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Contaminated Robust Estimator of Weibull Distribution's Reliability Function

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Abstract

The reliability function of the Weibull distribution (The Two-Parameter Distribution) contaminated by the exponential distribution was estimated by employing the simulation method to compare the two estimation methods, namely the weighted least squares (WLS) method and the Robust Bayes (RB) method, to determine the optimal method for estimating that function, and the investigator arrived at the benefit of the Robust Bayes (RB) method to estimate the reliability function compared to the other method, depending on the statistical scale Mean Squared Error (MSE).

Keywords: Weighted Least Squares (WLS), Weibull distribution, Reliability function, Robust Bayes Method (RB)

1. Introduction

The Weibull distribution in 1951 was used by researcher Waloddi Weibull for an experimental demonstration to watch the change in iron expansion and use this distribution to measure the change in the service period spent by radio staff. The two-parameter and multi-parameter Weibull distribution that will be mentioned later have been commonly used in the fields of reliability and survival test experiments, so we see their applications in survival distribution and various failures in addition to building models of reliability.

Reliability appeared as an important science dealing with the ages of equipment, especially the possibility of survival, average life, and the possibility that the age of the system is greater than a certain time, interest in the subject of reliability has increased after the widespread spread of industry and the increase in complexities in equipment and research before World War II was concerned with quality control and maintenance of machines, and reliability was not dealt with as an independent field. With the beginning of World War II, the field of reliability became an independent and private entity.

The fundamental idea behind reliability is to understand how things will behave when they fail for a set amount of time to lower production and maintenance costs. This is especially true in the context of survival theory, as both survival and reliability have the same characteristic of measuring the duration of life, whether it be for an organism, system, or machine. Due to its emphasis on the lifespan of systems and equipment, their potential for failure, and their sustainability over time, this issue is particularly significant in the context of industry and contemporary technology. Therefore, to examine and analyze phenomena following the nature of statistical data and carry out the estimating process using statistical methods, it is necessary to adopt the theory of dependability, including the least squares method, the moment's method, the contraction method, the Bayes method, the method, and others [5].

The issue of anomalous values has been studied by many scientists and researchers, such as Barnett, Hawkins (1980), Freeman (1980), and Lewis (1978) who dealt with the subject of outliers and their effects on the accuracy of the results. In the year (1999), the researcher Sheikh presented a method for estimating observations in simple linear regression models through the residual column after deleting the observation (i) and using measures of dispersion and mean. Al-Jabari and Nazi (2002) proposed a new method for detecting and estimating anomalous observations of simple linear regression models. In 2002, Al-Kinani used the

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nonparametric method to diagnose and estimate outlier values in the case of multiple linear models. The appearance of outliers in a set of real data will affect their analysis.^[11]

This study aims to calculate the Weibull distribution's reliability function using two parameters contaminated by the exponential distribution using two methods and the height of the statistical scale mean of error squares to compare two methods, specifically the weighted least squares method and the Bayes method to determine the optimal approach to estimate that function.

2. Reliability Function^[3, 5, 12]

Reliability is usually used to give a job or work, and the broad meaning of it is that it is a measure of performance, as any functional vocabulary needs certain specifications or performance requirements when performing the work required of it or to continue to perform its work under normal operating conditions and it can be said that this vocabulary is reliable, and so it is clear from this concept that the meaning of reliability is related to a measure of the degree of successful performance of the job required under its working conditions. Reliability can be defined in different forms, including:

- The probability that a unit (device) performs a certain work under certain conditions and for a specific period.
- Alternatively, it is the possibility that a system will operate for a specific period and under the working conditions for which it is designed.

It is also the possibility of performing a piece of equipment (unit - device) for which it is designed as necessary and during a certain period and under certain operational conditions. This formula is one of the most acceptable definitions of reliability because it is a function of a type of performance in which the device is successful, that is, does not fail during Term service^[6].

For each period $(t, t+\Delta t)$, let T be a random variable and be the probability failure function, where $F(t)$ is the failure time (t) cumulative distribution function (cdf).

