

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
NAAS Rating (2025): 4.49
Maths 2026; 11(5): 72-76
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<https://www.mathsjournal.com>
Received: 25-02-2026
Accepted: 30-03-2026

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New Ratio-Type and Product-Type Estimators Using Systematic Sampling Design

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DOI: <https://www.doi.org/10.22271/math.2026.v11.i5a.2319>

Abstract

This paper proposes improved ratio-type and product-type estimators for estimating the finite population mean of the study variable using systematic sampling design. The bias and mean squared error (MSE) of the proposed estimators have been obtained. Efficiency comparisons of the proposed estimators have been made with the competing estimators. Theoretical and empirical studies have been carried out to demonstrate the efficiencies of the proposed estimators over the existing estimators.

Keywords: Systematic sampling, auxiliary variable, efficiency comparison

1. Introduction

Use of the auxiliary variables is very important for the construction of the efficient estimators for the estimation of population parameters under different sampling designs. The importance of systematic sampling cannot be overemphasized, being one of the sampling schemes widely used in practice due to its simplicity. Systematic sampling has got the nice feature of selecting the whole sample with just one random start. Apart from its simplicity, which is of considerable importance, this method yields certain estimators which are more efficient than simple random sampling and stratified random sampling for certain types of populations, see Cochran (1946), Gautschi (1957), Hajeck (1959).

Consider a finite population $U = \{U_1, \dots, U_N\}$ of size N units, numbered from 1 to N . Let us assume that $N = nk$, where n and k are positive integers. Thus, there will be k samples, each of size n . Observe the study variate y and the auxiliary variates x and z for each and every unit selected in the sample. Let $y_{ij}, x_{ij}, z_{ij}; i = 1, 2, \dots, k; j = 1, 2, \dots, n$ denote the value of j th unit in the i th systematic sample. Let the systematic sample means be defined as $\bar{y}_{sy} = \frac{1}{n} \sum_{j=1}^n y_{ij}$, $\bar{x}_{sy} = \frac{1}{n} \sum_{j=1}^n x_{ij}$ and $\bar{z}_{sy} = \frac{1}{n} \sum_{j=1}^n z_{ij}$ which are the unbiased estimators of the population means $\bar{Y} = \frac{1}{N} \sum_{j=1}^N y_{ij}$, $\bar{X} = \frac{1}{N} \sum_{j=1}^N x_{ij}$ and $\bar{Z} = \frac{1}{N} \sum_{j=1}^N z_{ij}$ of y , x and z , respectively. Let $S_y^2 = \left(\frac{1}{N-1}\right) \sum_{i=1}^k \sum_{j=1}^n (y_{ij} - \bar{Y})^2$, $S_x^2 = \left(\frac{1}{N-1}\right) \sum_{i=1}^k \sum_{j=1}^n (x_{ij} - \bar{X})^2$ and $S_z^2 = \left(\frac{1}{N-1}\right) \sum_{i=1}^k \sum_{j=1}^n (z_{ij} - \bar{Z})^2$ be the population variances of y , x and z , respectively and let population covariances between y and x , y and z and x and z be $S_{yx} = \left(\frac{1}{N-1}\right) \sum_{i=1}^k \sum_{j=1}^n (y_{ij} - \bar{Y})(x_{ij} - \bar{X})$, $S_{yz} = \left(\frac{1}{N-1}\right) \sum_{i=1}^k \sum_{j=1}^n (y_{ij} - \bar{Y})(z_{ij} - \bar{Z})$ and $S_{xz} = \left(\frac{1}{N-1}\right) \sum_{i=1}^k \sum_{j=1}^n (x_{ij} - \bar{X})(z_{ij} - \bar{Z})$, respectively and C_y, C_x, C_z being the coefficients of variation of the variates y, x, z , respectively.

To obtain the properties of the estimators up to first order of approximation, let us consider

$e_0 = \frac{\bar{y}_{sy} - \bar{Y}}{\bar{Y}}$, $e_1 = \frac{\bar{x}_{sy} - \bar{X}}{\bar{X}}$ and $e_2 = \frac{\bar{z}_{sy} - \bar{Z}}{\bar{Z}}$ such that $E(e_i) = 0$, for $i = 0, 1$ and 2 . Then we have

