	DCC		aDCC	
		Estimate (Std Error)	t-value (p-value)	Estimate (Std Error)	t-value (p-value)
EXRR	Constant (ω)	0.0002 (0.0004)	0.5590 (0.5760)	0.0002 (0.0004)	0.5590 (0.5760)
	ARCH (α)	0.8760 (0.1060)	8.2620 (0.0000)	0.8760 (0.1060)	8.2760 (0.0000)
	GARCH (β)	0.1230 (0.0568)	2.1610 (0.0310)	0.1230 (0.0568)	2.1610 (0.0310)
	Shape (ν)	2.5210 (0.1120)	22.6030 (0.0000)	2.5210 (0.1120)	22.580 (0.0000)
OPR	Constant (ω)	12.2920 (9.2620)	1.3270 (0.1840)	12.2920 (9.2770)	1.3250 (0.1850)
	ARCH (α)	0.4840 (0.1580)	3.0560 (0.0020)	0.4840 (0.1590)	3.0390 (0.0020)
	GARCH (β)	0.4580 (0.1380)	3.3220 (0.0000)	0.4580 (0.1390)	3.3020 (0.0000)
	Shape (ν)	8.8380 (4.3330)	2.0400 (0.0410)	8.8380 (4.3320)	2.0400 (0.0410)
SER	Constant (ω)	0.0000 (0.0006)	0.0080 (0.9940)	0.0000 (0.0006)	0.0080 (0.9940)
	ARCH (α)	0.0000 (0.0000)	0.0000 (1.0000)	0.0000 (0.0000)	0.000 (1.0000)
	GARCH (β)	0.9990 (0.0000)	22758.6 (0.0000)	0.9990 (0.0000)	22764.2 (0.0000)
	Shape (ν)	4.6720 (0.7680)	6.0840 (0.0000)	4.6720 (0.7680)	6.0850 (0.0000)
INFR	Constant (ω)	1.5570	1.6910	1.5570 (0.9210)	1.6900

		(0.9210)	(0.0910)		(0.0910)
	ARCH (α)	0.3180 (0.1050)	3.0170 (0.0030)	0.3180 (0.1050)	3.0170 (0.0030)
	GARCH (β)	0.6810 (0.1120)	6.0620 (0.0000)	0.6810 (0.1120)	6.0620 (0.0000)
	Shape (ν)	3.7830 (0.6550)	5.7710 (0.0000)	3.7830 (0.6550)	5.7710 (0.0000)

Source: Researcher's Compilation

The Dynamic Conditional Correlation (DCC) model results in table 5 indicated significant volatility persistence and heavy-tailed distributions across all return series. For instance, the exchange rate returns exhibit high short-term volatility sensitivity with an ARCH term of 0.876, while stock returns show near-integrated GARCH behavior with a persistence parameter of 0.999. The low shape parameters, particularly for exchange rates and inflation returns, confirm the presence of extreme values and justify the use of a Student-t distribution. These findings collectively underscore the model's ability to capture time-varying volatility and leptokurtic characteristics, supporting its suitability for analyzing correlations under symmetric response assumptions.

The Asymmetric Dynamic Conditional Correlation (aDCC) model yields parameter estimates virtually identical to those of the standard DCC model across all equations and variables. This remarkable similarity, evident in both coefficient magnitudes and statistical significance, implies that asymmetric effects where negative and positive shocks differentially impact correlation dynamics are not empirically relevant in this dataset. Consequently, the more parsimonious symmetric DCC specification is sufficient, as the additional complexity of modeling asymmetry does not improve the fit or alter the interpretation of the volatility and dependence structure.

Table 6: Symmetric and Asymmetric Dynamic Conditional Correlation (DCC) Parameters

Parameter	Description	DCC		aDCC	
		Estimate (std Error)	t-value (P-value)	Estimate (std Error)	t-value (P-value)
α	Innovation	8.78×10^{-9} (1.85×10^{-5})	0.0005 (0.9996)	2.01×10^{-9} (2.71×10^{-3})	7.41×10^{-7} (0.9999)
β	Persistence	0.9070 (0.1070)	8.4960 (0.0000)	0.9560 (0.0685)	13.954 (0.0000)
γ	Asymmetry	-	-	8.32×10^{-8} (0.0148)	5.62×10^{-6} (0.9999)
ν	Joint Shape	4.0000 (0.5270)	7.5710 (0.0000)	4.0000 (0.5270)	7.5910 (0.000)

Source: Researcher's Compilation

The DCC and aDCC models both reveal that dynamic correlations are driven solely by high persistence ($\beta = 0.907$ and 0.956 , respectively), while the innovation (α) and asymmetry (γ) parameters are statistically insignificant and economically negligible. This indicates that correlations evolve gradually over time rather than reacting abruptly to new shocks or exhibiting asymmetric responses to positive versus negative returns. The significant joint shape parameter ($\nu = 4.000$) confirms heavy-tailed multivariate distributions, but the absence of meaningful asymmetry supports the use of the simpler symmetric DCC model for capturing the interdependence between these variables (Table 6). Given the insignificant asymmetry parameter, we conducted a diagnostic test on standard DCC model only.