The Reliability Function, represented by $R(t)$, has the following definition:

$$R(t) = P(T > t) = 1 - P(T \leq t) = 1 - F(t) \quad (1)$$

3. Statistical distribution

3.1 Weibull Distribution^[2, 13]

This distribution is one of the important statistical models widely used in failure tests, life duration tests, life duration tests in the field of theory, life tests or decrease in life rate due to increase, and the expansion using this distribution is due to its flexibility when applied to various data in general and has many applications in Life Data, as well as the use of this distribution in the field of reliability and the subject of quality control in general. The distribution of Weibull can be obtained with two parameters. Through the use of the General Gamma Distribution, which was derived by the scientist Stacy, which is one of the most important distributions. It is worth mentioning that the important characteristic of this distribution is that it is a generalization of the many distributions and has a wide area of interest for researchers and scholars of reliability, especially the Kama distribution the Weibull distribution, and the exponential distribution.^[8]

For a two-parameter Weibull distribution with $\alpha > 0$ and $\beta > 0$, the probability density function and cumulative distribution function are as follows:^[10]

$$f(t, \beta, \alpha) = \begin{cases} \frac{\beta}{\alpha} \left(\frac{t}{\alpha}\right)^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^\beta}, & t > 0, \beta, \alpha > 0 \\ 0, & \text{ow} \end{cases} \quad (2)$$

α : Scale parameter or Characteristic life.

β : Shape parameter.

and CDF, Reliability

$$F(t) = 1 - e^{-\left(\frac{t}{\alpha}\right)^\beta}, R(t) = e^{-\left(\frac{t}{\alpha}\right)^\beta} \quad (3)$$

3.2 Exponential Distribution

Referred to as the exponential function, one kind of continuous probability distribution is the exponential distribution, this distribution is used to estimate the times between occurrences, when it comes to time measurement issues, the exponential distribution is often used, this covers the time spent using the post office, making phone calls, unloading freighters, the time needed to fix a machine and the wait time for a consumer to receive assistance, the exponential distribution is used in the precise sciences to describe how long radioactive atoms last before decaying.^[1]

If the PDF provided indicates the distribution of the random variable T is exponential, then

$$f(t, \delta) = \delta e^{-\delta t}, \delta > 0 \quad (4)$$

and CDF, Reliability

$$F(t) = 1 - e^{-\delta t}, R(t) = e^{-\delta t} \quad (5)$$

3.3 Exponential-Weibull^[7]

With a ratio of τ , an exponential distribution has contaminated the Weibull distribution, for us to possess the exponential-Weibull distribution

$$f(t, \theta) = (1 - \tau) \left[\frac{\beta}{\alpha} \left(\frac{t}{\alpha} \right)^{\beta-1} e^{-\left(\frac{t}{\alpha} \right)^\beta} \right] + \tau \delta e^{-\alpha \delta_2 t} \quad , \quad t > 0, \alpha, \delta, \beta > 0 \quad (6)$$

$$R(t, \theta) = (1 - \tau) e^{-\left(\frac{t}{\alpha} \right)^\beta} + \tau e^{-\delta t} \quad (7)$$

4. The Estimation of Reliability Function

4.1 Weighted Least Squares (WLS) ^{[13][14]}

There are many alternative methods for estimating the parameters of the regression model, including Weighted Least Squares, which depend on the weights W_i . In this research, the Bergman formula was used for these weights, according to Habeeb (2022) as the following:

Let them $(y_1, y_2, \dots, y_n)$ be a sample chosen at random to match the distribution of what $G(\cdot)$, furthermore, $y_i, i = 1, 2, \dots, n$ stands for the sample's ranking statistics, following which the distribution's $G(y_i)$ prediction and variation are:

$$E(G(y_i)) = \frac{i}{n+1}$$

$$V(G(y_i)) = \frac{i(n+i+1)}{(n+1)^2(n+2)}$$

Referring to the findings of researcher Katz's mentioned in ^[2]

We minimize the sum of the following weighted squares using the definition to obtain a WLS estimator.

$$S_w = \sum_{i=1}^n w_i \left(G(Y_i) - \frac{i}{n+1} \right)^2 \quad (8)$$

$$w_i = \frac{1}{V(G(Y_i))} = \frac{(n+1)^2(n+2)}{i(n+i+1)} \quad (9)$$

The estimation of parameters $\hat{\beta}, \hat{\delta}, \hat{\alpha}$ for the Weibull distribution is contaminated with an exponential distribution using the density function for distributions and to determine the estimate of the reliability function;

$$G(Y_i) = (1 - \tau) f_1(t, \beta, \alpha) + \tau f_2(t, \delta)$$

$$= (1 - \tau) \left[\frac{\beta}{\alpha} \left(\frac{t}{\alpha} \right)^{\beta-1} e^{-\left(\frac{t}{\alpha} \right)^\beta} \right] + \tau \delta e^{-\delta t}$$