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$$E(e_0^2) = \theta \rho_y^* C_y^2, E(e_1^2) = \theta \rho_x^* C_x^2, E(e_2^2) = \theta \rho_z^* C_z^2,$$

$$E(e_0 e_1) = \theta k_{yx} C_x^2 \sqrt{\rho_y^* \rho_x^*}, E(e_0 e_2) = \theta k_{yz} C_z^2 \sqrt{\rho_y^* \rho_z^*}, E(e_1 e_2) = \theta k_{xz} C_z^2 \sqrt{\rho_x^* \rho_z^*},$$

where

$$\theta = \frac{N-1}{Nn}, \rho_y^* = [1 + (n-1)\rho_y], \rho_x^* = [1 + (n-1)\rho_x] \text{ and } \rho_z^* = [1 + (n-1)\rho_z],$$

$$C_y = \frac{S_y}{\bar{Y}}, C_x = \frac{S_x}{\bar{X}}, C_z = \frac{S_z}{\bar{Z}}, k_{yx} = \rho_{yx} \frac{C_y}{C_x}, k_{yz} = \rho_{yz} \frac{C_y}{C_z}, k_{xz} = \rho_{xz} \frac{C_x}{C_z}, \rho_{yx} = \frac{S_{yx}}{S_y S_x}, \rho_{yz} = \frac{S_{yz}}{S_y S_z}, \rho_{xz} = \frac{S_{xz}}{S_x S_z},$$

and ρ_y, ρ_x and ρ_z are the intraclass correlation coefficients among the pairs of units for the variables y, x and z , respectively, defined by

$$\rho_y = \frac{E(y_{ij} - \bar{Y})(y'_{ij} - \bar{Y})}{E(y_{ij} - \bar{Y})^2}, \rho_x = \frac{E(x_{ij} - \bar{X})(x'_{ij} - \bar{X})}{E(x_{ij} - \bar{X})^2}, \rho_z = \frac{E(z_{ij} - \bar{Z})(z'_{ij} - \bar{Z})}{E(z_{ij} - \bar{Z})^2}.$$

The variance of the usual unbiased estimator for the population mean in systematic sampling is given by

$$Var(\bar{y}_{sy}) = \theta \bar{Y}^2 \rho_y^* C_y^2. \quad (1.1)$$

The ratio and product estimators for finite population mean suggested by Swain (1964) and Shukla (1971) are, respectively, given by

$$\bar{y}_{Rsy} = \bar{y}_{sy} \left(\frac{\bar{X}}{\bar{x}_{sy}} \right) \quad (1.2)$$

$$\text{and } \bar{y}_{Psy} = \bar{y}_{sy} \left(\frac{\bar{z}_{sy}}{\bar{Z}} \right). \quad (1.3)$$

The mean square errors of the estimators $\bar{y}_{Rsy}$ and $\bar{y}_{Psy}$, to the first order of approximation, are, respectively, given as

$$M(\bar{y}_{Rsy}) = \theta \bar{Y}^2 [\rho_y^* C_y^2 + \rho_x^* C_x^2 (1 - 2k_{yx}^*)] \quad (1.4)$$

$$\text{and } M(\bar{y}_{Psy}) = \theta \bar{Y}^2 [\rho_y^* C_y^2 + \rho_z^* C_z^2 (1 + 2k_{yz}^*)] \quad (1.5)$$

$$\text{where } k_{yx}^* = k_{yx} \sqrt{\frac{\rho_y^*}{\rho_x^*}} \text{ and } k_{yz}^* = k_{yz} \sqrt{\frac{\rho_y^*}{\rho_z^*}}.$$

It is evident from (1.2) and (1.4) that the estimator $\bar{y}_{Rsy}$ will perform better than $\bar{y}_{sy}$ if the condition $k_{yx}^* \geq \frac{1}{2}$ is satisfied. Similarly from (1.3) and (1.5), $\bar{y}_{Psy}$ becomes more efficient than $\bar{y}_{sy}$ if the condition $k_{yz}^* \leq -\frac{1}{2}$ is met.

Utilizing information on auxiliary variables, Singh *et al.* (2011) suggested the modified exponential- type ratio and product estimators for the population mean under systematic sampling as follows

$$\bar{y}_{Resy} = \bar{y}_{sy} \exp \left(\frac{\bar{X} - \bar{x}_{sy}}{\bar{X} + \bar{x}_{sy}} \right) \quad (1.6)$$

and

$$\bar{y}_{Pesy} = \bar{y}_{sy} \exp \left(\frac{\bar{z}_{sy} - \bar{Z}}{\bar{z}_{sy} + \bar{Z}} \right). \quad (1.7)$$

