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Dr. Rashmirekha Shial
P.G. Department of
Mathematics, F.M. Autonomous
College, Balasore, Odisha, India

An inventory model for Pareto deterioration with parabolic time varying holding cost and inflation under trade credit

Rashmirekha Shial

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Abstract

This paper presents a detailed analysis of inventory systems in which deteriorating items having Pareto rate of decay and constant demand. In this model we have considered inflation and parabolic time varying holding cost, and trade credit. Shortages assured which is completely backlogged. Supplier offers a credit limit to the customer and during that span of time no interest is charged, but after the expiry of the specified time limit, the supplier will charge some interest. This study has main focus to establish the mathematical model for an inventory system under the above conditions. At the end of the paper, a numerical example is provided to illustrate the problem and sensitivity analysis has been carried out for showing the effect of variation in the parameter.

Keywords: Trade credit, Pareto deterioration, inflation, parabolic time varying holding cost, shortages

1. Introduction

Inventory modeling is a prerequisite for developing the optimal ordering and pricing policies in several places such as ware houses, market yards production process, cargo handling. When one is concerned with the inventory control, it is essential to identify two important factors namely the nature of the demand. When we consider the nature of procurement, we have to consider life time of the commodity along with the demand pattern. The Pareto life time distribution is capable of charactering age dependent rate of deterioration. The Pareto distribution is extensively used in reliability and life testing experiments. Recently much work has been reported in inventory models for deteriorating items. In enveloping the inventory model it is needed to ascribe a probability distribution to the life time of the commodity.

K. Srinivas Rao & J.K. Begum (2007) ^[1, 2], Viv. Vs kesavara, K. Srivinas a rao, Y. Sirinivas rao (2012) ^[14], assumed that the lifetime of commodity following Pareto distribution. Recently G.C Mahanta, P Mahanta (2009) ^[7], R.P. Tripathy, S.S. Mishra (2012) ^[8], Vipin Kumar, Gopal Pathak and C. B. Gupta (2013) ^[3] developed on Deterministic inventory model under trade credit. Trade credit is an important economic phenomenon. It is most effective tool for the supplier more and to attract more retailers. A supplier will allow a certain period for settling the amount owed to him for the items supplied. Usually there is no charge, if the outstanding amount is settled within the permitted fixed settlement period. Beyond this period interest in charged. Secondly due to high inflation and consequent sharp decline in the purchasing power of money, especially in the developing countries the financial situation has been charged, so it has been impossible to ignore the effect of inflation. C.K. Jaggi & A. Khana (2008) ^[11] developed on impact of inflation & credit policies on a production lot size model.

H.C. Liao, C.H. T Sai, C.T. Su (2000) ^[5] developed on an inventory model for deteriorating items under inflation when a delay in payments is permissible. R.P. Tripathi & Neha Sang (2012) ^[6] developed on EOQ model for constant demand rate with completely backlogged shortage. Vipin Kumar, Gopal Pathak, C.B. Gupta (2013) ^[3] developed on a Deterministic inventory model for deteriorating items with selling price dependent demand and parabolic time varying holding cost under trade credit. In this paper we have developed an inventory model for

Corresponding Author:
Dr. Rashmirekha Shial
P.G. Department of
Mathematics, F.M. Autonomous
College, Balasore, Odisha, India

Deteriorating items in which the deterioration rate is Pareto distribution and demand is constant but shortages are allowed which is fully backlogged and holding cost is parabolic time varying. The effects of inflation of delay in payments have been discussed. Using the differential equal the instantaneous state of inventory is derived. A numerical illustration is also discussed. The sensitivity of the model with respect to the costs and parameters is also discussed.