Table 7: Conditional Correlation Matrix

Variables	EXRR	OPR	SER	INFR
EXRR	1			
OPR	-0.0985	1		
SER	-0.0620	0.2608	1	
INFR	0.0356	0.0064	-0.0308	1

Source: Researcher's Compilation

The conditional correlation matrix in Table 7 revealed generally weak interdependence among the return series. The strongest correlation observed is a modest positive relationship between oil price returns and stock exchange returns (0.2608). Exchange rate returns show a slight negative correlation with oil prices (-0.0985) and stock returns (-0.0620), while inflation returns exhibit near-zero correlations with all other variables, particularly oil prices (0.0064) and stock returns (-0.0308). The low correlation magnitudes suggest limited short-term co-movement and diversified risk across these asset classes.

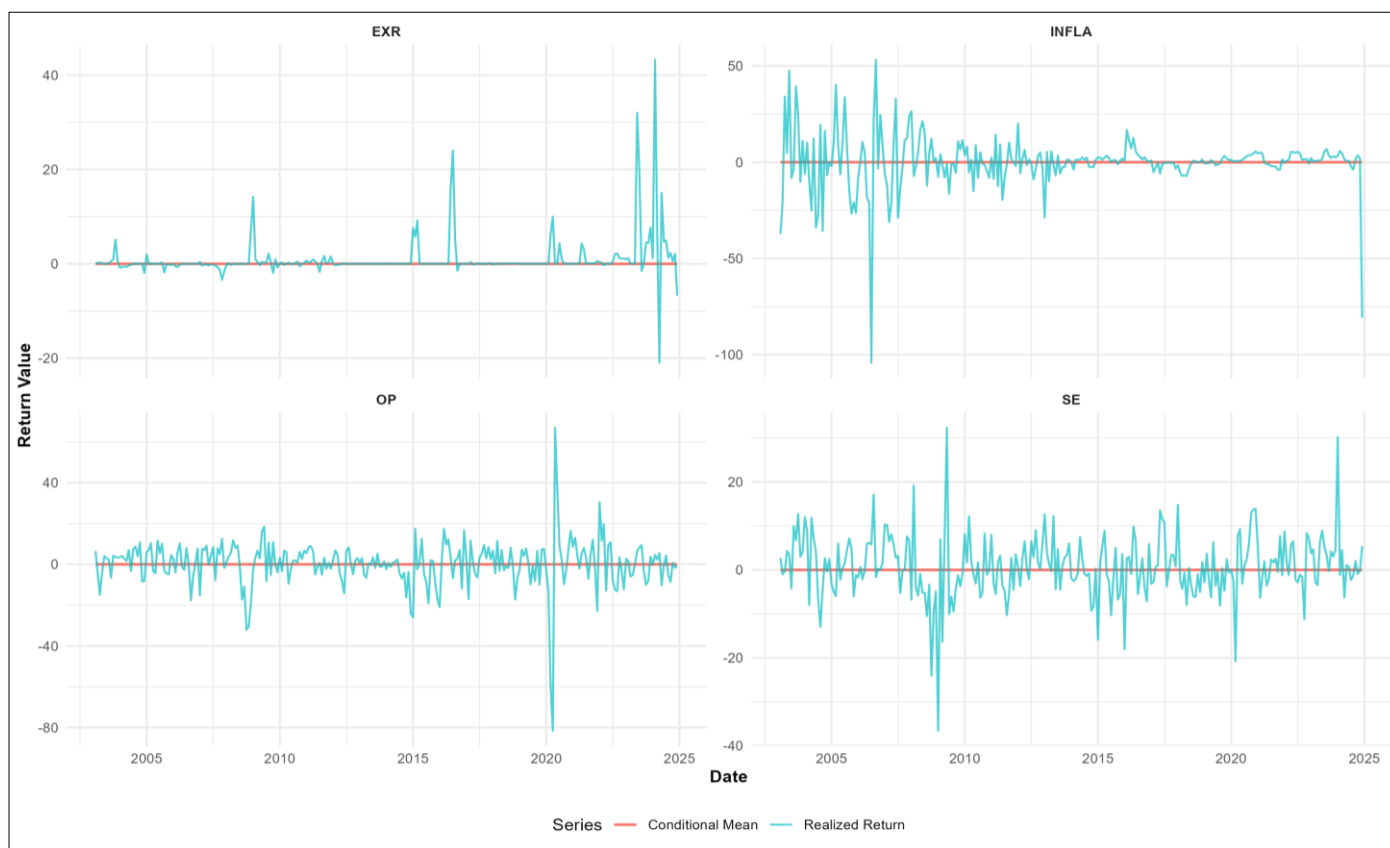


Fig 3: Conditional means versus realized returns for each variable

Figure 3 compares the model's conditional mean forecasts against the actual realized returns for each variable over time. The plot shows the model's ability to track the general direction and volatility of the returns, though some deviations are evident, particularly during periods of extreme market movements. The conditional mean appears to smooth out the high-frequency