

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452

NAAS Rating (2025): 4.49

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Maths SP-11(3): 06-12

www.mathsjournal.com

Received: 07-01-2026

Accepted: 10-02-2026

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A systematic review of ranked set sampling and its developments

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DOI: <https://www.doi.org/10.22271/math.2026.v11.i3Sa.2277>

Abstract

Ranked Set Sampling is becoming more prevalent since this method proposes more accurate estimates without increasing sample size. It is especially useful when the process of ranking is simple but measurement is costly or difficult. Ranked Set Sampling enhances productivity and decision-making accuracy by reducing variance and Mean Squared Error. As a highly relevant and cost-effective sampling method, it is currently being used extensively for dependability analysis, industrial quality control, medical, agricultural, and environmental investigations. This review paper provides a structured discussion on the Ranked set sampling framework, ranking mechanisms, major modifications, inferential procedures, and practical applications.

Keywords: Ranked set sampling, Statistical process control, Sampling method, Process monitoring

1. Introduction

Sampling is essential when the entire population cannot be observed due to time, cost, or measurement constraints. Sampling theory's strategic objective is to obtain effective estimators of population parameters with the lowest possible sampling error by using auxiliary data and the ratio, product, or regression estimator. The traditional Simple Random Sampling (SRS) design demands precise measurement of each selected unit, which may be costly or impractical in fields such as industrial quality control, environmental studies, agriculture, and reliability theory. To overcome these problems, RSS was proposed by McIntyre (1952) ^[1], who demonstrated that making inexpensive initial rankings prior to precise measurement can significantly decrease variance without increasing sample sizes. Subsequently, Takahashi and Wakimoto established the mathematical theory of RSS, proving its unbiasedness and superiority over SRS in terms of efficiency under ideal conditions of ranking. Later studies by Stokes, Dell, Clutter, Samawi, Muttalak, Al-Saleh, Al-Kadiri, and others have expanded RSS to incorporate auxiliary information, imperfect rankings, and robustness. RSS has evolved to become a versatile theory with different models, such as Extreme RSS, Median RSS, Double RSS, and Min-Max RSS, which have been successfully applied to estimation problems, composite sampling, and more recent developments in statistical process control methods such as CUSUM and EWMA charts, proving its superiority over traditional methods in terms of efficiency.

2. Enhancements and Extensions

The effectiveness and usefulness of RSS applications in modern statistical inference have been significantly enhanced through improvements and enhancements. Various forms of RSS have been developed to improve the precision of the estimators for symmetric and skewed populations, such as Double Ranked Set Sampling (DRSS), Extreme Ranked Set Sampling (ERSS), Median Ranked Set Sampling (MRSS), and Percentile Ranked Set Sampling (PRSS). More advanced forms of RSS, such as Neoteric Ranked Set Sampling (NRSS), have also been developed to further improve the performance of the estimators. Using probabilistic misclassification models, the developments also incorporate imperfect models for ranking in the RSS framework. In the context of regression, reliability, and quality control,

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RSS estimators outperform the traditional SRS estimators, especially for populations that follow the Normal, Gamma, and other distributions, as well as for populations with the presence of outliers in the data set. RSS is a robust and powerful sampling technique because of the improvements and enhancements in the methodology, primarily focused on increasing Percent Relative Efficiency (PRE) and reducing the Mean Square Error (MSE).

3. Theory and Role of RSS in SPC

The integration of RSS into SPC represents a major advancement in improving process monitoring efficiency under cost and measurement constraints. Traditional SPC procedures were primarily developed under SRS, where all selected units must be precisely measured. However, in many industrial and environmental settings, exact measurement is expensive or time-consuming, while ranking units based on judgment or auxiliary information is relatively easy. RSS addresses this limitation by incorporating a preliminary ranking step before measurement, thereby reducing variance and yielding more efficient estimators without increasing sample size.