2. Assumptions and Notations:

- A. The demand rate is assumed to be constant which is $D(t) = D$
- B. The lead time is zero
- C. Shortages allowed which is completely backlogged
- D. Life time of the commodity is random and follows a Pareto distribution having probability density function of the form

$$f(t) = \frac{\alpha \delta^\alpha}{t^{\alpha+1}}, t \geq \delta, \delta \geq 0$$

$$\text{and } F(t) = 1 - \left(\frac{\delta}{t}\right)^\alpha$$

The instantaneous rate of deterioration is

$$\theta(t) = \frac{f(t)}{1 - F(t)} = \frac{\alpha}{t}, \alpha > 0, t > \delta$$

- E. T : is the length of the cycle
- F. A = Ordering cost at time $t = A_0 e^{kt}$
Where A_0 is the ordering cost at time zero and k is the constant Rate of inflation
- G. C = unit purchase at time $t = C_0 e^{kt}$
- H. h : The holding cost is parabolic with time dependent $h(t) = h + \beta t^2, h > 0, \beta > 0$
- I. s : Shortages cost per unit per unit time
- J. M : Permissible delay in settling the accounts
- K. I_c, I_e : Interest charge and Interest earn respectively where $I_c \geq I_e$
- L. C : Purchase cost per unit per unit time
- M. TC_1, TC_2 : Total average cost per unit time for $M \leq t_1$ and $M > t_1$ respectively.

3. Mathematical formulation of the model

Let $I(t)$ be the inventory level at time " t " ($0 \leq t \leq T$). The differential equations governing the system in the cycle time $[0, T]$ are

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D \quad 0 \leq t \leq t_1 \quad (1)$$

$$\frac{dI(t)}{dt} = -D \quad t_1 \leq t \leq T \quad (2)$$

With boundary condition $I(t) = 0$ at $t = t_1$

Solving equation (1) and (2)

$$I(t) = \frac{D}{\alpha + 1} \left(\frac{t_1^{\alpha+1}}{t^\alpha} - t \right) \quad (3)$$

$$I(t) = D(t_1 - t) \quad (4)$$

The expected Deterioration cost per unit time is

$$D.C = C_0 e^{kt} \left[I(0) - \int_0^{t_1} D dt \right] = -C_0 e^{kt} D t_1 \quad (5)$$

Ordering cost per unit is

$$O.C = A_0 e^{kt} \quad (6)$$

Expected inventory holding cost per unit per unit time is

$$\begin{aligned} H.C &= \int_0^{t_1} (h + \beta t^2) I(t) dt \\ &= \frac{hD}{\alpha+1} \left(\frac{t_1^2}{1-\alpha} - \frac{t_1^{\alpha+3}}{2} \right) + \frac{D}{\alpha+1} \left(\frac{t_1^4}{3-\alpha} - \frac{t_1^{\alpha+4}}{3} \right) \end{aligned} \quad (7)$$

Total cost per unit time is

$$TC(t_1, T) = \frac{1}{T} (A + H.C + D.C + S.C + IC_1 - IE_1) \quad (8)$$

CASE-I: ($M \leq t_1$)

For $M \leq t_1$, the buyer has stock on hand beyond M and so he can use the sale revenue to earn interest at an annual rate I_e up to t_1 . The interest earned denoted by IE_1 , is therefore

$$IE_1 = CI_e \int_0^{t_1} D t dt = CI_e D \frac{t_1^2}{2} \quad (9)$$

However beyond the fixed credit period M , the unsold stock is assumed to be financed with annual charge I_c and the interest charge, denoted by IC_1 , is given by

$$IC_1 = CI_c \int_M^{t_1} D t dt = CI_c D (t_1 - M) \quad (10)$$

Therefore the total average cost in this case comes out to be

$$TC_1(t_1, T) = \frac{A + HC + DC + SC + IC_1 - IE_1}{T} \quad (11)$$

Substituting the equations of HC , DC , SC , IC_1 and IE_1 in equation (11), we have

$$\begin{aligned} TC_1(t_1, T) &= \frac{1}{T} [A_0 e^{kt} - C_0 D t_1 e^{kt} + \frac{hD}{\alpha+1} \left(\frac{t_1^2}{1-\alpha} - \frac{t_1^{\alpha+3}}{2} \right) + \frac{\beta D}{\alpha+1} \left(\frac{t_1^4}{3-\alpha} - \frac{t_1^{\alpha+4}}{3} \right) \\ &\quad + SD(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}) + CI_c D(t_1 - M) - CI_e D \frac{t_1^2}{2}] \end{aligned} \quad (12)$$