fluctuations, capturing the persistent components while occasionally underestimating the magnitude of sharp peaks and troughs. This suggests the model provides a reasonable fit for the central tendency of the data but may struggle with predicting the timing and intensity of extreme events, highlighting the inherent challenge in forecasting volatile financial returns.

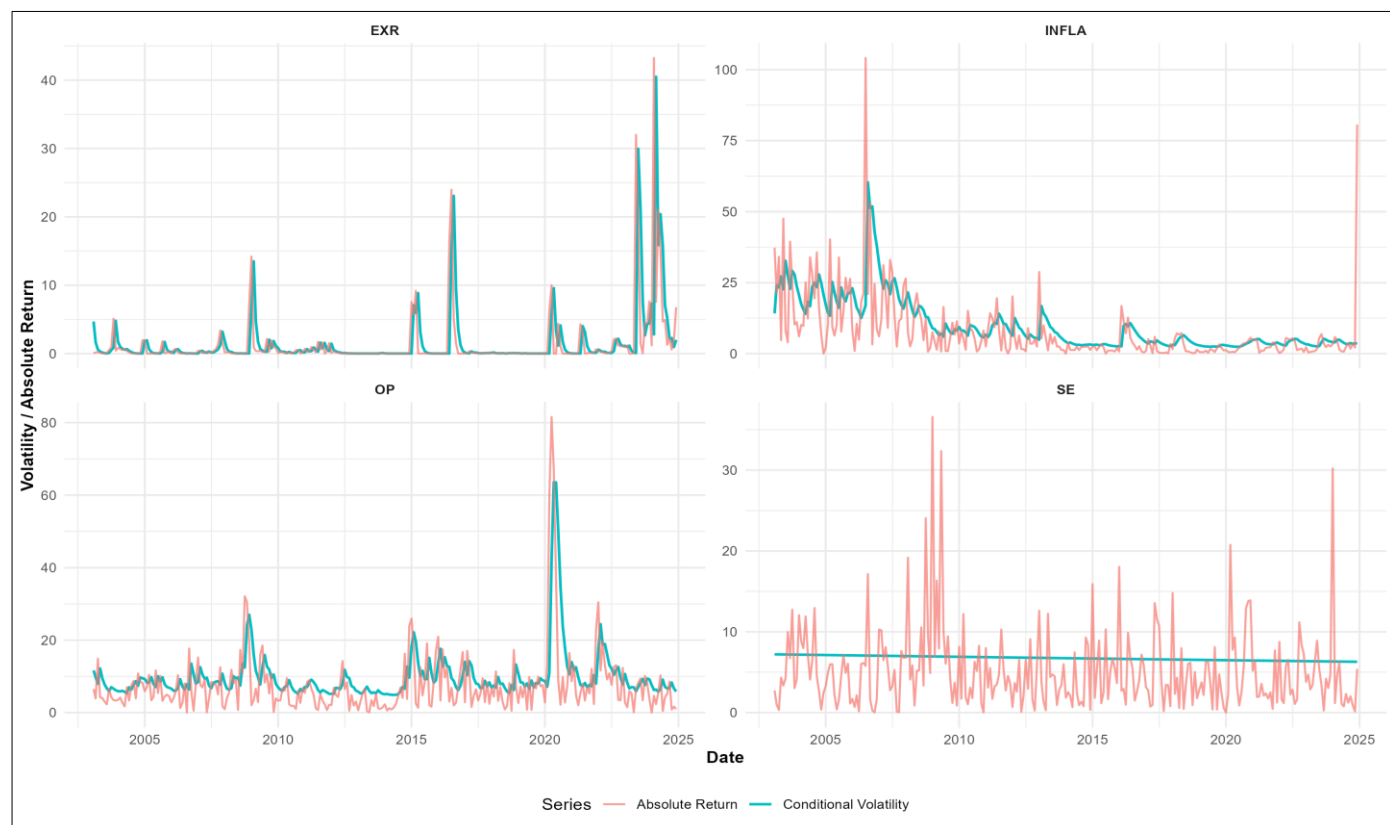


Fig 4: Conditional volatilities versus absolute returns

The plot in Figure 4 presents the DCC model's estimated conditional volatilities against the observed absolute returns for each variable. The conditional volatility series likely displays smoother, persistent trends that capture the underlying time-varying risk, while the absolute returns exhibit more erratic, peaky behavior reflecting immediate market reactions. The model appears to successfully track the broad evolution of volatility, with spikes in absolute returns generally coinciding with higher estimated conditional volatilities. This alignment confirms the model's effectiveness in capturing volatility clustering. However, the smoother nature of the conditional volatility lines suggests the model may underestimate the intensity of the most extreme short-term market shocks, a common characteristic of GARCH-type models.