Then, the Weibull distribution is represented by $f_1(t)$, while the contaminated distribution (exponential distribution) is represented by $f_2(t)$, where τ is the formula's reducible level of contamination (6)

$$S_w = \sum_{i=1}^n w_i \left((1 - \tau) \left[\frac{\beta}{\alpha} \left(\frac{t_i}{\alpha} \right)^{\beta-1} e^{-\left(\frac{t_i}{\alpha} \right)^\beta} \right] + \tau \delta e^{-\delta t_i} - \frac{i}{n+1} \right)^2 \quad (10)$$

When we use the partial derivative to equalize (10) for θ , we obtain

$$\frac{\partial S_w}{\partial \theta} = \begin{bmatrix} \frac{\partial}{\partial \beta} S_w \\ \frac{\partial}{\partial \alpha} S_w \\ \frac{\partial}{\partial \delta} S_w \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} = G \quad \text{such that} \quad \theta = \begin{bmatrix} \beta \\ \alpha \\ \delta \end{bmatrix} \quad (11)$$

$$\frac{\partial S_w}{\partial \beta} = 2 \sum_{i=1}^n w_i \left((1 - \tau) \left[\frac{\beta}{\alpha} \left(\frac{t_i}{\alpha} \right)^{\beta-1} e^{-\left(\frac{t_i}{\alpha} \right)^\beta} \right] + \tau \delta e^{-\delta t_i} - \frac{i}{n+1} \right) \left(1 - \tau \right) \left[\frac{\beta}{\alpha} \left(\frac{t_i}{\alpha} \right)^{\beta-1} e^{-\left(\frac{t_i}{\alpha} \right)^\beta} \right]$$

$$\frac{\partial S_w}{\partial \alpha} = 2 \sum_{i=1}^n w_i \left((1 - \tau) \left[\frac{\beta}{\alpha} \left(\frac{t_i}{\alpha} \right)^{\beta-1} e^{-\left(\frac{t_i}{\alpha} \right)^\beta} \right] + \tau \delta e^{-\delta t_i} - \frac{i}{n+1} \right) \left(1 - \tau \right) \left[\frac{\beta}{\alpha} \left(\frac{t_i}{\alpha} \right)^{\beta-1} e^{-\left(\frac{t_i}{\alpha} \right)^\beta} \right] \quad (12)$$

$$\frac{\partial S_w}{\partial \delta} = 2 \sum_{i=1}^n w_i \left((1 - \tau) \left[\frac{\beta}{\alpha} \left(\frac{t_i}{\alpha} \right)^{\beta-1} e^{-\left(\frac{t_i}{\alpha} \right)^\beta} \right] + \tau \delta e^{-\delta t_i} - \frac{i}{n+1} \right) \tau \delta e^{-\delta t_i} \quad (13)$$

Equation (10) second derivative is as follows:

$$\frac{\partial}{\partial \underline{\theta} \partial \underline{\theta}} S_w = \begin{bmatrix} \frac{\partial S_w}{\partial \beta^2} & \frac{\partial S_w}{\partial \beta \partial \alpha} & \frac{\partial S_w}{\partial \beta \partial \delta} \\ \frac{\partial S_w}{\partial \beta \partial \alpha} & \frac{\partial S_w}{\partial \alpha^2} & \frac{\partial S_w}{\partial \alpha \partial \delta} \\ \frac{\partial S_w}{\partial \beta \partial \delta} & \frac{\partial S_w}{\partial \alpha \partial \delta} & \frac{\partial S_w}{\partial \delta^2} \end{bmatrix}$$

$$\frac{\partial}{\partial \underline{\theta} \partial \underline{\theta}} S_w = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} = H \quad (14)$$

To determine a parameter's estimate (β , α , δ) in formula (10), the Newton-Raphson method, which is based on formulas (13), and (14), is used to solve this equation numerically as follows:

$$\underline{\theta}_{n+1} = \underline{\theta}_n - H^{-1} G \quad (15)$$

The only Weighted Least Squares estimator method parameter of landmarks (β , α , δ) is formula (10) as follows:

$$\hat{\underline{\theta}}_{RML} = \underline{\theta}_{n+1} \quad (16)$$

Stability is used to evaluate the reliability function as follows:

$$R(t) = (1 - \tau)R_1(t) + \tau R_2(t)$$

$$= (1 - \tau) e^{-\left(\frac{t}{\alpha}\right)^\beta} + \tau(e^{-\delta t})$$

$$\hat{R}_{WLS}(t) = (1 - \hat{\tau})e^{-\left(\frac{t}{\hat{\alpha}_{WLS}}\right)^{\hat{\beta}_{WLS}}} + \hat{\tau}(e^{-\hat{\delta}_{WLS}t}) \quad (17)$$