The mean square errors of the estimators $\bar{y}_{Resy}$ and $\bar{y}_{Pesy}$, to the first order of approximation, are, respectively, given as

$$M(\bar{y}_{Resy}) = \theta \bar{Y}^2 \left[\rho_y^* C_y^2 + \frac{\rho_x^* C_x^2}{4} (1 - 4k_{yx}^*) \right], \quad (1.8)$$

$$M(\bar{y}_{Pesy}) = \theta \bar{Y}^2 \left[\rho_y^* C_y^2 + \frac{\rho_z^* C_z^2}{4} (1 + 4k_{yz}^*) \right]. \quad (1.9)$$

While the exponential ratio estimator $\bar{y}_{Resy}$ performs better than $\bar{y}_{sy}$ when the condition $k_{yx}^* \geq \frac{1}{4}$ remains satisfied, the same is found to fare better than $\bar{y}_{Rsy}$ if $k_{yx}^* \leq \frac{3}{4}$. Similarly, $\bar{y}_{Pesy}$ is found to be more efficient than the estimators $\bar{y}_{sy}$ and $\bar{y}_{Psy}$ when the

conditions $k_{yz}^* \leq -\frac{1}{4}$ and $k_{yz}^* \geq -\frac{3}{4}$, hold good, respectively. Thus, $\bar{y}_{Resy}$ will be better than both $\bar{y}_{sy}$ and $\bar{y}_{Rsy}$ when $\frac{1}{4} \leq k_{yx}^* \leq \frac{3}{4}$. Analogously, $\bar{y}_{Pesy}$ can be shown to be better than both $\bar{y}_{sy}$ and $\bar{y}_{Psy}$ provided $-\frac{3}{4} \leq k_{yz}^* \leq -\frac{1}{4}$.

2. New Ratio-Type Estimator Under Systematic Sampling

We suggest the following ratio-type estimator for the population mean using systematic sampling design as

$$\bar{y}'_{Resy} = \alpha \bar{y}_{sy} + (1 - \alpha) \bar{y}_{Resy} \quad (2.1)$$

where α is a real constant to be determined such that the MSE of $\bar{y}'_{Resy}$ is minimized. It is observed that the estimator reduces to

1. usual unbiased estimator $\bar{y}_{sy}$ for $\alpha = 1$,
2. Singh *et al.* (2011) ratio estimator $\bar{y}_{Rsy}$ for $\alpha = 0$.

Taking the expectations in (2.1), the *bias* of the estimator is obtained as

$$B(\bar{y}'_{Resy}) = \theta \bar{Y} (1 - \alpha) C_x^2 \rho_x^* \left[\frac{3}{8} - \frac{k_{yx}^*}{2} \right]. \quad (2.2)$$

The MSE of the estimator, upto the first order of approximation, is given by

$$M(\bar{y}'_{Resy}) = \theta \bar{Y}^2 \left[\rho_y^* C_y^2 + \frac{1}{4} \rho_x^* C_x^2 \{ (1 - \alpha)^2 - 4(1 - \alpha) k_{yx}^* \} \right]. \quad (2.3)$$

2.1: Optimal Value of the Weight α

The optimal value of the weight α , for which the MSE of the estimator is minimized, is obtained by using the following condition

$$\frac{\partial}{\partial \alpha} M(\bar{y}'_{Resy}) = 0. \quad (2.1.1)$$

On solving (2.1.1), we have

$$\alpha^* = 1 - 2k_{yx}^*. \quad (2.1.2)$$

Using the above optimal value of α in (2.3) we obtain the minimum attainable MSE of the estimator $\bar{y}'_{Resy}$ as

$$M(\bar{y}'_{Resy})_{opt} = \theta \bar{Y}^2 \rho_y^* C_y^2 [1 - \rho_{yx}^2]. \quad (2.1.3)$$

2.2: Efficiency Comparison

For making efficiency comparisons of the estimator $\bar{y}'_{Resy}$ with the existing estimators, we have from (1.1), (1.4), (1.8), (2.1.3)

1. $M(\bar{y}'_{Resy}) < V(\bar{y}_{sy})$ if

$$\alpha > 1 - 4k_{yx}^*, \quad (2.2.1)$$

2. $M(\bar{y}'_{Resy}) < M(\bar{y}_{Rsy})$ if

$$\alpha < 3 - 4k_{yx}^*, \quad (2.2.2)$$

3. $M(\bar{y}'_{Resy}) < M(\bar{y}_{Pesy})$ if

$$\alpha > 2 - 4k_{yx}^*. \quad (2.2.3)$$

2.3: Numerical Illustration

To examine the efficiency of the proposed estimator over the existing estimators, we have considered the following numerical illustration:

Population I - [Source: Tailor *et al.* (6)]

$N = 15, n = 3, \bar{X} = 44.47, \bar{Y} = 80, C_y = 0.56, C_x = 0.28,$

$S_y^2 = 2000, S_x^2 = 149.55, S_{yx} = 538.57,$

$\rho_{yx} = 0.9848, \rho_y = 0.6652$ and $\rho_x = 0.707$.