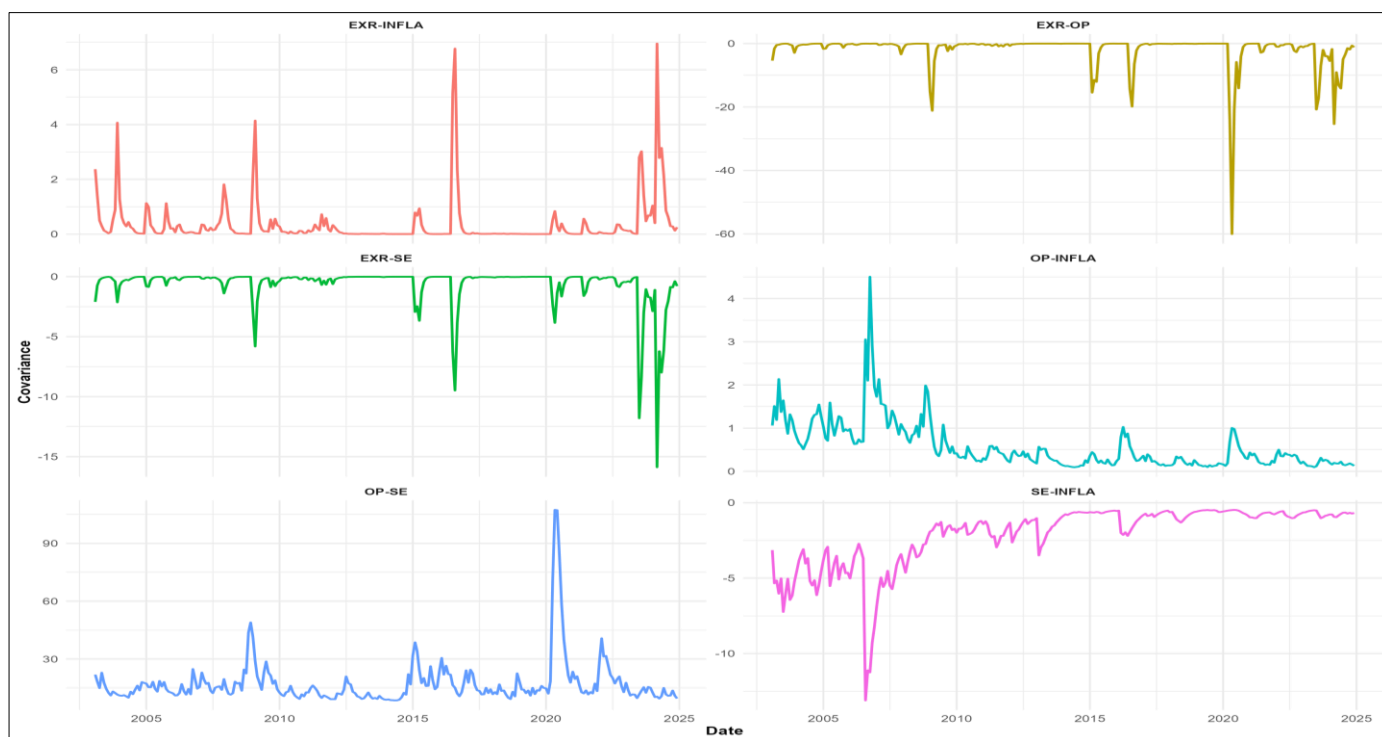


Fig 5: Conditional covariances between variable pairs

Figure 5 displays the time-varying conditional covariances between pairs of variables, such as exchange rates, oil prices, stock returns, and inflation. The plot showed periods of heightened covariance, particularly during times of market stress or economic turbulence, indicating increased co-movement between the assets. These spikes suggest that diversification benefits may diminish during volatile episodes when asset returns become more correlated. The overall evolution of the covariances aligns with the dynamic correlation structure captured by the DCC model, reinforcing the presence of time-varying interdependence and shared volatility risks among the variables in the system.