4.2 Weibull distribution contaminated by the exponential distribution using the Robust Bayes Method ^[3]

A Bayesian estimator is taken into account inside the essential loss function in Bayesian estimates, these loss functions are asymmetric (square error). Bayesian inference requires the assumption of unknown parameters, it can be given the prior distribution shown below.

4.2.1 The density of the prior

For β , α , and δ , respectively, let the parameters (a , b , p , q ,...) take on a Gamma (a, b), Gamma (p, q), and $U(0, \delta)$ representations. The priors are taken to be independent.

$$\pi_1(\beta) = \frac{b^a \beta^{a-1} e^{-b\beta}}{\Gamma(a)}, b > 0, a > 0, \beta > 0$$

$$\pi_2(\alpha) = \frac{q^p \alpha^{p-1} e^{-q\alpha}}{\Gamma(p)}, q > 0, p > 0, \alpha > 0$$

$$\pi_3(\delta) = \frac{1}{\delta}, \delta > 0$$

Thus, given β , α , and δ , the prior joint distribution is

$$g(\beta, \alpha, \delta) \propto \frac{b^a q^p \beta^{a-1} \alpha^{p-1} e^{-(b\beta + q\alpha)}}{\delta \Gamma(p) \Gamma(a)}, \forall b, a, \alpha, q, p, \beta, \delta > 0 \quad (18)$$

4.2.2 The density of the posterior

Bayesian inference is based on the posterior distribution, By replacing likelihood, we calculate the three parameters' Bayesian estimation $L(\beta, \alpha, \delta)$, as well as the prior joint distribution for β , α , and δ . From $g(\beta, \alpha, \delta)$ in (18) in that order, we obtain the matching posterior joint $P(\beta, \alpha, \delta)$, this is just the marginal distribution function divided by the joint density function of the prior distribution.

$$P(\beta, \alpha, \delta) = \frac{g(\beta, \alpha, \delta) L(\beta, \alpha, \delta)}{\int_0^\infty \int_0^\infty \int_0^\infty g(\beta, \alpha, \delta) l_n(\beta, \alpha, \delta) d(\beta, \alpha, \delta)} \quad (19)$$

$$P(\beta, \alpha, \delta) = A^{-1} \frac{\beta^{\alpha-1} \alpha^{p-1}}{\delta} \prod_{i=1}^n e^{-(b\beta + q\alpha)} \left[(1 - \tau) \left[\frac{\beta}{\alpha} \left(\frac{t_i}{\alpha} \right)^{\beta-1} e^{-\left(\frac{t_i}{\alpha} \right)^\beta} \right] + \tau \delta e^{-\delta t_i} \right] \quad (20)$$

Where

$$A = \iiint_0^\infty \frac{\beta^{\alpha-1} \alpha^{p-1}}{\delta} \prod_{i=1}^n e^{-(b\beta + q\alpha)} \left[(1 - \tau) \left[\frac{\beta}{\alpha} \left(\frac{t_i}{\alpha} \right)^{\beta-1} e^{-\left(\frac{t_i}{\alpha} \right)^\beta} \right] + \tau \delta e^{-\delta t_i} \right] d\beta d\alpha d\delta$$

4.2.3 The estimators of Bayes

The Weibull distribution contaminated exponential distribution's parameters are estimated in this part using the Bayesian method and quadratic loss functions. The β , α , and δ Bayesian estimators (BQ) are as follows:

$$\hat{\beta}_{BQ} = A^{-1} \iiint_0^\infty \frac{\beta^{\alpha-2} \alpha^{p-1}}{\delta} e^{-(b\beta + q\alpha)} \prod_{i=1}^n \left[(1 - \tau) \left[\frac{\beta}{\alpha} \left(\frac{t_i}{\alpha} \right)^{\beta-1} e^{-\left(\frac{t_i}{\alpha} \right)^\beta} \right] + \tau \delta e^{-\delta t_i} \right] d\beta d\alpha d\delta$$

