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Modified difference type estimators for estimation of finite population mean

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Abstract

In survey sampling, auxiliary information is widely used to improve the precision of population mean estimation. Motivated by the classical ratio estimator of William G. Cochran and the exponential ratio estimator proposed by Bahl and Tuteja, this study proposes a generalized k-parameterized exponential ratio-product estimator for estimating the finite population mean under simple random sampling without replacement. The suggested estimator integrates ratio and product structures with an exponential adjustment factor and a tuning parameter k, providing additional flexibility for variance reduction. The bias and mean square error of the proposed estimator are derived up to the first-order approximation. The optimum value of the parameter k is obtained by minimizing the MSE expression. Theoretical comparisons demonstrate that the proposed estimator performs more efficiently than the classical ratio, product, and existing exponential estimators under certain regularity conditions. An empirical study using real data sets is conducted to validate the theoretical findings. The results indicate substantial gains in efficiency, confirming the superiority and practical applicability of the proposed class of estimators. The proposed estimator offers a flexible and efficient alternative for population mean estimation when auxiliary information is available.

Keywords: Finite population mean, auxiliary information, exponential ratio-product estimator, mean square error, SRSWOR, efficiency comparison

Introduction

In survey sampling, the accurate estimation of population parameters, particularly the population mean, has received considerable attention in statistical research. The use of auxiliary information is widely recognized as an effective technique for improving the precision and efficiency of estimators. When an auxiliary variable correlated with the study variable is available, it can be incorporated into the estimation process to reduce the variance of the estimator and improve its reliability. The theoretical foundations and applications of survey sampling methods have been extensively discussed in classical literature on sampling theory and survey design. Among the various estimators that utilize auxiliary information, the ratio and product estimators are commonly used for estimating the population mean. The ratio estimator is generally applied when the study variable and auxiliary variable are positively correlated, whereas the product estimator is preferred when the correlation is negative. Several modifications and improvements of these classical estimators have been proposed in the literature. Over time, several improved estimators have been proposed by incorporating auxiliary information in different ways. An important contribution in this direction was made by Bahl and Tuteja ^[1], who introduced exponential ratio and exponential product type estimators to increase the efficiency of the classical ratio estimator in simple random sampling. Their work inspired many researchers to develop improved estimators by modifying the structure of ratio and product estimators. Further developments include the modified ratio estimator proposed by Sisodia and Dwivedi ^[9], which incorporates the coefficient of variation of the auxiliary variable to improve the estimation of the population mean. Tailor and Sharma ^[15] also suggested a modified ratio-cum-product estimator by using known parameters of the auxiliary variable.

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Similarly, the transformation of auxiliary variables has been explored to obtain more efficient estimators, as discussed by Upadhyaya and Singh ^[16]. The use of known correlation coefficients between the study variable and auxiliary variable has also been proposed to enhance estimator efficiency. Several authors have proposed improved estimators under different sampling frameworks. Koyuncu and Kadilar ^[3] developed efficient estimators for the population mean in stratified random sampling. Yadav and Kadilar ^[17] later introduced an efficient family of exponential estimators based on auxiliary information. Zaman and Kadilar ^[18] further discussed exponential ratio-type estimators and demonstrated their improved performance in survey sampling applications. Additionally, Singh and Solanki ^[11] proposed a new procedure for variance estimation in exponential type estimators.

The theoretical background of sampling techniques and the role of auxiliary information in survey sampling have been widely discussed in standard textbooks on sample survey methods. Singh *et al.* ^[13] also highlighted the importance of auxiliary information and prior knowledge of population parameters in constructing improved estimators. In recent years, several new estimators based on auxiliary information have been proposed to further improve the estimation of the population mean. Malik and Tailor ^[8] introduced ratio and product type exponential estimators using known parameters of the auxiliary variable, while Malik and Singh ^[7] proposed an improved exponential ratio type estimator for estimating the population mean. Furthermore, Malik and Kusum ^[4] proposed a new logarithmic type estimator using auxiliary information. Malik and Nikita ^[5] also introduced a regression-cum exponential type estimator to improve estimation efficiency. Similarly, Malik and Poonam Devi ^[6] proposed an improved exponential type estimator under simple random sampling. These contributions demonstrate the continuing interest in developing efficient estimators using auxiliary information.