$$\begin{aligned} \frac{\partial TC_1(t_1, T)}{\partial t_1} &= \frac{1}{T} \left[\frac{hD}{\alpha+1} \left(\frac{2t_1}{1-\alpha} - \frac{\alpha+3}{2} t_1^{\alpha+2} \right) + \frac{\beta D}{\alpha+1} \left(\frac{4t_1^3}{3-\alpha} - \frac{\alpha+4}{3} t_1^{\alpha+3} \right) + SD(t_1 - M) \right. \\ &\quad \left. + CI_c D - CI_e D t_1 - C_0 D e^{kt} \right] \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{\partial TC_1(t_1, T)}{\partial T} = & -\frac{1}{T^2} [A_0 e^{kt} - C_0 D t_1 e^{kt} + \frac{hD}{\alpha+1} (\frac{t_1^2}{1-\alpha} - \frac{t_1^{\alpha+3}}{2}) + \frac{\beta D}{\alpha+1} (\frac{t_1^4}{3-\alpha} - \frac{t_1^{\alpha+4}}{3})] \\ & - \frac{1}{2} SD - \frac{1}{T^2} CI_c D(t_1 - M) + \frac{1}{2T^2} I_e D t_1^2 \end{aligned} \quad (14)$$

The optimal values of t_1 and T which minimize total average cost per unit per time, can be obtained by solving following equations

$$\frac{\partial TC_1(t_1, T)}{\partial t_1} = 0 \quad \text{and} \quad \frac{\partial TC_1(t_1, T)}{\partial T} = 0.$$

Provided they satisfy the sufficient conditions

$$\frac{\partial^2 TC_1(t_1, T)}{\partial t_1^2} > 0, \quad \frac{\partial^2 TC_1(t_1, T)}{\partial T^2} > 0$$

And

$$\left[\left\{ \frac{\partial^2 TC_1(t_1, T)}{\partial t_1^2} \right\} \left\{ \frac{\partial TC_1(t_1, T)}{\partial T^2} \right\} - \left\{ \frac{\partial^2 TC_1(t_1, T)}{\partial t_1 \partial T} \right\} \right] > 0.$$

CASE-II: ($M > t_1$)

Since $M > t_1$, the retailer pays no interest but earns interest at an annual rate I_e during the period $(0, M)$. But $[0, T]$, the retailer sells product at selling price P /unit and deposits the revenue into interest earning account at the rate of I_e /year. In the period $[T, M]$, the retailer deposits only the total revenue into an account that earns I_e /year. Hence, interest earned per time unit is

$$IE_2 = CI_e \left[\int_0^{t_1} (t_1 - t) D dt + \int_0^{t_1} (M - t_1) D dt \right] = CI_e D t_1 \left(M - \frac{t_1}{2} \right) \quad (15)$$

and interest charge per unit is $IC_2 = 0$

Total average cost per unit time is

$$TC_2(t_1, T) = \frac{A + HC + DC + SC + IC_2 - IE_2}{T} \quad (16)$$

$$\begin{aligned} TC_2(t_1, T) = & \frac{1}{T} [A_0 e^{kt} - C_0 D t_1 e^{kt} + \frac{hD}{\alpha+1} (\frac{t_1^2}{1-\alpha} - \frac{t_1^{\alpha+3}}{2}) + \frac{\beta D}{\alpha+1} (\frac{t_1^4}{3-\alpha} - \frac{t_1^{\alpha+4}}{3})] \\ & + SD(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}) - CI_e D t_1 (M - \frac{t_1}{2}) \end{aligned} \quad (17)$$

$$\begin{aligned} \frac{\partial TC_2(t_1, T)}{\partial t_1} = & \frac{1}{T} [-C_0 D e^{kt} + \frac{hD}{\alpha+1} (\frac{2t_1}{1-\alpha} - \frac{\alpha+3}{2} t_1^{\alpha+2}) + \frac{\beta D}{\alpha+1} (\frac{4t_1^3}{3-\alpha} - \frac{\alpha+4}{3} t_1^{\alpha+3})] \\ & + SD(T - t_1) - CI_e D(M - t_1) \end{aligned} \quad (18)$$