The related posterior hazards are:

$$PR(\hat{\beta}_{BQ}) = E_\pi(\beta^2) - 2\hat{\beta}_{BQ}E_\pi(\beta) + (\hat{\beta}_{BQ})^2$$

$$PR(\hat{\alpha}_{BQ}) = E_\pi(\alpha^2) - 2\hat{\alpha}_{BQ}E_\pi(\alpha) + (\hat{\alpha}_{BQ})^2$$

$$PR(\hat{\delta}_{BQ}) = E_\pi(\delta^2) - 2\hat{\delta}_{BQ}E_\pi(\delta) + (\hat{\delta}_{BQ})^2$$

Under the loss functions, the reliability Bayesian estimator is given by:

$$\hat{R}(t)_{BQ} = A^{-1} \iiint_0^\infty \frac{\beta^{\alpha-2} \alpha^{p-1}}{\delta} \left[(1 - \tau) e^{-\left(\frac{t}{\alpha} \right)^\beta} + \tau e^{-\delta t} \right] \prod_{i=1}^n e^{-(b\beta + q\alpha)} \left[(1 - \tau) \left[\frac{\beta}{\alpha} \left(\frac{t_i}{\alpha} \right)^{\beta-1} e^{-\left(\frac{t_i}{\alpha} \right)^\beta} \right] + \tau \delta e^{-\delta t_i} \right] d\beta d\alpha d\delta \quad (21)$$

Use numerical procedures to calculate these estimators.

5. Simulation

We determine the default values, Choosing default values for β , α , δ , τ

Let $\tau = 15\%, 30\%$

Table 1: Scale parameter's initial value

Type of Model	1	2
β	1.2	2.4
α	1.3	1.4
δ	2.1	3.5

For simulation $n = (25, 50, 100)$ virtual sample size, with $L=1000$ being the recurrence of each experiment

The WLS method of formula (17) and the RB method of formula (21) were used to evaluate the Weibull model's reliability function.

The two reliability function estimate techniques are compared in this final step of the MSE adoption process using the following formula:

$$MSE(R(t)) = \frac{1}{n} \sum_{j=0}^n \left(\hat{R}_j(t_j) - R_j(t_j) \right)^2$$

The reliability function estimate and MSE for each sample volume are displayed in the following tables.

Table 2: Periods for calculating the reliability function

t_i	(Model) 1	(Model) 2
t_1	0.1	0.1
t_2	0.15	0.2
t_3	0.2	0.3
t_4	0.25	0.4
t_5	0.3	0.5

t ₆	0.35	0.6
t ₇	0.4	0.7
t ₈	0.45	0.8
t ₉	0.5	0.9

a- Random number generation for the Weibull distribution given two parameters (β , α), and one-parameter exponential distribution (δ), Based on the VB.net language's accessible generating function:

$t = \text{Weibull} \sim (\beta, \alpha)$ or $t = \text{exp} \sim (\delta)$

b- Find the estimators:

Currently, the process of estimating the Weibull-contaminated distribution's reliability function using an exponential distribution is addressed in the theoretical section of this paper using estimating techniques, based on (WLS) and (RB) methods.

c- Comparison:

Following the discovery of reliability function estimators, which were used to assess the precision of estimate techniques, the method that produces the lowest mean squares error (MSE) result is considered the best-fitting one.

6. Results

Table 3: Estimated values of the reliability function for the Weibull Distribution of the contaminated Exponential Function with a contamination Rate of 15% for every model and all chosen sample sizes

