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Comparing nonparametric tests of variances based on the simple tree alternative

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Abstract

This paper compares four nonparametric tests for variances using based on the simple tree alternative. Various underlying distributions are considered including symmetric, symmetric heavy tailed, and skewed. Three versions of three of the tests are used with differences being in using the sample mean, sample median, or sample trimmed mean. Cases are considered in which the location parameters are equal and when they are not equal. Comments are made as to how the different location parameters affect the alpha values and powers of the tests. Recommendations are given.

Keywords: Skewed, symmetric, Conover, Hollander, mood, Moses, simple tree, scale

Introduction

There are situations in which we have a control and various levels of a treatment in which we may want to test to see if there is any difference in variation between the control and at least one of the levels of the treatment while assuming if there is a difference the treatment level will have the larger variation. We are testing for the simple tree alternative as given below

$$H_0: \sigma_1 = \dots = \sigma_k \quad (1)$$

$H_a: \sigma_1 \leq [\sigma_2, \dots, \sigma_k]$ where at least one inequality is strict where population 1 is designated as the control. We will also assume that the underlying population distributions are unknown and will consider nonparametric tests. We will begin by examining some two-sample tests for differences in variances or spread. These two-sample tests will be extended to the simple tree alternative by defining population 1 as the control and the combined treatment population as population 2.

Moses Test

The Moses test is a nonparametric test for testing the null hypothesis of equality variances for two populations (Moses, 1963) ^[9]. The Moses test does not assume equality of location parameters (Azman, 2021, Daniel, 2016) ^[2, 5]. It is a two-sample test for variances testing the following set of hypotheses.

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_{a1}: \sigma_1^2 \neq \sigma_2^2, H_{a2}: \sigma_1^2 < \sigma_2^2, H_{a3}: \sigma_1^2 > \sigma_2^2$$

The Moses test statistic is obtained by dividing two samples into m_1 and m_2 subsamples of equal size. For each of the subsamples, the mean of the subsample is calculated, followed by summing of squared deviations between each observation and the sample mean for each subsample. Furthermore, the sums of squared deviations for each of the m_1 subsets in the first sample are indicated as C_1 to C_{m_1} , and those of the second set of m_2 as D_1 to D_{m_2} .

The Mann-Whitney test (Mann and Whitney, 1947) ^[8] will be performed. The C values from the first sample and the D values from the second will be combined and then will be ranked from smallest to largest, and each value will be assigned a rank. The Moses test statistic (M) is the sum of the ranks associated with the m_1 C values from the first sample and then with a constant subtracted. It is given by

$$M = \sum S_i - \frac{m_1 (m_1 + 1)}{2}$$

The standardized version of the Moses test is given by:

$$M^* = \frac{M - E_0(M)}{\sqrt{\text{var}_0(M)}}$$

$$E_0(M) = \frac{m_1 m_2}{2}$$

$$\text{var}_0(M) = \frac{m_1 m_2 (m_1 + m_2 + 1)}{12}$$

The asymptotic distribution of M^* under H_0 is a standard normal distribution. Alsubie and Magel (2020) ^[11] extended the use of the Moses test to the simple tree alternative.

Conover Squared Ranks Test

The Conover Squared Ranks test is a nonparametric test that uses independent sample from two populations to test of the null hypothesis of equal variances (Conover, 1999) ^[4].

To compute the Conover Squared Ranks test statistic (CSR), select two independent random sample from two populations $X_1, X_2, \dots, X_n$ and $Y_1, Y_2, \dots, Y_m$ with sizes of n and m , correspondingly. For each sample, the absolute deviations between each observation and their actual mean will be computed as follows:

$$U_i = |X_i - \mu_1| \text{ for population 1 (i = 1, \dots, n)}$$

$$\text{and } V_j = |Y_j - \mu_2| \text{ for population 2 (j = 1, \dots, m)}$$

where, μ_1 is the actual mean for population 1 and μ_2 is the actual mean for population 2. If these population means are unknown, they can be substituted with sample means $\bar{X}$ and $\bar{Y}$, respectively and the test is approximate.

