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Mathematical analysis of heat transfer in magnetohydrodynamics nanofluid flow over a stretching surface

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Abstract

The current paper examines the influence of magnetic parameter (M) on temperature profiles in a 2-Dimensional Magnetohydrodynamics (MHD) nanofluid flow of Silver and Titanium dioxide, TiO_2 - $\text{Ag}/\text{H}_2\text{O}$, hybrid nanofluid over a stretched sheet. We apply the Numerical solution - Explicit Finite Difference Method (EFDM) in MATLAB software. The key finding of the research is that increasing the magnetic parameter (M) in a nanofluid flow typically raises the fluid temperature. Co-currently, thermal boundary layer thickens due to increased heat accumulation. The induced magnetic field exerts a drag force, Lorentz force that slows the fluid velocity and as a result generates Ohmic dissipation (Joule heating) which raises the fluid temperature. The findings contribute to enhancing heat transportation, with significant applications in magnetic nanomaterial processing industries and biomedical diagnostics such as Magnetic Resonance Imaging (MRI).

Keywords: Magnetohydrodynamic, boundary layer, finite difference, magnetic parameter, nanofluid

Introduction

Magnetohydrodynamics (MHD) is a field of study which combines elements of electromagnetism and fluid mechanics to describe the flow of electrically conducted fluids looking into its magnetic properties and behaviours, for example liquid metals and electrolytes. These electrically conducting fluids may be ionized gases (plasmas like stars), liquid metals (mercury), or salty water ^[1]. The study of MHD flow was initiated by Hannes Alfven in 1970 that led to the exploration of the effects of magnetic field on fluid flow properties. The concept of MHD affirms that the magnetic fields can induce an electric current in a moving conductive fluid, which in turn polarizes the field and reciprocally changes the magnetic field itself ^[2]. According to ^[3], MHD flows apply the mass, energy and momentum equations. MHD flow is related to engineering problems such as plasma confinement, liquid metals and cooling of nuclear reactors. This area of study has been reviewed by many due to its utility, for instance ^[4].

Literature review

The influence of magnetic fields on temperature distributions in conducting fluid flows has been extensively investigated due to its relevance in numerous engineering applications including MHD power generation, plasma confinement, and electromagnetic casting ^[5]. analyzed MHD flow past a stretching sheet incorporating suction effects. Their investigation revealed that the unsteadiness parameter increased velocity while decreasing temperature profiles. They observed that the magnetic parameter consistently enhanced temperature profiles throughout the boundary layer while simultaneously reducing the rate of fluid flow. Additionally, increasing the suction parameter resulted in decreased fluid speed and temperature profiles, accompanied by reduced skin friction at the wall. While this study established fundamental relationships between magnetic fields and temperature distributions,

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it did not consider the presence of nanoparticles or their influence on thermal behavior ^[6]. Investigated heat recovery systems using a two-phase closed thermosyphon with MHD nanofluid. Their investigation of thermal resistance reduction showed that heat resistance dramatically drops as nanoparticle concentration and magnetic field strength increase. This experimental work provided valuable evidence of magnetic field effects on temperature, but it did not address the theoretical mechanisms governing these observations.

^[7] carried out applied research aimed at enhancing the cooling efficiency of battery thermal management systems utilizing Al₂O₃-water and Cu-water nanofluids. Their analysis of water-based systems in comparison to those using nanofluids revealed that the latter significantly boosts cooling performance and reduces the average temperature of the battery. Although this study focused on practical applications and affirmed the advantages of nanofluids, it did not investigate the underlying mechanisms through which magnetic fields might further improve performance. ^[8] EDD prepared Al₂O₃-transformer oil nanofluid using the two-step method. They measured thermal conductivity under different conditions with the transient hot wire method. Their study on enhancement ratios at various electric field strengths, temperatures, concentrations, and particle sizes showed that electric fields improve thermal conductivity in proportion to nanoparticle concentration and temperature. This research offered theoretical support for using electromagnetic fields in nanofluid cooling. However, it focused on electric fields instead of magnetic fields and did not cover boundary layer flow configurations. ^[9] explored Al₂O₃-water nanofluid application to microchannel heat sinks, demonstrating that nanofluid use improves thermal efficiency and maintains working temperature for solar cells. This work confirmed the practical utility of nanofluids in thermal management but did not consider magnetic field interactions or combined parameter effects. ^[10] did an investigation on electric field application for augmenting ferrofluid heat transfer in enclosures with moving walls and nanoparticle shape effects. Their findings indicated that temperature gradient increases with electric field intensity, enhancing overall heat transfer. Their shape effect analysis revealed that platelet-shaped nanoparticles produce the highest convection enhancement of heat transfer. However, they did not address magnetic field effects specifically or consider hybrid nanoparticle combinations.