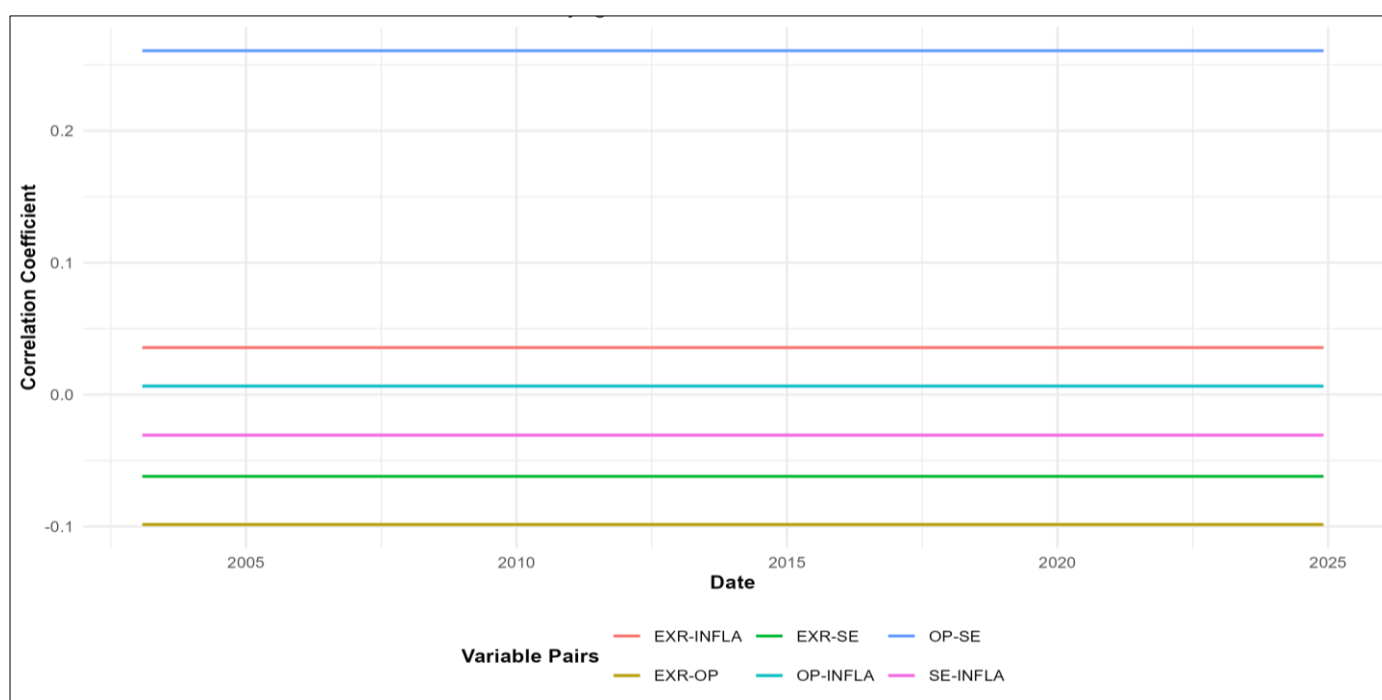


Fig 6: Dynamic conditional correlations

Figure 6 illustrates the evolution of dynamic conditional correlations between variable pairs over time, spanning from 2003 to 2024. The correlations fluctuate within a moderate range, roughly between -0.1 and 0.2, indicating generally weak but time-varying interdependence. The trajectories show periods of increased correlation, particularly around market stress events, as well as phases of divergence or negative correlation, suggesting shifting co-movement patterns. This variability underscores the importance of using a dynamic model like DCC-GARCH, as constant correlation assumptions would fail to capture these evolving relationships. The results highlight that diversification benefits and spillover risks are not static but change over time.