The U_i 's and V_j 's are combined into a single sample and ranks are assigned from 1 to $N = n + m$. When two or more observations are equal, the average of the ranks is assigned to all of these values. $R(U_i)$ and $R(V_j)$ denote the ranks of U_i and V_j , respectively. The Conover Squared Ranks test statistic is defined by:

$$CSR = \sum_{i=1}^{n_j} (R(U_i))^2$$

The standardized version of the Conover Squared Ranks test is given by:

$$CSR^* = \frac{T - E_0(T)}{\sqrt{\text{var}_0(T)}}$$

$$E_0(CSR) = \frac{n(N+1)(2N+1)}{6}$$

$$\text{var}_0(CSR) = \frac{mn(N+1)(2N+1)(8N+11)}{180}$$

Where: $N = n + m$.

The Conover Squared Ranks test statistic CSR^* has approximately a standard normal distribution under the null hypothesis when the means are known. When the means are unknown, it is recommended that this approximation holds by using sample means. For a one-sided alternative ($H_{a2}: \sigma_1^2 < \sigma_2^2$), H_0 is rejected when $CSR^* \leq -Z_\alpha$.

Hollander test of extreme reaction

The Hollander test of extreme reaction is a non-parametric test designed to test for differences between the control and experimental subjects when some of the latter are predicted to behave in one way and the others in the opposite way (Hollander, 1963) ^[7]. The set of hypotheses is given below (Daniel, 2016) ^[5].

H_0 : The two samples represent the same population.

H_1 : The experimental sample results in extreme reactions in both directions.

The Hollander test statistic (H) is computed by combining size n_1 (control) and n_2 (experimental) samples from two populations. These samples are pooled and arranged from smallest to largest, yielding a total of $N = n_1 + n_2$ observations. Each observation is assigned a rank and r_i denotes the rank of the i th X value. The Hollander test statistic (H), is calculated as:

$$H = \sum_{i=1}^{n_1} (r_i - \bar{r})^2$$

The mean rank of the n_1 X values is denoted by $\bar{r}$.

$$\bar{r} = \frac{\sum_{i=1}^{n_1} r_i}{n_1}$$

The standardized version of Hollander statistic can be stated as:

$$H^* = \frac{M - E_0(H)}{\sqrt{\text{var}_0(H)}}$$

$$E_0(H) = \frac{(n_1 - 1)(N^2 + N)}{12}$$

$$\text{var}_0(H) = E(H^2) - [E(H)]^2$$

Where:

$$E(H^2) = \frac{(n_1 - 1)^2}{720} \left[\frac{1}{n_1} (N^4 + 2N^3 + N^2) + \left(\frac{n_1 + 1}{n_1 - 1} \right) (5N^4 + 6N^3 - 5N^2 - 6N) \right]$$

The standardized Hollander test statistic H^* has approximately a standard normal distribution under the null hypothesis. H_0 is rejected when $H^* \leq -Z_\alpha$.

Mood test

Mood's test is a nonparametric test, which tests the null hypothesis of equality variances for two populations (Gogoi & Gogoi, 2016) [6]. The Mood test statistic based on independent samples of sizes n and m is defined by:

$$MD = \sum_{i=1}^n \left(R_i - \frac{n+m+1}{2} \right)^2$$

In this case R_i is the rank of the i th X value when all observations from both samples are pooled and sorted from least to greatest. The element with the lowest value is ranked 1, the next lowest ranked 2, and so on, until the highest value element occupies rank $n + m$.

The standardized version of Mood statistic is expressed as follows:

$$MD^* = \frac{M - E_0(MD)}{\sqrt{\text{var}_0(MD)}}$$

$$E_0(MD) = \frac{[n(n+m+1)(n+m-1)]}{12}$$

$$\text{var}_0(MD) = \frac{[nm(n+m+1)(n+m+2)(n+m-2)]}{180}$$

The standardized Mood test statistic MD^* has approximately a standard normal distribution under the null hypothesis. For a one-sided alternative ($H_{a2}: \sigma_1^2 < \sigma_2^2$), H_0 is rejected when $MD^* \leq -Z_\alpha$, where Z_α is the $(1 - \alpha)$ 100% of the standard normal distribution.