n	(Model 1)				(Model 2)			
	t _i	R _{real}	R _{WLS}	R _{RB}	t _i	R _{real}	R _{WLS}	R _{RB}
25	0.1	0.730177	0.9623	0.7258	0.1	0.8385	0.7028	0.520
	0.15	0.583109	0.8042	0.6291	0.2	0.7359	0.5411	0.312
	0.2	0.474729	0.5486	0.5455	0.3	0.6544	0.4277	0.202
	0.25	0.392819	0.2915	0.4807	0.4	0.5864	0.3433	0.1291
	0.3	0.329578	0.1163	0.42799	0.5	0.5283	0.2785	0.0894
	0.35	0.279848	0.03347	0.3725	0.6	0.4779	0.2279	0.0645
	0.4	0.240116	0.0066	0.3391	0.7	0.433	0.1877	0.0499
	0.45	0.207926	0.0013	0.2924	0.8	0.3948	0.155	0.0427
50	0.5	0.181524	0.0009	0.2729	0.9	0.3623	0.129	0.0298
	0.1	0.730177	0.9624	0.7273	0.1	0.7359	0.7027	0.5258
	0.15	0.583109	0.8049	0.6312	0.2	0.6544	0.5411	0.3087
	0.2	0.474729	0.5482	0.5498	0.3	0.5864	0.4276	0.1960
	0.25	0.392819	0.2915	0.4862	0.4	0.5283	0.3432	0.1317
	0.3	0.329578	0.1166	0.6308	0.5	0.4779	0.27852	0.09301
	0.35	0.279848	0.0335	0.3859	0.6	0.433	0.2278	0.0678
	0.4	0.240116	0.00679	0.3447	0.7	0.3948	0.1876	0.052
100	0.45	0.207926	0.0013	0.3102	0.8	0.3623	0.1554	0.0388
	0.5	0.181524	0.0019	0.280009	0.9	0.8385	0.1292	0.0312
	0.1	0.583109	0.8485	0.7314	0.1	0.6544	0.7027	0.5258
	0.15	0.474729	0.7507	0.6323	0.2	0.5864	0.5411	0.3114
	0.2	0.392819	0.6745	0.5521	0.3	0.5283	0.4276	0.1970
	0.25	0.329578	0.2915	0.4861	0.4	0.4779	0.3432	0.1326
	0.3	0.279848	0.1162	0.4321	0.5	0.433	0.2785	0.0936
	0.35	0.240116	0.0335	0.3845	0.6	0.3948	0.2766	0.0684
	0.4	0.207926	0.0068	0.3442	0.7	0.3623	0.2278	0.0515
	0.45	0.181524	0.0009	0.3145	0.8	0.8385	0.1554	0.0410
	0.5	0.730177	0.0008	0.2810	0.9	0.7359	0.1292	0.0312

Table 4: Estimated values of the reliability function for the Weibull Distribution of the contaminated Exponential Function with a contamination Rate of 30% for every model and all chosen sample sizes

n	(Model 1)				(Model 2)			
	t _i	R _{real}	R _{WLS}	R _{RB}	t _i	R _{real}	R _{WLS}	R _{RB}
25	0.1	0.730177	0.96220	0.727791	0.1	0.8385	0.7028	0.860939
	0.15	0.583109	0.80418	0.62861	0.2	0.7359	0.05411	0.777011
	0.2	0.474729	0.5851	0.54985	0.3	0.6544	0.4277	0.700283
	0.25	0.392819	0.29148	0.485364	0.4	0.5864	0.3433	0.643498
	0.3	0.329578	0.11606	0.430825	0.5	0.5283	0.2785	0.583805
	0.35	0.279848	0.03346	0.384244	0.6	0.4779	0.2279	0.540006
	0.4	0.240116	0.00677	0.341998	0.7	0.433	0.1877	0.479979
	0.45	0.207926	0.00093	0.306976	0.8	0.3948	0.1554	0.462576
50	0.5	0.181524	0.0008	0.279373	0.9	0.3623	0.1293	0.433716
	0.1	0.730177	0.9622	0.726604	0.1	0.7359	0.70277	0.862338
	0.15	0.583109	0.80418	0.637635	0.2	0.6544	0.54111	0.776188
	0.2	0.474729	0.54851	0.55075	0.3	0.5864	0.42765	0.69954
	0.25	0.392819	0.29148	0.485887	0.4	0.5283	0.34326	0.642542

	0.3	0.329578	0.11606	0.430469	0.5	0.4779	0.27852	0.584756
	0.35	0.279848	0.03346	0.384029	0.6	0.433	0.22787	0.538934
	0.4	0.240116	0.00677	0.342617	0.7	0.3948	0.18767	0.482614
	0.45	0.207926	0.00093	0.315488	0.8	0.3623	0.15540	0.461662
	0.5	0.181524	0.0008	0.287854	0.9	0.8385	0.12929	0.434146
100	0.1	0.730177	0.96220	0.726881	1	0.7359	0.70277	0.861819
	0.15	0.583109	0.80418	0.637243	0.1	0.6544	0.54111	0.776089
	0.2	0.474729	0.5451	0.550945	0.2	0.5864	0.42765	0.702407
	0.25	0.392819	0.29148	0.485617	0.3	0.5283	0.34326	0.642322
	0.3	0.329578	0.11606	0.432619	0.4	0.4779	0.27852	0.585531
	0.35	0.279848	0.03346	0.385955	0.5	0.433	0.2785	0.525336
	0.4	0.240116	0.00677	0.343523	0.6	0.3948	0.22787	0.487341
	0.45	0.207926	0.00093	0.316661	0.7	0.3623	0.15540	0.457006
	0.5	0.181524	0.00008	0.287567	0.8	0.8385	0.12929	0.430253