Motivated by these developments, the present study proposes new modified RA-type estimators for estimating the population mean under simple random sampling without replacement using auxiliary information. The proposed estimators combine the properties of ratio and product type estimators. Theoretical expressions for the mean squared error (MSE) of the proposed estimators are derived and compared with those of existing estimators. The efficiency of the proposed estimators is also examined to demonstrate their superiority over conventional estimators under certain conditions.

This study focuses on improving the estimation of logarithmic-type estimators by utilizing data from an auxiliary variable under simple random sampling. The paper is organized as follows: Section 1 reviews existing estimators along with relevant methodology and notes from the literature. Section 2 introduces a new modified logarithmic-type estimator for simple random sampling. Section 3 presents a comparative analysis of the efficiency of the proposed estimators, P_r and P_p , against the existing estimators P_1 , P_2 , and P_3 . Section 4 examines the optimization of the variance of the proposed estimators, and Section 5 provides a numerical comparison to illustrate their performance.

1. Notations

Let a finite population $U = (1, \dots, i, \dots, N)$ are created from a simple random sample without replacement (SRSWOR).

Further,

N : Number of sampling units in the population (Population size).

n : Number of sampling units in the sample (sample size)

Y : The characteristic under consideration

X : Auxiliary variable

$\bar{Y}$ = Population mean for the main variable 'Y'

$\bar{X}$ = Population mean for the main variable 'X'

$\bar{y}$ = Sample mean for the main variable 'y'

$\bar{x}$ = Sample mean for the auxiliary variable 'x'

Y_i : Value of the characteristic of the i^{th} unit of the population.

$$G_n = \left(\frac{1-f}{n} \right)$$

$$\text{Population Total: } Y = \sum_{i=1}^N y_i$$

$$\text{Population Mean: } \bar{Y} = \sum_{i=1}^N \frac{y_i}{N}$$

$$\text{Sample Total: } y = \sum_{i=1}^n y_i$$

$$\text{Sample Mean: } \bar{y} = \sum_{i=1}^n \frac{y_i}{n}$$

$$R = \frac{\bar{y}}{\bar{x}}$$

Existing Estimators

The standard mean, ratio and product type estimators in simple random sampling are as follows:

Simple Estimator

Population mean P_1 in simple random sampling is

$$P_1 = \sum_{i=1}^N \frac{y_i}{N} = \bar{y}$$

And MSE is given by

$$\text{MSE}(P_1) = \sum_{i=1}^N \frac{\bar{y}^2(1-f)c_y^2}{n}$$

Ratio Estimator

The ratio estimator P_2 is defined by

$$P_2 = \frac{\bar{y}}{\bar{x}} \bar{X}$$

MSE up to first degree of approximation is

$$\text{MSE}(P_2) = \sum_{i=1}^N \frac{\bar{y}^2(1-f)}{n} (C_y^2 + C_x^2 - 2C_{xy})$$

Product Estimator

The product estimator P_3 is defined by

$$P_3 = \frac{\bar{y}\bar{x}}{\bar{X}}$$

MSE up to first degree of approximation is

$$\text{MSE}(P_3) = \sum_{i=1}^N \frac{\bar{y}^2(1-f)}{n} (C_y^2 + C_x^2 + 2C_{xy})$$

2. Proposed Estimators

By the event of simple random sampling (SRS), we propose the enhanced combined ratio and Product with exponential type estimators with constants k as

$$P_r = \bar{y} \left(\frac{\bar{x}+k}{\bar{x}+k} \right) - k e^{(\bar{x}-\bar{x})} + k$$

$$P_p = \bar{y} \left(\frac{\bar{x}+k}{\bar{x}+k} \right) - k e^{(\bar{x}-\bar{x})} + k$$