$$\frac{\partial TC_2(t_1, T)}{\partial T} = -\frac{1}{T^2} [A_0 e^{kt} - C_0 D t_1 e^{kt} + \frac{hD}{\alpha+1} (\frac{t_1^2}{1-\alpha} - \frac{t_1^{\alpha+3}}{2}) + \frac{\beta D}{\alpha+1} (\frac{t_1^4}{3-\alpha} - \frac{t_1^{\alpha+4}}{3})]$$

$$-\frac{1}{2}SD + \frac{CI_e Dt_1}{T^2} \left(M - \frac{t_1}{2}\right) \quad (19)$$

The optimal values of t_1 and T which minimize total average cost per unit per time, can be obtained by solving following equations

$$\frac{\partial TC_2(t_1, T)}{\partial t_1} = 0 \quad \text{and} \quad \frac{\partial TC_2(t_1, T)}{\partial T} = 0$$

Provided they satisfy the sufficient conditions

$$\frac{\partial^2 TC_2(t_1, T)}{\partial t_1^2} > 0, \frac{\partial^2 TC_2(t_1, T)}{\partial T^2} > 0$$

and

$$\left[\left\{ \frac{\partial^2 TC_2(t_1, T)}{\partial t_1^2} \right\} \left\{ \frac{\partial TC_2(t_1, T)}{\partial T^2} \right\} - \left\{ \frac{\partial^2 TC_2(t_1, T)}{\partial t_1 \partial T} \right\} \right] > 0$$

4. Numerical investigations

Illustration-I: ($M \leq t_1$)

To illustrate the inventory model we consider the following hypothetical values: $A_0=150, \beta=0.95, C_0=2, e=2.718, t=0.5, k=0.1, \alpha=0.5, h=0.2, s=7, D=10, C=12, I_c=0.15, I_e=0.11$

Based on above data and using the software Mathematica5.1, we calculate the optimal value of $M=50$ where time $t_1=2.0089$, optimal values of cycle length $T=4.4277$, optimal values of total average cost $TC_1=240.395$

Illustration-II: ($M > t_1$)

To illustrate the inventory model we consider the following hypothetical values:

$A_0=150, \beta=0.95, C_0=2, e=2.718, t=0.5, k=0.1, \alpha=0.5, h=0.2, s=7, D=10, C=12, I_c=0.15, I_e=0.11$