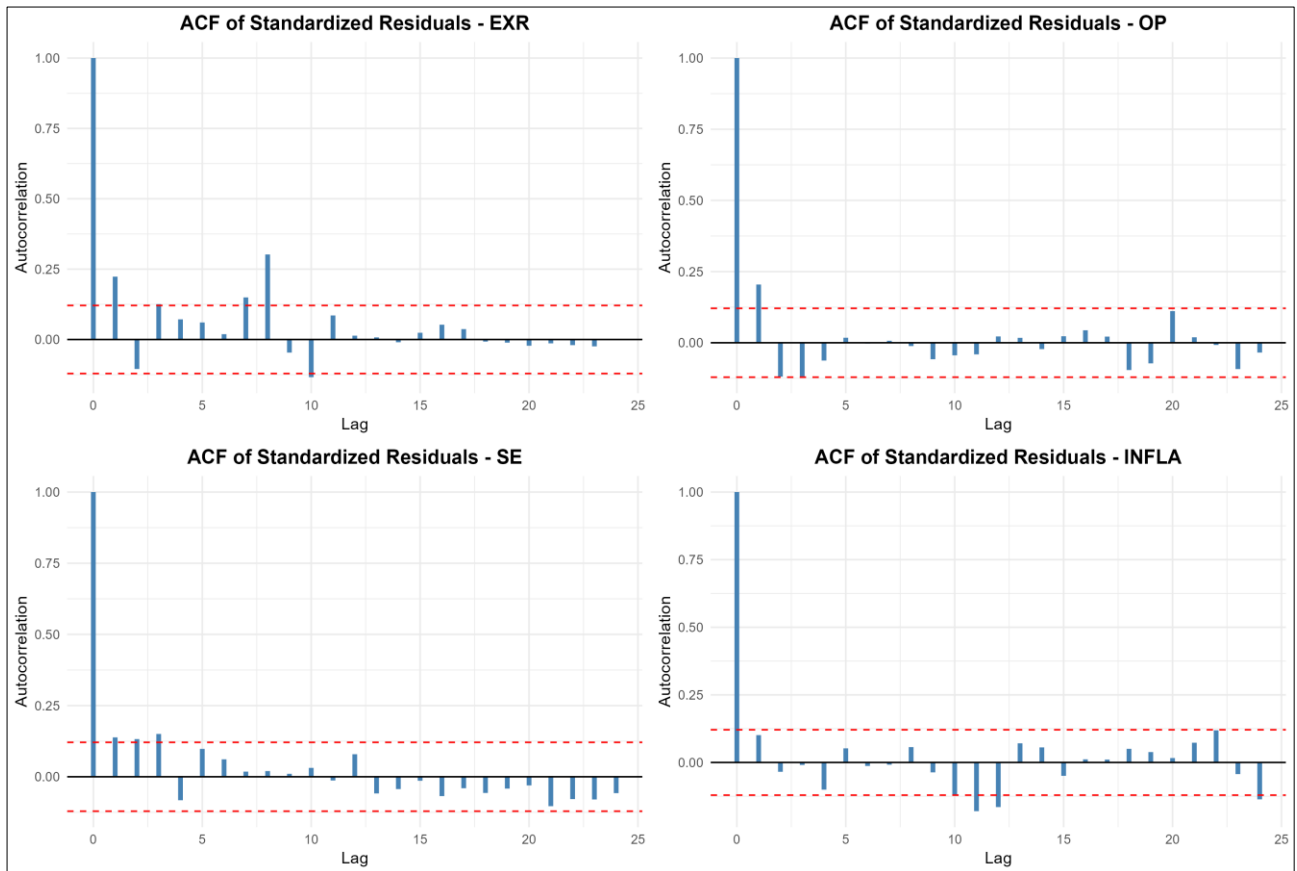


Fig 7: Autocorrelation functions of standardized residuals

The autocorrelation function (ACF) plots for the standardized residuals of EXR, OP, SE, and INFLA in figure 7 indicated that the model has effectively captured the linear temporal dependencies in the series. The vast majority of autocorrelation coefficients across all lags are statistically insignificant, falling within the 95% confidence bounds and clustering around zero. This absence of significant serial correlation suggests the residuals approximate white noise, validating the model's specification.