The application of RSS in SPC gained significant attention in the early 2000s. Bouza (2002) was among the first to construct RSS-based Shewhart control charts, demonstrating improved Average Run Length (ARL) performance compared to classical SRS-based charts. This marked the beginning of systematic research on RSS in control chart design. Later, Abbas *et al.* (2013) introduced EWMA control charts using RSS, demonstrating enhanced sensitivity to small process shifts. Similarly, Abu-Shaaban *et al.* (2017) developed CUSUM charts based on RSS, reporting superior detection ability for persistent shifts. Lee and Abujiya (2013) provided a comprehensive comparison of RSS-based Shewhart, CUSUM, and EWMA charts, concluding that RSS significantly improves chart performance while maintaining the same sampling effort.

In recent years, the role of RSS in SPC has expanded further with the development of hybrid and memory-type schemes such as EWMA, CUSUM and HWMA charts. These advanced designs combine the variance reduction property of RSS with the accumulation feature of memory charts, resulting in excellent detection capability for small and moderate shifts. Today, RSS plays a vital role in modern SPC by offering a cost-effective, efficient, and flexible framework for process monitoring, particularly in industries where measurement is expensive but ranking is practical, thereby strengthening both theoretical development and real-world quality improvement practices.

4. Procedures of Ranked Set Sampling Methods

4.1 Ranked Set Sampling (RSS)

In the classical ranked set sampling procedure ^[1], a simple random sample of size

m^2 is drawn from the population and randomly divided into m sets, each containing m units. Without actually measuring anything, the units in each set are ordered according to the variable of interest through expert judgment, visual examination, or a cheap auxiliary variable. The smallest ranked unit is chosen for measurement from the first set, followed by the second smallest ranked unit from the second set, and so on until the largest ranked unit is taken from the m^2 set. This concludes one RSS cycle. A ranked set sample of size $n = rm$ is obtained by repeating the cycle r times.

4.2 Extreme Ranked Set Sampling (ERSS)

ERSS ^[14] median randomly selects n sets, each with n units, from the population. Each set's units are ranked using visual judgment or a reasonable way to determine the smallest and largest units. The smallest unit is measured from the first group, followed by the largest unit from the second. This alternating procedure of selecting the smallest and largest units is repeated for the remaining sets. The final set selection utilizes on whether n is even or odd. If n is even, only the largest unit has been measured. If n is odd, the average of the smallest and largest units, or the median unit, is measured.

4.3 Median Ranked Set Sampling (MRSS)

In MRSS ^[15], n sets, each containing n units, are randomly drawn from the population. Within each set, the units are ranked according to the variable of interest using visual judgment or any low-cost ranking method. The unit selected for actual measurement depends on whether the set size n is odd or even. If n is odd, the median-ranked unit, that is, the $\frac{n+1}{2}$ th smallest unit, is chosen from each set for measurement. If n is even, the selection is divided equally: from the first $n/2$ sets, the $n/2$ th ranked unit is measured, and from the remaining $n/2$ sets, the $\frac{n+1}{2}$ th ranked unit is measured. This technique generates one complete cycle of MRSS with n measured observations, which can be repeated as needed.

4.4 Double Ranked Set Sampling (DRSS)

DRSS ^[31] is an extension of the usual RSS designed to improve estimation efficiency without increasing the set size. Assume the set size is m (with one cycle, $n = 1$). First, select m^3 units randomly from the population and divide them into m groups, each containing m^2 units. For each group, apply the standard Ranked Set Sampling (RSS) procedure: divide the m^2 units into m subsets of size m , rank the units within each subset, and select one unit from each subset according to the RSS rule. This step produces m ranked set samples, each of size m . Finally, apply the RSS procedure again to the m ranked set samples obtained in Step 2. After ranking and selecting according to the RSS rule, we obtain a DRSS of size m . This method effectively applies the ranking process twice, increasing the estimator's precision relative to ordinary RSS while keeping the set size unchanged.