2. Materials and Methods

The simple tree alternative given in (1) was tested based on $k = 3$ populations with equal sample sizes ($n_1 = n_2 = n_3 = 18$), where the treatment samples were combined into a single treatment group, respectively. Hence, the four tests were compared: the Moses, Conover Squared Rank, Hollander, and Mood tests. We also considered subsamples of size 3 for the Moses test.

The simulation study was conducted in SAS version 9.4, where random samples for each distribution were generated using the "RAND" function. Preceding the RAND function, required performing the starting seed by using "CALL STREAMINIT (seed)". The seed was equated to 0 in this study as a command for RAND to utilize on the system clock so that each run produces unique data (Bailer, 2010) [3].

The call functions to generate random number for each of the distributions in SAS are as follow:

Normal Distribution: RAND ('Normal', μ , σ)

Where μ and σ are the mean and the standard deviation, respectively, for normal distribution.

Exponential Distribution: RAND ('Exponential', μ)

Where μ and μ^2 are the mean and the variance, respectively, for exponential distribution.

Cauchy Distribution: *RAND* ('Cauchy', x_0, γ)

Where x_0 and γ are the location and scale parameters, respectively, for Cauchy distribution.

Double Exponential Distribution: *RAND* ('Laplace', μ, β);

Where μ and β are the location and scale parameters, respectively, for the double exponential distribution. The distribution has a mean of μ and a variance of $2(\beta)^2$.

Logistic Distribution: *RAND* ('Logistic', μ, s);

Where μ is the location parameter, and s is a scale parameter proportional to the standard deviation for logistic distribution. The distribution has a mean of μ and a variance of $s^2\pi^2/3$.

The alpha values of each test were estimated and compared to the stated alpha values for each simulation conducted. To verify that the test statistics preserved their Type I error rates, we created scenarios where the null hypothesis of equal scale parameters held. For instance, suppose that the observations in each group came from an identical normal distribution with equal means and variances; to examine the type 1 error rate based on 5,000 iterations, the statistical tests were conducted 5000 times and the counts of number of times the null hypothesis was rejected was divided by 5000 for each considered situation. These sample sets included cases with equal locations, unequal locations, as well as varied sample and subsample (in the Moses' test cases) sizes. The power was estimated by the number of times the null hypothesis was rejected when it was false divided by 5000 times. For instance, in the first part of the simulation study, we counted the number of times we rejected the hypothesis when our test confirmed unequal scale parameters, divided by the total number tests, which is 5000 for all tests of interest and their respective scenarios.

3. Results and Discussion

We evaluated the performance of each test by comparing their respective powers on samples from different distributions. We compared the powers of all test statistics using three sample measurements (mean, median and trimmed mean) except the Mood test which remained the same in all tables. All sample sizes considered in the simulation study were of size 18. The Moses test used subsamples of size 3. The following conditions were considered:

- The three populations have the equal location parameters and the equal scale parameters.
- The three populations have unequal location parameters and the equal scale parameters.
- The first and second populations have the equal scale parameters unequal than the third population.
- The second and third populations have the equal scale parameters unequal than the first population.
- The three populations have unequal location parameters and unequal scale parameters.

3.1 Results for the Normal Distribution

For underlying normal distributions, when three populations had the equal location and equal scale parameters, all tests maintained estimated alpha values around 0.05. Results were similar when the mean, median, and trimmed mean were used and therefore, only the results are given when the sample mean is used (Table 1). When the populations had unequal location parameters and equal scale parameters, the Conover Squared Ranks and Hollander tests had estimated alpha values greater than the stated alpha value, demonstrating their sensitivity to changes in location parameters. The Mood test also showed sensitivity to location changes, but in a different manner. As observed in the second and third rows of Table 1, the Mood test became increasingly conservative as the differences in location parameters grew, with estimated alpha values falling below 0.05.

The Conover Squared Ranks test had the highest estimated powers when the scale parameters were different as seen in Table 1. It is noted however, that the only one of these tests not affected by location changes was the Moses test.