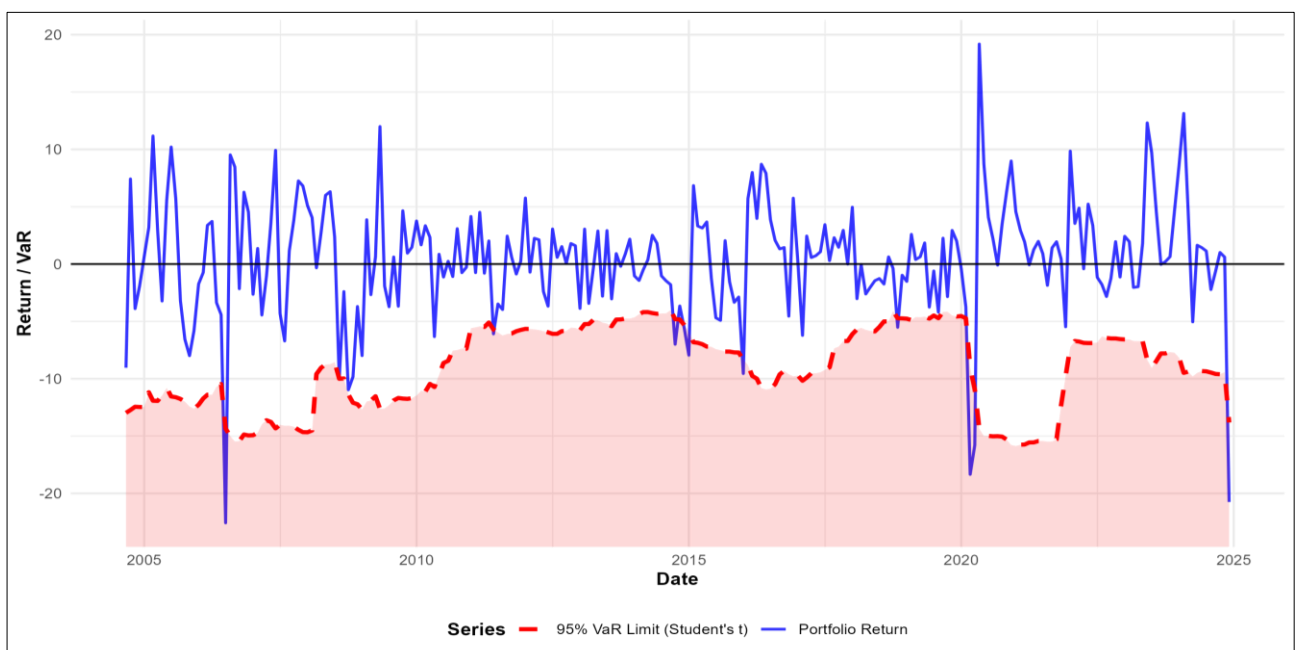


Fig 8: Equal-Weighted Portfolio VaR analysis

Figure 8 presents the Value-at-Risk (VaR) analysis for an equal-weighted portfolio constructed from the included variables in the standard DCC GARCH model. The VaR estimates fluctuate over time, with periods of heightened risk (e.g., sharp negative spikes around -20) coinciding with market turmoil, such as the 2008 financial crisis or the 2020 pandemic. The dynamic evolution of VaR reflects the time-varying volatility and correlation structure captured by the model, highlighting episodes where portfolio downside risk intensified significantly. This demonstrates the model's utility in quantifying tail risk and stress periods, emphasizing the importance of accounting for changing market conditions in risk management.

Table 8: Summary Statistics of Conditional Volatilities

Variables	Mean	Max	Min	Volatility ratio
EXRR	1.5702	40.5013	0.0167	2431.8122
OPR	9.7871	63.5829	4.8782	13.0342
SER	6.7396	7.2107	6.2895	1.1465
INFR	9.8877	60.3639	2.3945	25.2089

Source: Researcher's Compilation

The summary statistics of conditional volatilities in table 8 revealed substantial differences in risk profiles across variables. Exchange rate returns exhibit the highest relative volatility, with an extreme volatility ratio of 2431.8, reflecting intense fluctuations between its maximum (40.50) and minimum (0.017) values. Oil price returns and inflation returns also show significant volatility, with ratios of 13.0 and 25.2, respectively, indicating pronounced time-varying risk. In contrast, stock exchange returns demonstrate remarkably stable volatility, with a narrow range between 6.29 and 7.21 and a low ratio of 1.15, suggesting persistent, moderate risk levels with minimal variation over time.

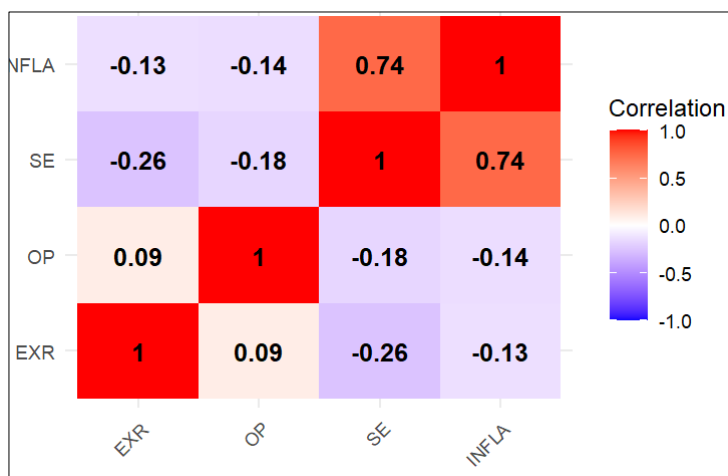


Fig 9: Correlation between Conditional Volatilities

The heatmap in figure 9 presents the correlation matrix between the conditional volatilities of the variables EXR, OP, INFLA, and SE. The plot reveals generally weak interdependencies among volatility series, with most correlations being low or negative. Notably, a moderate positive correlation (0.74) exists between the volatilities of SE and INFLA, suggesting some shared uncertainty dynamics between stock exchange and inflation rates. Conversely, SE volatility shows slight negative correlations with EXR (-0.26) and OP (-0.18), indicating divergent volatility patterns during certain periods. The overall weak correlations imply that volatility shocks are largely idiosyncratic, with limited evidence of strong volatility spillovers across these markets.