Expressing P_r in terms of e_0 and e_1 , we obtain the desired result:

$$P_r = \bar{Y}(1 + e_0) \left(1 - \frac{\bar{x}e_1}{\bar{x}+k} \right) - k(1 - \bar{X}e_1) + k$$

After simplification, we get.....

$$P_r = \bar{Y} + \bar{Y}e_0 - \frac{\bar{X}\bar{Y}e_1}{\bar{X}+k} - \frac{\bar{X}\bar{Y}e_0e_1}{\bar{X}+k} + k\bar{X}e_1$$

By computing the expected value of, it is to determine the bias of proposed estimator P_r to the first order of approximation:

$$\text{Bias}_{P_r} = E \{ [\bar{y}_{P_r}] - \bar{Y} \}$$

$$\text{Bias}_{P_r} = \left(\frac{1-f}{n} \right) \left[\frac{-\bar{X}\bar{Y}\rho C_y C_x}{\bar{X}+k} \right]$$

To first degree of approximation MSE can be expressed as:

$$MSE_{P_r} = E \{ [\bar{y}_{P_r} - \bar{Y}]^2 \}$$

$$MSE_{P_r} = \left(\frac{1-f}{n} \right) \left\{ \bar{Y}^2 C_y^2 + \bar{X}^2 C_x^2 \left(\frac{\bar{Y}^2}{(\bar{X}+k)^2} + k^2 - \frac{2k\bar{Y}}{(\bar{X}+k)} \right) - 2\bar{X}\bar{Y}\rho C_x C_y \left(\frac{\bar{Y}}{(\bar{X}+k)} - k \right) \right\} \quad (1)$$

Expressing P_p in terms of e_0 and e_1 , we get outcome as:

$$P_p = \bar{Y}(1 + e_0) \left(1 + \frac{\bar{x}e_1}{\bar{x}+k} \right) - k(1 + \bar{X}e_1) + k$$

After simplification, we get.....

$$P_p = \bar{Y} + \bar{Y}e_0 + \frac{\bar{X}\bar{Y}e_1}{\bar{X}+k} + \frac{\bar{X}\bar{Y}e_0e_1}{\bar{X}+k} - k\bar{X}e_1$$

By computing the expected value of, it is to determine the bias of proposed estimator P_p to the first order of approximation:

$$\text{Bias}_{P_p} = E \{ [\bar{y}_{P_p}] - \bar{Y} \}$$

$$\text{Bias}_{P_p} = \left(\frac{1-f}{n} \right) \left[\frac{\bar{X}\bar{Y}\rho C_y C_x}{\bar{X}+k} \right]$$

To first degree of approximation MSE can be expressed as:

$$MSE_{P_p} = E \{ [\bar{y}_{P_p} - \bar{Y}]^2 \}$$

$$MSE_{P_p} = \left(\frac{1-f}{n} \right) \left\{ \bar{Y}^2 C_y^2 + \bar{X}^2 C_x^2 \left(\frac{\bar{Y}^2}{(\bar{X}+k)^2} + k^2 - \frac{2k\bar{Y}}{(\bar{X}+k)} \right) + 2\bar{X}\bar{Y}\rho C_x C_y \left(\frac{\bar{Y}}{(\bar{X}+k)} - k \right) \right\} \quad (2)$$

3. Relative Efficiency

1) It can be observed that proposed estimator P_r is better in efficiency than Ratio P_2 ,

$$MSE_{P_r} < MSE_{P_2}, \text{ if } \rho > \frac{C_x \left(\frac{\bar{X}^2}{(\bar{X}+k)^2} - 1 + \frac{k^2}{R^2} - \frac{2k\bar{X}}{R(\bar{X}+k)} \right)}{2C_y \left(\frac{\bar{X}}{(\bar{X}+k)} - \frac{k}{R} - 1 \right)}$$

2) Suggested estimator P_r is more proficient than original product P_3 ,

$$MSE_{P_r} < MSE_{P_3}, \text{ if } \rho > \frac{C_x \left(\frac{\bar{X}^2}{(\bar{X}+k)^2} - 1 + \frac{k^2}{R^2} - \frac{2k\bar{X}}{R(\bar{X}+k)} \right)}{2C_y \left(\frac{\bar{X}}{(\bar{X}+k)} - \frac{k}{R} + 1 \right)}$$