Table 2.3.1: Mean Square Error (MSE) and Percent Relative Efficiency (PRE) of Different Estimators of Population Mean

Estimator	MSE	PRE
$\bar{y}_{sy}$	1454.5843	100
$\bar{y}_{Rsy}$	373.1425	389.82
$\bar{y}_{Resy}$	820.0032	177.387
$\bar{y}'_{Resy}$	44.429	3273.952

3. New Product-Type Estimator in Systematic Sampling

We suggest the following product-type estimator for the population mean using systematic sampling design as

$$\bar{y}'_{Pesy} = \beta \bar{y}_{sy} + (1 - \beta) \bar{y}_{Pesy} \quad (3.1)$$

where β is a real constant to be determined such that the MSE of $\bar{y}'_{Pesy}$ is minimized. It is observed that the estimator reduces to

1. usual unbiased estimator $\bar{y}_{sy}$ for $\beta = 1$,
2. Singh *et al.* (2011) ratio estimator $\bar{y}_{Psy}$ for $\beta = 0$.

Taking the expectations in (3.1) the *bias* of the estimators is obtained, respectively, as

$$B(\bar{y}'_{Pesy}) = \theta \bar{Y} (1 - \beta) C_y^2 \rho_z^* \left[\frac{k_{yz}^*}{2} - \frac{1}{8} \right]. \quad (3.2)$$

The MSE of the estimator, upto the first order of approximation, is given by

$$M(\bar{y}'_{Pesy}) = \theta \bar{Y}^2 \left[\rho_y^* C_y^2 + \frac{1}{4} \rho_z^* C_z^2 \{ (1 - \beta)^2 + 4(1 - \beta) k_{yz}^* \} \right]. \quad (3.3)$$

3.1: Optimal Value of β

The optimal value of β , for which the MSE of the estimator is minimized, is obtained by using the following condition

$$\frac{\partial}{\partial \beta} M(\bar{y}'_{Pesy}) = 0. \quad (3.1.1)$$

On solving (3.1.1), we have

$$\beta^* = 1 + 2k_{yz}^*. \quad (3.1.2)$$

Using the optimal value of β in (2.5) we obtain the minimum attainable MSE of the estimator $\bar{y}'_{Pesy}$ as

$$MSE(\bar{y}'_{Pesy})_{opt} = \theta \bar{Y}^2 \rho_y^* C_y^2 [1 - \rho_{yz}^2]. \quad (3.1.3)$$

3.2: Efficiency Comparison

For making efficiency comparisons of the estimator $\bar{y}'_{Pesy}$ with the existing estimators, we have from (1.1), (1.5), (1.9), (3.1.3)

1. $M(\bar{y}'_{Pesy}) < V(\bar{y}_{sy})$ if

$$\beta > 1 + 4k_{yz}^*, \quad (3.2.1)$$

2. $M(\bar{y}'_{Pesy}) < M(\bar{y}_{Psy})$ if

$$\beta < 3 + 4k_{yz}^*, \quad (3.2.2)$$

3. $M(\bar{y}'_{Pesy}) < M(\bar{y}_{Pesy})$ if

$$\beta > 2 + 4k_{yz}^*. \quad (3.2.3)$$

3.3: Numerical Illustration

To examine the efficiencies of the proposed estimator over the existing estimators, we have considered the following numerical illustration:

Population II - [Source: Tailor *et al.* (6)]

$N = 15, n = 3, \bar{Y} = 80, \bar{Z} = 48.40, C_y = 0.56, C_z = 0.43,$

$$S_y^2 = 2000, S_z^2 = 427.83, S_{yz} = -902.86,$$

$\rho_{yz} = -0.9760, \rho_y = 0.6652$ and $\rho_z = 0.5487$.

Table 3.3.1: Mean Square Error (MSE) and Percent Relative Efficiencies (PRE) of Different Estimators of Population Mean

Estimator	MSE	PRE
$\bar{y}_{sy}$	1454.5843	100
$\bar{y}_{psy}$	158.1395	919.81
$\bar{y}_{pesy}$	614.6941	236.635
$\bar{y}'_{pesy}$	71.1507	2044.371

4. Conclusion

The newly proposed estimators, both ratio-type and product-type, based on systematic sampling strategies, perform better than the corresponding competing estimators already available in literature under conditions that hold good in practice. The empirical investigation bears a testimony to the theoretical findings.

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