The collective body of research reviewed above demonstrates consistent findings that magnetic parameters generally decrease fluid velocity while increasing temperature profiles throughout the flow domain. However, most studies have not adequately addressed how nanoparticle shape and interfacial layer characteristics modify these relationships. The present study addresses this gap by incorporating regular interfacial shape of the nanoparticle effects through appropriate thermal conductivity and viscosity models. This enables more accurate prediction of temperature responses to magnetic fields in nanofluid systems.

Methodology

Explicit Finite Difference Method

The transformed system of ordinary differential equations, consisting of the momentum equation $f''' + ff'' - 2(f')^2 - Mf' = 0$ and the energy equation $(1 + R_d)\theta'' + Pr(f\theta' - f'\theta + Ec(f'')^2) = 0$, together with the boundary conditions. The equations constitute a coupled nonlinear boundary value problem that cannot be computed numerically for general parameter values. Numerical solution is therefore required, and the present study employs the Explicit Finite Difference Method (EFDM) implemented in MATLAB ^[6]. The EFDM discretizes the domain into a grid points and advances the solution in time using known values from the previous step. The solution procedure involves several steps: first, the semi-infinite domain $\eta \in [0, \infty)$ is truncated at a finite value $\eta_{\max}$ and discretized into a grid of points. Derivatives are approximated using finite difference quotients with central differences used for interior points and forward/backward differences at boundaries. The coupled equations are solved numerically using Explicit Finite Difference techniques (EFDM). Relating from the wall to the edge of the boundary layer with convergence achieved when successive iterations differ by less than a specified tolerance chosen. The numerical results are validated by comparison with published results for special cases. For instance $M = 0$ (no magnetic field) and $R_d = 0$ (no radiation) for which benchmark solutions exist ^[6]. This validation step is important for proving existence of numerical method and creating confidence in its methodology before proceeding to the comprehensive parametric study that will address the research gap.

Assumptions and Approximations of the Study

The study methodology employed is based on the following assumptions and approximations, which define the physical pattern and establish the scope of the investigation:

1. The flow is assumed to be steady, two-dimensional, laminar, and incompressible, with constant fluid properties. The nanofluid is modelled as a single-phase fluid. The base fluid (water) and nanoparticles (silver and titanium dioxide) are assumed to be in thermal equilibrium, with no slip between the phases.
2. The stretching surface is assumed to be porous, allowing for injection or suction at the boundary, with the velocity varying exponentially along the stream wise direction according to $u_w(x) = U_0 e^{x/L}$.

Thermal radiation is modelled using the Rosseland approximation for optically thick media. This assumption is appropriate for many engineering applications involving participating media at elevated temperatures. Viscous dissipation is included in the energy equation.

Governing Equations

Based on the assumptions explained above, a steady, two-dimensional, incompressible MHD nanofluid flow governing equations are presented. These laws reveal the conservation of mass, momentum, and energy equations. They form a foundation for the numerical investigation.