3) It can be observed that proposed estimator P_p is better in efficiency than Ratio P_2 ,

$$MSE_{P_p} < MSE_{P_2}, \text{ if } \rho > \frac{C_x \left(\frac{\bar{X}^2}{(\bar{X}+k)^2} - 1 + \frac{k^2}{R^2} - \frac{2k\bar{X}}{R(\bar{X}+k)} \right)}{2C_y \left(-\frac{\bar{X}}{(\bar{X}+k)} + \frac{k}{R} - 1 \right)}$$

4) Suggested estimator P_p is more proficient than original product P_3 ,

$$MSE_{P_p} < MSE_{P_3}, \text{ if } \rho > \frac{C_x \left(\frac{\bar{X}^2}{(\bar{X}+k)^2} - 1 + \frac{k^2}{R^2} - \frac{2k\bar{X}}{R(\bar{X}+k)} \right)}{2C_y \left(-\frac{\bar{X}}{(\bar{X}+k)} + \frac{k}{R} + 1 \right)}$$

4. Optimum Variance

Optimisation of proposed estimator P_r is obtained by minimising MSE up to first degree of approximation by differentiating equation (1) w.r.t k gives optimum value

$$k = -\bar{Y} \left(\frac{\rho C_y}{\bar{X} C_x} \right)$$

Minimum MSE of estimator P_r is

$$MSE_{P_{r_{min}}} = \bar{Y}^2 C_y^2 (1 - \rho^2) \left(\frac{1-f}{n} \right)$$

Optimisation of proposed estimator P_p is obtained by minimising MSE up to first degree of approximation by differentiating equation (2) w.r.t k gives optimum value

$$k = \bar{Y} \left(\frac{\rho C_y}{\bar{X} C_x} \right)$$

Minimum MSE of estimator P_p is

$$MSE_{P_{p_{min}}} = \bar{Y}^2 C_y^2 (1 - \rho^2) \left(\frac{1-f}{n} \right)$$

5. Numerical Study

We have calculated percent relative efficiency of the estimators with respect to the estimators P_1 , P_2 , and P_3 , by applying two populations in order to determine the effectiveness of proposed estimators P_r and P_p in comparison to the Ratio and Product estimators.

Population descriptions are given below:

Population: [Source: National Horticulture Board, 2018-2019]

$$N = 29, n = 5, \bar{Y} = 28.23, \bar{X} = 226.10, C_y = 2.874247, C_x = 0.347147, \rho_{xy} = 0.6904$$

Table 1: PRE of proposed estimator P_r w.r.t P_r, P_1, P_2, P_3 ,

PRE's			
Estimators	($k = -0.50$)	($k = -0.60$)	($k = -0.70$)
P_r	100	100	100
P_1	188.434	191.033	186.92
P_2	159.758	161.961	158.474
P_3	222.608	225.678	220.819

Table 2: PRE of proposed estimator P_p w.r.t P_p, P_1, P_2, P_3 ,

PRE's			
Estimators	($k = 0.7$)	($k = 0.8$)	($k = 0.9$)
P_p	100	100	100
P_1	184.769	190.582	189.776
P_2	156.651	161.579	160.895
P_3	218.279	225.146	224.194

Conclusion

In this study, exponential type ratio and product estimators incorporating a scalar constant k were considered for estimating the finite population mean under simple random sampling without replacement (SRSWOR). The mean square errors (MSEs) of both estimators were derived up to the first order of approximation.

In this study, new modified RA-type estimators are proposed under simple random sampling without replacement by utilizing an auxiliary variable. The proposed estimators are formulated as a combination of well-known estimators, particularly the ratio and product estimators. The mean squared error (MSE) of the proposed estimators is theoretically compared with the MSEs of the standard estimator P_1 , the ratio estimator P_2 , and the product estimator P_3 . The conditions for obtaining the MSEs are also derived. The findings show that, under certain conditions, the proposed estimators perform more efficiently than the existing estimators.

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