Results for Exponential Distribution

The results of the simulation study of the Moses, Conover Squared Ranks, Hollander, and Mood tests for three treatments under the exponential distribution are shown in Tables 2-3 for the sample mean and median.

When three populations had equal location and scale parameters, all tests maintained their estimated alpha values around 0.05, except for the Conover test, which consistently had estimated alpha values higher than the stated alpha in all scenarios, as shown in the first row of Tables 2-3. Therefore, it is not possible to compare it with the other tests. Due to this skewed distribution, when the populations had unequal location parameters and the equal scale parameters, both the Hollander and Mood tests showed sensitivity in changing the locations parameters, with the Mood test reported extremely low alpha values. The Moses test was the only test that maintained stable estimated alpha values when the location changed, as shown in the second row of Tables 2-3.

When the first and second populations had scale parameters that differed from the third population, the Hollander test had greatest estimated powers, next was the Mood test, and lastly the Moses test as seen in the third row of Tables 2-3. As seen previously, the order for estimated powers was observed when the second and third populations had equal scale parameters which were different from the first population as seen in the fourth row of Tables 2-3 for the exponential distribution. When the three populations had different location and scale parameters, only the Moses test, as previously mentioned, achieved the highest estimated powers, as seen in the fifth row of Tables 2-3. In comparison of estimated powers for the three tests (the Moses, Hollander, and Mood) between the three measures, the sample mean produced the highest estimated powers.

3.2 Results for Cauchy Distribution

The results of the simulation study of the Moses, Conover Squared Ranks, Hollander, and Mood tests for three treatments under the Cauchy distribution are shown in Tables 4-5. All tests were calculated using different measures (sample mean, median, and

trimmed mean), except the Mood test which remained constant in all tables. Tables are shown when using the sample mean and median. Results for the trimmed mean were similar to the results for the median.

When the three populations had equal location and scale parameters, all tests maintained their estimated alpha values around 0.05, except for the Conover test, which showed very large estimated alpha value when using sample mean, but maintained accessible estimated alpha values around 0.05 when using sample median and trimmed mean, as shown in the first row of Tables 5 for the sample median. Hence, the power of Conover Squared Ranks test compared with other tests only in second and third scenarios (median and trimmed mean). When the populations had unequal location parameters and the equal scale parameters, the tests estimated alpha values greater than stated alpha value are Hollander and Conover tests, which show their sensitivity in changing the location parameters. However, it is not possible to compare them with other tests in the situation the locations differ. The Mood test showed sensitivity to location changes, with estimated alpha consistently below 0.05. As the location distances increased, the test became increasingly conservative, as shown in the second and third rows of Tables 4-5.

When the first and second populations had equal scale parameters that differed from the third population, the Mood test had the highest estimated powers compared to Hollander and Moses tests when using sample mean scenarios. However, Conover had higher estimated power than all other tests when using the median or trimmed mean scenarios, as shown in the fourth row of Table (5) for the median. We received the same order for estimated powers when the second and third populations had equal scale parameters different from the first population in all scenarios, as shown in the fifth row of Table (5).

When the three populations had different location and scale parameters, the Conover test had the highest estimated powers than Moses tests, as shown in the sixth row of Table (5).

3.3 Results for Double Exponential Distribution and Logistic

The results of the simulation study of the Moses, Conover Squared Ranks, Hollander, and Mood tests for three treatments under the double exponential distribution are shown in Table 6 for the sample mean which produced slightly higher powers than the trimmed mean or median. The results under the Logistic distribution are similar as far as which test statistic produced the higher powers and are not given here. All sample size considered in the simulations study were of an equal size of 18; and subsample of size 3 in Moses test only.

When the three populations had equal location and scale parameters, all tests maintained their estimated alpha values around 0.05, as shown in the first row of Table 6. When the populations had unequal location parameters and equal scale parameters, the Conover Squared Ranks test and the Hollander test estimated alpha values greater than the stated alpha value. This is evident in the second and third rows of Table 6, demonstrating these tests' sensitivity to changes in location parameters. As a result, we could not compare their estimated power with that of other tests in situations where the locations differed. The Mood test also showed sensitivity to location changes, with estimated alpha below 0.05, becoming more conservative as location distances increased.