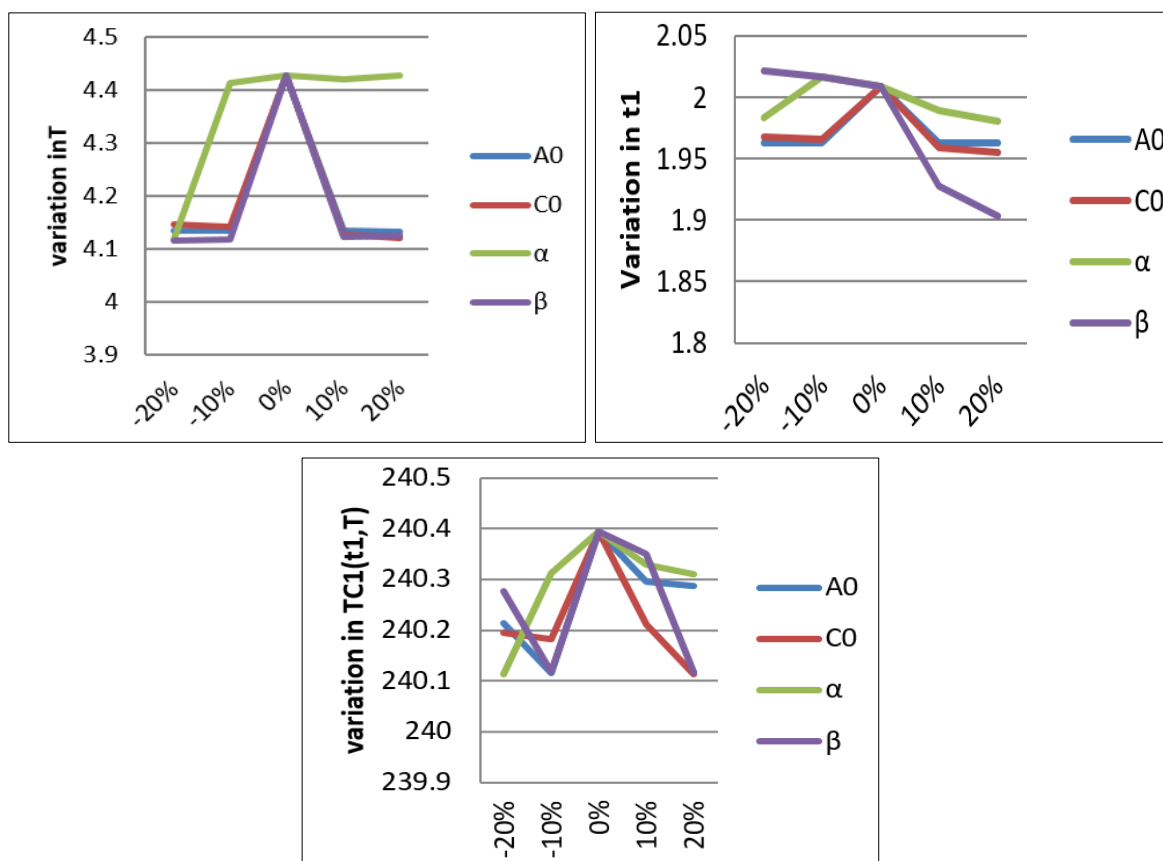
Based on above data and using the software Mathematica5.1, we calculate the optimal value at $M=35$ where time $t_1=1.6162$, optimal values of cycle length $T=3.699907$, optimal values of total average cost $TC_2=222.786$.

5. Sensitivity analysis

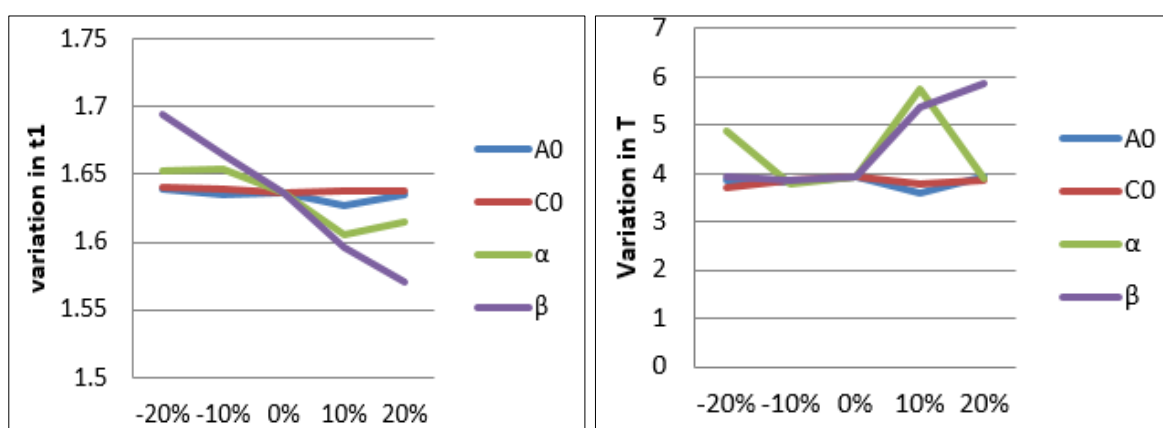
The sensitivity analysis is carried to explore the effect of changes in model parameters and total average cost of the optimal policies, by varying each parameter (-20%, -10%, 0%, 10%, 20%) at a time for the model under study for section 3. The results are presented in Table-1, 2. The relationship between the parameters, optimal cycle length T and optimal total average cost TC_1, TC_2 is shown in figure