Table 9: DCC Ljung-Box Portmanteau Test for Autocorrelation

Variables	Lag	Test For	Q-Statistic	P-value
EXRR	5	AC in residuals	22.80	0.0000
	5	AC in squared residuals	12.75	0.0260
	10	AC in residuals	59.45	0.0000
	10	AC in squared residuals	52.48	0.0000
OPR	5	AC in residuals	19.82	0.0010
	5	AC in squared residuals	168.10	0.0000
	10	AC in residuals	21.33	0.0190
	10	AC in squared residuals	168.24	0.0000
SER	5	AC in residuals	20.23	0.0010
	5	AC in squared residuals	23.89	0.0010
	10	AC in residuals	21.74	0.0160
	10	AC in squared residuals	28.34	0.0020
INFR	5	AC in residuals	6.52	0.0230
	5	AC in squared residuals	13.02	0.2940
	10	AC in residuals	11.87	0.0360
	10	AC in squared residuals	19.34	0.0080

Source: Researcher's Compilation, AC - Autocorrelation

The significant p-values ($p < 0.05$) for the squared residuals across all four variables in Table 9 indicated the presence of remaining autocorrelation, suggesting that the applied GARCH specification may not have fully captured all volatility clustering patterns. This implies that some predictable structure in squared returns persists post-modeling, a common outcome in financial time series analysis. To improve the model's adequacy, extensions such as higher-order GARCH terms or alternative error distributions (e.g., skewed or heavy-tailed formulations) could be explored to better account for these residual nonlinear dependencies.

Table 10: Multivariate Ljung-Box test for cross correlation

Lag	Q-statistic	DF	P-value
1	52.4	16	0.0000
2	77.8	32	0.0000
3	100.1	48	0.0000
4	112.1	64	0.0000
5	120.5	80	0.0000
6	133.3	96	0.0000
7	150.6	112	0.0000
8	189.0	128	0.0000
9	202.5	144	0.0000
10	226.8	160	0.0000

Source: Researcher's Compilation

The significant Q-statistics ($p\text{-value} < 0.05$) across lags 1 to 10 in Table 10 provided strong evidence against the null hypothesis of no cross-autocorrelation, indicating persistent linear dependence in the multivariate residuals. This suggests that while the univariate GARCH models adequately captured within-series volatility dynamics, the DCC specification failed to fully account for lead-lag interactions between the series.

5. Conclusion

Based on the empirical analysis conducted in this study, several conclusive findings emerge regarding the dynamic interrelationships among Nigeria's key macro-financial variables. The investigation reveals that while significant volatility persistence and heavy-tailed characteristics are present across all series exchange rates, oil prices, stock returns, and inflation the conditional correlations between these variables exhibit relatively weak interdependence with dynamic but symmetric evolution over time. The optimal model specification, identified through comprehensive information criteria, employs a symmetric Dynamic Conditional Correlation (DCC) framework with multivariate Student-t distribution, as the more complex asymmetric DCC (ADCC) formulation provided no statistically significant improvement in capturing correlation dynamics. This indicates that negative and positive shocks exert quantitatively similar effects on the co-movement patterns among these variables, contrary to what might be expected in an emerging economy subject to substantial external shocks and structural vulnerabilities.

The empirical results demonstrate distinct volatility characteristics across the analyzed series. Exchange rate returns display the highest relative volatility with intense fluctuations, while stock returns showed remarkable stability with persistent, moderate risk levels. Oil price and inflation returns occupy an intermediate position with significant time-varying volatility. The dynamic correlation analysis reveals that co-movements remain generally modest, with the strongest positive association observed between oil prices and stock returns, while inflation maintains notably weak connections with other variables. Crucially, the time-varying nature of these correlations underscores the limitation of constant correlation assumptions and highlights the value of the DCC framework in capturing evolving market interrelationships. The identified periods of heightened correlation, particularly during episodes of market stress, emphasize the conditional nature of diversification benefits and risk transmission mechanisms in Nigeria's financial markets.

From a methodological perspective, while the DCC model effectively captures the core volatility and correlation dynamics, diagnostic tests indicate some remaining cross-autocorrelation in the residuals, suggesting that additional lead-lag interactions or nonlinear dependencies may not be fully accounted for in the current specification. This presents opportunities for future research to explore more sophisticated modeling approaches, such as regime-switching correlation models or higher-order multivariate GARCH specifications. Nevertheless, the current study provides valuable insights for financial practitioners and policymakers by offering a robust framework for assessing time-varying risks and interdependencies in Nigeria's macro-financial system. The findings emphasize the importance of dynamic risk management strategies that account for evolving market conditions and highlight the relatively symmetric nature of correlation responses to market shocks in this specific economic context, contributing to enhanced portfolio allocation decisions, risk management practices, and policy formulation for maintaining financial stability in oil-dependent emerging economies.

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