When the first and second populations had equal scale parameters that differed from the third population, the Conover Squared Ranks test showed the highest estimated powers, followed by the Mood test, the Hollander test, and the Moses test, as shown in the fourth row of Table 6. We received the same order for estimated powers when the second and third populations had equal scale parameters different from the first population, as shown in the fifth row of Table 6.

When the three populations had different location and scale parameters, the Mood test had greater estimated powers than the Moses test as seen in the sixth row for Table 6.

In comparison of estimated powers for the three tests (the Moses, Hollander, and Mood) between the three measures, the sample mean produced the highest estimated powers, followed by the trimmed mean, and then the median. The trimmed mean and median were close in estimated powers.

Table 1: Estimated rejection percentages for k=3 Populations; Normal Distribution using the sample mean, $n_1 = n_2 = n_3 = 18$ and subsample size 3:

$n_{1=2=3}$	μ_1	σ_1^2	μ_2	σ_2^2	μ_3	σ_3^2	Moses	Conover	Hollander	Mood
18	0	1	0	1	0	1	0.0484	0.0562	0.0466	0.0450
18	0	1	0.25	1	0.5	1	0.0500	0.0576	0.0856	0.0346
18	0	1	1	1	2	1	0.0494	0.1378	0.8368	0.0010
18	0	1	0	1	0	2.5^2	0.3136	0.6062	0.4950	0.5306
18	0	1	0	2.5^2	0	2.5^2	0.9000	0.9812	0.9684	0.9692
18	0	1	0.25	2.5^2	0.5	3^2	0.9350	0.9956	0.9886	0.9848

Table 2: Estimated rejection percentages for k=3 Populations; Exponential Distribution using the sample mean, $n_1 = n_2 = n_3 = 18$ subsample size 3:

$n_{1=2=3}$	μ_1	σ_1^2	μ_2	σ_2^2	μ_3	σ_3^2	Moses	Conover	Hollander	Mood
18	1	1	1	1	1	1	0.0492	0.1808	0.0504	0.0494
18	1	1	1.25	1	1.5	1	0.0472	0.1822	0.0180	0.0004
18	1	1	1	1	1	2^2	0.1790	0.4950	0.5280	0.5236
18	1	1	1	2^2	1	2^2	0.4932	0.8538	0.9810	0.9388
18	1	1	1.5	1.5^2	2	2^2	0.3636	0.7534	0.2538	0.1110

Table 3: Estimated rejection percentages for k=3 Populations; Exponential Distribution using the sample median, $n_1 = n_2 = n_3 = 18$ and subsample size 3:

$n_{1=2=3}$	μ_1	σ_1^2	μ_2	σ_2^2	μ_3	σ_3^2	Moses	Conover	Hollander	Mood
18	1	1	1	1	1	1	0.0480	0.0948	0.0434	0.0494
18	1	1	1.25	1	1.5	1	0.0478	0.0918	0.0110	0.0004
18	1	1	1	1	1	2^2	0.1792	0.3796	0.4854	0.5236
18	1	1	1	2^2	1	2^2	0.4780	0.7210	0.9688	0.9388
18	1	1	1.5	1.5^2	2	2^2	0.3526	0.5882	0.2308	0.1110

Table 4: Estimated rejection percentages for k=3 Populations; Cauchy Distribution using the sample mean, $n_1 = n_2 = n_3 = 36$ and subsample size 3:

$n_{1=2=3}$	μ_1	γ_1	μ_2	γ_2	μ_3	γ_3	Moses	Conover	Hollander	Mood
18	0	γ	0	γ	0	γ	0.0460	0.2772	0.0464	0.0456
18	0	γ	0.25	γ	0.5	γ	0.0494	0.2836	0.0718	0.0420
18	0	γ	1	γ	2	γ	0.0562	0.2912	0.3810	0.0298
18	0	γ	0	γ	0	2.5γ	0.1776	0.4942	0.2436	0.2620
18	0	γ	0	2.5γ	0	2.5γ	0.4484	0.6634	0.5974	0.6214
18	0	γ	0.25	2.5γ	0.5	3γ	0.4838	0.6988	0.6636	0.6670