4.5 Quartile Ranked Set Sampling (QRSS)

In the QRSS technique ^[31], n random samples of size n units are chosen from the population, and each sample's units are ranked according to a variable of interest. Choose the $(q_1(n+1))$ th smallest rank from the first $n/2$ samples and the $(q_1(n+1))$ th smallest rank from the second $n/2$ samples for measurement if the sample size is even, where $q_1 = 0.25$ and $q_u = 0.75$. Note we will take the nearest integer of $q_1(n+1)$ th and $q_u(n+1)$ th. If the sample size is odd, choose the $q_1(n+1)$ th smallest rank from the first $(n-1)/2$ samples, the $q_u(n+1)$ th smallest rank from the second $(n-1)/2$ samples, and the median for that sample for the measurement. The QRSS data is made up of these number units.

4.5 Partial Ranked Set Sampling (PRSS)

PRSS ^[19] is a combination of Simple Random Sampling and Ranked Set Sampling and is preferred when ranking all units in the set is difficult or costly. To get a partial ranked set sample of size m , a coefficient k is calculated as the largest integer less than or equal to αm , where $0 \leq \alpha < 0.5$. In a single sampling cycle, $2k$ units are directly sampled from the population using simple random sampling without any ranking. For the remaining $m - 2k$ units, $m - 2k$ sets, each of size m , are sampled from the population, and the units in each set are ranked according to judgment or an auxiliary

variable. From the i^{th} set, the i^{th} ranked unit is selected for $i = k + 1, \dots, m - k$, and a total of m units are measured in a single cycle. This is repeated independently r times to get a final sample of size $n = mr$. The total number of units from the population involved in the selection of a PRSS sample of size m is given by $m^2 - 2k(m - 1)$, and when $k = 0$, the PRSS method is the same as the standard ranked set sampling method.

4.6 Mixed Ranked Set Sampling (MxRSS)

MxRSS^[20] is a flexible sampling technique that combines the concepts of SRS and RSS. It is especially helpful when partial ranking is possible or when the cost of ranking is high. To collect a mixed ranked set sample of size m , the following procedure is followed: First, k_1 units are chosen directly from the desired population using the simple random sampling (SRS) method, where $0 \leq k_1 \leq m$. These units are selected without any ranking. Next, a constant k_2 is specified as the largest integer less than or equal to $\beta(m - k_1)$, where $0 \leq \beta < 0.5$. Then, $(m - k_1)(m - k_1 - k_2)$ units are selected from the population and allocated into $m - k_1 - k_2$ sets, consisting of $m - k_1$ units in each set. Without evaluating the actual values, the units in each set are ranked according to the study variable using an inexpensive or judgment sampling method.

From these ranked sets, the smallest ranked unit is chosen from the first $m - k_1 - k_2$ sets. Additionally, from the first k_2 sets, the $(m - k_1 - i + 1)^{th}$ smallest ranked unit is chosen for $i = 1, 2, \dots, k_2$. Along with the initially chosen k_1 SRS units, this marks the end of one cycle of a mixed ranked set sample of size m . Lastly, the above procedure may be repeated independently r times to derive a final mixed ranked set sample of size.

4.7 Neoteric Ranked Set Sampling (NRSS)

Neoteric Ranked Set Sampling (NRSS)^[33] is an improvement of the Double ranked set sampling design, which uses a large ranked set instead of several small sets to increase the efficiency of estimation. In this method, a simple random sample of size n^2 units is first taken from the study population and ranked using a pre-defined ordering criterion based on the variable of interest through judgment or any inexpensive method without measurement. Then, from the ranked units, one unit is chosen for measurement at the position $[(i - 1)n + l]^{th}$, where $i = 1, 2, \dots, n$, and the value of l depends on whether n is odd or even. If n is odd, then $l = (n + 1)/2$; and if n is even, then $l = (n + 2)/2$ when i is odd, and $l = n/2$ when i is even. This process will provide a sample of size n in one cycle, and the process can be repeated independently m times to provide a final NRSS sample of size mn .