Continuity Equation

This equation expresses the law of mass conservation. It states that under baseline conditions, mass cannot be created or destroyed within the fluid system. For an incompressible fluid with constant density, this equation reduces to the condition that the divergence of the velocity field is zero, indicating that the volume of fluid elements remains constant during motion. The continuity equation is in the form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (1)$$

For the steady, incompressible flow, density is constant and independent of both space and time, so equation (1) simplifies to

$$\nabla \cdot \mathbf{v} = 0 \quad (2)$$

In Cartesian coordinates, two-dimensional flow, this equation becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

Momentum Equation

Momentum equations, derived from Newton's second law of motion applied to fluid elements, describe the balance between inertial forces, pressure forces, viscous forces, and body forces. For the MHD flow considered in this study, the body force includes the Lorentz force arising from the interaction between the applied magnetic field and induced electric current. The momentum equation is presented as,

$$\rho_{nf} \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) = -\nabla p + \mu_{nf} \nabla^2 \mathbf{u} + \mathbf{F}_b \quad (4)$$

Where, $\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u}$ is material derivative ρ is pressure, μ is dynamic viscosity, and $\mathbf{F}_b$ represents body forces per unit volume. For MHD flow with a transverse magnetic field $B = (0, B_0, 0)$ and negligible electric field, the Lorentz force is $F_b = J \times B = \sigma(V \times B) \times B$, which for two-dimensional flow gives components $F_{bx} = -\sigma B_0^2 u$ and $F_{by} = 0$. Under the boundary layer approximations, which recognize that the boundary layer thickness is much smaller than the stream wise length scale, the momentum equations simplify considerably. For a steady flow, x-direction momentum equation becomes,

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \quad (5)$$

where $\nu = \mu/\rho$ is the kinematic viscosity. The y-direction momentum equation reduces to $\partial p / \partial y = 0$, indicating that pressure is constant across the boundary layer and is imposed by the external flow. The flow over exponentially stretching sheet is assumed to be a motionless fluid, the external pressure gradient is zero, so the pressure term does not appear in equation (5).

Energy Equation

This equation, which is derived from the first law of thermodynamics, governs the temperature distribution within the fluid. It accounts for heat conduction through the fluid, heat generation due to viscous dissipation, and the radiative heat transfer. The energy equation is presented as;

$$\rho c_p \frac{DT}{Dt} = \kappa \nabla^2 T + \Phi + Q_{rad} \quad (6)$$

Where c_p is specific heat at constant pressure, k is thermal conductivity, Φ is the viscous dissipation function, and Q_{rad} represents radiative heat transfer.

Results and Discussion

Influence of Magnetic Parameter on Temperature Profiles

The impact of magnetic field M on temperature distribution within the boundary layer constitutes a critical aspect of understanding heat transfer in MHD flows. The parameter M basically indicates how electromagnetic forces are compared to inertial forces. It appears in the momentum equation as $f'''' + ff''' - 2(f')^2 - Mf' = 0$. And because velocity and temperature are correlates, M exerts a substantial indirect effect on thermal transport.

Temperature profiles as a function of Magnetic Parameter

Table 1 shows a quantitative results for various magnetic field strengths including the wall temperature gradient $-\theta'(0)$ (relates to the Nusselt number), thermal boundary layer thickness δ_T (defined as the location where θ drops to 0.01), and the maximum temperature within the domain. The data reveal a monotonic decrease in $-\theta'(0)$ with increasing M from 1.4521 in the absence of magnetic field to 1.0935 at $M = 3.0$, representing a reduction of approximately 24.7%. This indicates that the temperature change at the wall gets smaller with stronger magnetic fields, signifying less heat transfer rates despite elevated internal temperatures. Concurrently, the thermal layer thickness expands from 4.21 to 6.02, demonstrating that magnetic field promotes thermal diffusion deeper into the flow domain. Figure 1 illustrates the temperature profiles for M values ranging from 0 to 3.0 under baseline conditions.