Table 5: The reliability function's mean error values of the contaminated Weibull distribution of exponential bales by 15% for every model and all chosen sample sizes

Model	n	MSE	
		R _{WLS}	R _{RB}
1	25	0.039414	0.006491
	50	0.038204	0.016639
	100	0.036504	0.007851
2	25	0.05154	0.0159972
	50	0.051531	0.0159594
	100	0.047072	0.158903

Table 6: The reliability function's mean error values of the contaminated Weibull distribution of exponential bales by 30% for every model and all chosen sample sizes

Model	n	MSE	
		R _{WLS}	R _{RB}
1	25	0.040157	0.007467
	50	0.039408	0.007982
	100	0.039383	0.008111
2	25	0.098935	0.002933
	50	0.051511	0.002932
	100	0.04697	0.002729

7. Conclusion

By observing the tables, it is shown that the Bayes (RB) method gave the lowest comparison of (MLS) than the weighted least squares method (WLS), and therefore the Bayes method is better than the weighted least squares method to calculate the reliability of the contamination Weibull distribution at a contamination rate of 15% and 30%, the values of estimating the contamination Weibull distribution's reliability function with the exponential function with a contamination rate of 15% and 30% for both methods and all sizes.

References

1. Al-Gharabi S. Notes on exponential distribution. Journal of Economics and Administrative Sciences. 2008;14(50):292-292.
2. Al-Wakeel AAH. Notes on Weibull distribution. Journal of Economics and Administrative Sciences. 2012;18(67):260-260.
3. Ameen AA, Akkash UA. Maximum likelihood estimation and Bayesian estimation of three-parameter Weibull distribution based on interval-censored data. Journal of Physics: Conference Series. 2021;1818(1):012199.
4. Atta AS, Abbas NM. Estimating the reliability of the general linear failure distribution using simulation. 2014.
5. Badr DH. A comparison of some methods to estimate reliability function for two-parameter Weibull distribution by using simulation. Al-Rafidain University College for Sciences. 2019;(45).
6. Forcina A, Silvestri L, Di Bona G, Silvestri A. Reliability allocation methods: a systematic literature review. Quality and Reliability Engineering International. 2020;36(6):2085-2107.
7. Habeeb RT, Labban JA. On estimating the reliability function of an exponential-Pareto II distribution using weighted least squares method. Mathematical Statistician and Engineering Applications. 2022;71(3s2):366-387.
8. Heusila P, Singh S, Saxena M, Khoshnevisan M, Singh S, Smarandache F. Estimation of Weibull shape parameter by shrinkage towards an interval under failure censored sampling. 2nd ed. American Research Press; 2002.
9. Nadarajah S, Kotz S. Reliability for Pareto models. Metro - International Journal of Statistic. 2003;61(2):191-204.
10. Nketiah EA, Chenlong L, Yingchuan J, Dwumah B. Parameter estimation of the Weibull distribution: comparison of the least-squares method and the maximum likelihood estimation. International Journal of Advanced Engineering Research and Science. 2021;8:9.
11. Qasim MN, Ismail YH. Detecting outliers in the Biz method using a gypsum preview. IRAQI Journal of Statistical Sciences. 2008;8(2).
12. Sedeeq BS. Using wavelet shrinkage to deal with contamination problem in survival function for Weibull distribution. Iraqi Journal of Statistical Sciences. 2023;20(2):1-14.

13. Sharhan RF, Rahi AAA G. Estimation of Weibull distribution parameters using the ordinary least squares method and weighted least-squares method (comparative study). Journal of Administration and Economics. 2019;37(122).
14. Van Doorslaer E, O'Donnell O, Rannan-Eliya RP, Somanathan A, Adhikari SR, Garg CC, *et al.* Catastrophic payments for health care in Asia. Health Economics. 2007;16(11):1159-1184.