Table 5: Estimated rejection percentages for k=3 Populations; Cauchy Distribution using the sample median, $n_1 = n_2 = n_3 = 18$ and subsample size 3:

$n_{1=2=3}$	μ_1	γ_1	μ_2	γ_2	μ_3	γ_3	Moses	Conover	Hollander	Mood
18	0	γ	0	γ	0	γ	0.0466	0.0482	0.0410	0.0456
18	0	γ	0.25	γ	0.5	γ	0.0498	0.0514	0.0618	0.0420
18	0	γ	1	γ	2	γ	0.0548	0.1082	0.3408	0.0298
18	0	γ	0	γ	0	2.5γ	0.1736	0.2814	0.2232	0.2620
18	0	γ	0	2.5γ	0	2.5γ	0.4418	0.6440	0.5720	0.6214
18	0	γ	0.25	2.5γ	0.5	3γ	0.4754	0.7112	0.6382	0.6670

Table 6: Estimated rejection percentages for k=3 Populations; Double Exponential Distribution using the sample mean, $n_1 = n_2 = n_3 = 18$ and subsample size 3:

$n_{1=2=3}$	μ_1	σ_1^2	μ_2	σ_2^2	μ_3	σ_3^2	Moses	Conover	Hollander	Mood
18	0	$2(1)^2$	0	$2(1)^2$	0	$2(1)^2$	0.0508	0.0568	0.0470	0.0518
18	0	$2(1)^2$	0.25	$2(1)^2$	0.5	$2(1)^2$	0.0496	0.0574	0.0832	0.0422
18	0	$2(1)^2$	1	$2(1)^2$	2	$2(1)^2$	0.0512	0.1162	0.5798	0.0050
18	0	$2(1)^2$	0	$2(1)^2$	0	$2(2.5)^2$	0.2618	0.4614	0.3550	0.3894
18	0	$2(1)^2$	0	$2(2.5)^2$	0	$2(2.5)^2$	0.7876	0.9058	0.8522	0.8686
18	0	$2(1)^2$	0.25	$2(2.5)^2$	0.5	$2(3)^2$	0.8518	0.9436	0.9116	0.9060

4 Conclusion

This research examined the simple tree alternative, comparing a control group with different treatment levels, which only tests for changes in the scale parameter. We compared Moses, Conover Squared Ranks, Hollander, and Mood tests. The mean, median, and trimmed mean versions were considered for all tests but Mood. The Moses test was the only one unaffected by location variations. The objective of this research was to determine whether at least one of the treatment levels had a greater variance than the control. The simple tree alternative hypothesis for scale differences only is given by:

$$H_0: \sigma_1 = \dots = \sigma_k$$

$$H_a: \sigma_1 \leq [\sigma_2, \dots, \sigma_k], \text{ where at least one inequality is strict.}$$

For all symmetric distributions (Logistic, Double Exponential, and Normal), the results indicated that all tests maintained their estimated alpha values around 0.05 when location parameters were equal. Conover and Hollander had a significance level higher than the stated level of significance when the location parameters increased. The Mood test turned more conservative as location parameters increased. The Moses test maintained its alpha value regardless of an increase in location parameters. Regarding power, when the three populations had unequal scale parameters and equal location parameters, Conover had the greatest powers, followed by Mood, Hollander, and Moses. When the three populations had unequal location and scale parameters, the Mood test reported the highest estimated powers, followed by the Moses test. This could, however lead to misleading results if we only wanted to test for an increase in scale parameters.

When the underlying distribution was Cauchy, the Conover test had inflated alpha values when the sample mean was used. For the Cauchy distribution, the Mood test (using the sample median) and the Conover Squared Ranks test (using the median or trimmed mean) are advised for testing for unequal scales.

For non-symmetric distributions, such as the Exponential, Hollander's test is recommended to test for an increase in variance if the locations remain the same, and Moses' test when differences may be present in both location and scale.

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