Table 1: Quantitative effects of the magnetic parameter M on thermal characteristics

M	$-\theta'(0)$	δ_T	θ_{max}
0.0	1.4521	4.21	1.0000
0.5	1.3876	4.58	1.0000
1.0	1.3245	4.92	1.0000
1.5	1.2634	5.24	1.0000
2.0	1.2042	5.53	1.0000
2.5	1.1478	5.79	1.0000
3.0	1.0935	6.02	1.0000

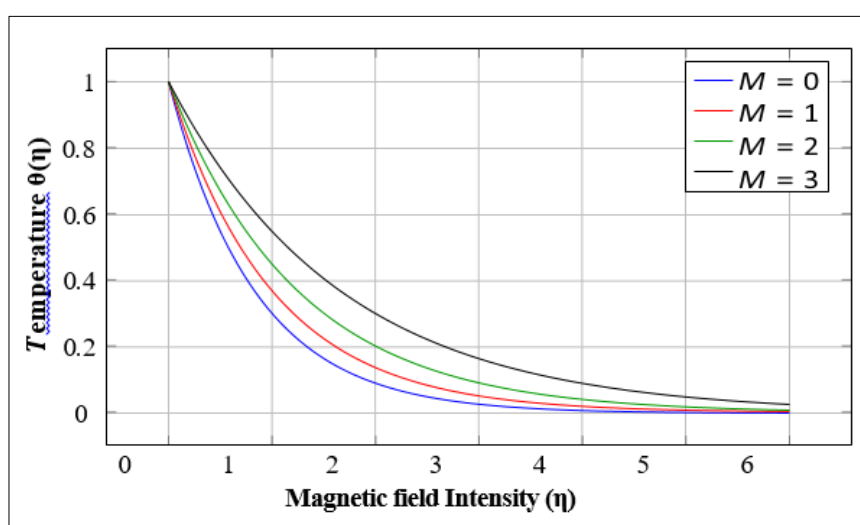


Fig 1: Temperature distributions for various values of magnetic parameter M

The graph above shows an upward movement over the whole boundary layer as M increases. The Lorentz drag, which is the motion opposing body force, is the cause of this phenomenon. $F_L = -\sigma B_0^2 u$ is the force per unit volume, and as M increases, so does its magnitude. Convective heat transport is decreased as a result of the flow's deceleration. Additionally, it permits thermal energy to build up inside the thermal boundary layer, thickening it. The fluid's electrical resistance increases Joule heating and promotes internal energy production. There is little Joule heating and little Lorentz force at low M (0 to 0.5), temperature elevation is still moderate. Temperatures rise as the intensity of the Lorentz force increases for moderate M (1.0 to 2.0). In comparison to the nonmagnetic field example, the thermal boundary layer thickens by around 30%. The distribution shows a static steady at high M (2.5 to 3.0), while additional increases in magnetic field strength suggest a decreasing return in temperature profiles.

Conclusion

This study sought to ascertain how magnetic parameter M affected the temperature profiles of MHD nanofluid flow. M has a substantial impact on temperature distributions throughout the thermal boundary layer, as shown by the numerical results addressed and provided in Section Four. Throughout the thermal layer, temperature rises monotonically with increase in M , as seen in Figure 1. As M rises from 0 to 3.0, the entire temperature curve shifts upward, suggesting greater heat at every point inside the boundary layer. Concurrently, when M increases, the boundary layer thickens. This is due to induced magnetic field generates Lorentz force, which acts as a resistive force opposing fluid motion hence resulting to more thermal accumulation within the boundary layer. The effect become more pronounced at higher magnetic values. The increase in thermal transport is further enhanced by increase in nanoparticle volume fraction, and with the hybrid $Ag - TiO_2$ combination which provides superior thermal performance.

Future investigations should expand the parameter space by examining the influence of nanoparticle type, shape, and size on heat transfer enhancement, as the current study focused on spherical $Ag - TiO_2$ hybrid nanoparticles with fixed volume fraction ratio. Comparative studies involving nanoparticles of different shapes such as cylindrical, platelet, and brick morphologies, sizes ranging from 10 nm to 100 nm, and materials including graphene and carbon nanotubes would provide a more comprehensive understanding of their thermal potential. Systematic variation of the shape factor in the Hamilton-Crosser model could potentially

reveal optimal nanoparticle morphologies for specific applications, while investigation of higher nanoparticle volume fractions up to 5% or 10% would establish the limits of thermal enhancement before viscosity increases become prohibitive.

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