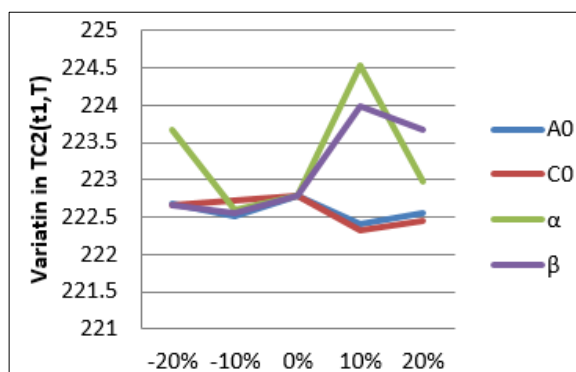
Table 1: (Case - I) Sensitivity analysis of the model ($M \leq t_1$)

Variation Parameters	Optimal Policies	Change in parameters				
		-20%	-10%	0%	10%	20%
A_0	t_1	1.9628	1.9628	2.0089	1.9627	1.9626
	T	4.1343	4.1340	4.4277	4.1331	4.1329
	$TC_1(t_1, T)$	240.215	240.115	240.395	240.295	240.287
C_0	t_1	1.9678	1.9662	2.0089	1.9592	1.9556
	T	4.147	4.1403	4.4277	4.1270	4.1203
	$TC_1(t_1, T)$	240.195	240.183	240.395	240.212	240.113
α	t_1	1.9833	2.0172	2.0089	1.9896	1.9803
	T	4.1181	4.4132	4.4277	4.4189	4.4268
	$TC_1(t_1, T)$	240.113	240.313	240.395	240.329	240.311
β	t_1	2.0216	2.0172	2.0089	1.9281	1.9034
	T	4.1162	4.1184	4.4277	4.1222	4.1239
	$TC_1(t_1, T)$	240.276	240.117	240.395	240.351	240.118

Table 2: (CASE-II) Sensitivity analysis of the model ($M > t_1$)

Variation Parameters	Optimal Policies	Change in parameters				
		-20%	-10%	0%	10%	20%
A_0	t_1	1.6387	1.6344	1.6362	1.6272	1.6353
	T	3.8617	3.8340	3.9410	3.5728	3.9150
	$TC_2(t_1, T)$	222.673	222.514	222.786	222.412	222.543
C_0	t_1	1.6403	1.6393	1.6362	1.6375	1.6382
	T	3.7192	3.8481	3.9410	3.7984	3.8388
	$TC_2(t_1, T)$	222.657	222.721	222.786	222.321	222.451
Alpha	t_1	1.6532	1.6535	1.6362	1.6058	1.6148
	T	4.8710	3.7904	3.9410	5.7408	3.9097
	$TC_2(t_1, T)$	223.678	222.598	222.786	224.534	222.984
Beta	t_1	1.6944	1.6646	1.6362	1.5960	1.5702
	T	3.9215	3.8601	3.9410	5.3577	5.8446
	$TC_2(t_1, T)$	222.659	222.561	222.786	223.983	223.659





6. Results and Discussions

The two numerical investigations for case-I ($M \leq t_1$) and case-II ($M > t_1$) are considered to study the effect of changes of the effect of changes of the system parameters A_0 , C_0 , α and β on the total average cost. Whenever one parameter is changing by some percentage all other parameters are kept at their original values. Investigation has been done for positive and negative changes of these four parameters. The results obtained are discussed in Table-I (case-I) and Table-II(case-II). A sensitivity analysis has also been conducted to reflect the effects of varied system performance. It is evident that the parameter β is comparatively less sensitive than other parameters in Table-I. Where as in table-II parameter α is more sensitive for positive changes than negative changes. Further it can examine various policy scenarios that are associated with different levels of economic inputs.

7. Conclusion

In this paper, we have developed an inventory model for Pareto deterioration with parabolic time varying holding cost and inflation under trade credit. Shortages are allowed. It is assumed that the supplier offers credit period to the retailer who has the reserve money to make the payments at the time of receiving items from supplier. Finally different costs have been illustrated through the numerical example and inferences of the theoretical results.

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