4.8 Minimax Ranked Set Sampling (MMRSS)

In the MiniMax Ranked Set Sampling plan^[22], initial simple random samples of progressively larger sizes are obtained from the population. To be specific, for one cycle, m simple random samples are obtained, with the i^{th} sample consisting of i units for $i = 1, 2, \dots, m$. The units in each sample are then arranged in ascending order within the sample according to the variable of interest through judgment or any inexpensive method of ranking. From the first sample, the minimum unit is chosen, from the second sample the maximum unit is chosen, and this pattern of choosing the minimum and maximum alternates among samples; in the last sample, the minimum unit is chosen if m is odd and the maximum unit if m is even. This completes one MiniMax RSS cycle of size m . Independently repeating the process r times will then obtain a final MiniMax RSS sample of size $n = rm$.

4.9 Modified MiniMax Ranked Set Sampling (MMMRSS)

In the MMMRSS design^[23], a series of simple random samples of growing sizes is first chosen from the target population. More specifically, m samples are chosen, with the i^{th} sample consisting of i units for $i = 1, 2, \dots, m$. If the set size m is odd, the process follows the MiniMax RSS design. But if m is even, the final sample of size m is replicated to prevent inefficiency due to asymmetry in distribution. The units in each sample are then ranked from the smallest to the largest using intuition or any other cheap method of ranking. The smallest ranked unit is chosen from the first sample, the largest ranked unit from the second sample, the smallest ranked unit from the third sample, and this pattern of choosing the minimum and maximum continues. For even m , the final observation is derived by calculating the average of the two middle ranked units from the replicated samples. This marks the end of one cycle in an MMMRSS of size m . The whole process can be carried out independently r times to yield a final MMMRSS sample of size $n = rm$.

4.10 Hybrid Ranked Set Sampling (HRSS)

In the HRSS method^[24], three constants k_1, k_2 , and k_3 are chosen such that $k_2 = \lfloor \alpha k_1 \rfloor$ and $k_3 = \lfloor \alpha(m - k_1) \rfloor$, where $0 \leq \alpha \leq 0.5$. For constructing one HRSS sample of size m , $k_1(k_1 - k_2)$ units are sampled from the population and allocated into $k_1 - k_2$ sets of size k_1 . The units in each set are ranked, and the r^{th} smallest unit is selected from the r^{th} set, while the $(k_1 - r + 1)(k_1 - r + 1)^{th}$ smallest unit is selected from the first k_2 sets. Next, $(m - k_1)2(m - k_1 - k_3)$ units are chosen and formed into $m - k_1 - k_3$ sets, to which the traditional RSS method is applied. From these sets, the r^{th} smallest unit is selected from the r^{th} set and the $(m - k_1 - r + 1)^{th}$ smallest unit from the first k_3 sets. These units are combined to form one HRSS sample of size m . This process is repeated

4.11 Neutrosophic Ranked Set Sampling (NRSS)

NRSS^[34] is an extension of the traditional ranked set sampling method for dealing with imprecise, vague, or interval data in a neutrosophic setting. In this method, m neutrosophic random samples, each of size m , are drawn from the population. Each item in the sample is represented as an interval to account for any indeterminacy. Each sample is then ranked according to an auxiliary variable by an inexpensive judgment method, where ranking occurs according to the neutrosophic interval values' midpoints. If there are any ties, lower and upper bounds are employed for resolution. From the first sample, the smallest item from the sample is selected for measurement, the second smallest from the second sample, and so on, until the largest item from the m^{th} sample. This process constitutes one NRSS cycle, which produces m neutrosophic data points. This process is replicated independently r times to generate a neutrosophic ranked set sample of size $n = mr$, which can be employed for estimating population parameters such as the mean in a neutrosophic setting.

4.12 Minimum Ranked Set Sampling (MinRSS)

In MinRSS^[27] method, one cycle of sampling is conducted as follows: First, n simple random samples are collected from the population of interest, where the sample size in the i^{th} sample is i , for $i = 1, 2, \dots, n$. The i^{th} sample is then ranked from lowest to highest value for the variable of interest using judgment or other inexpensive ranking techniques without measurement. The minimum (smallest) element from each sample is then selected for measurement. The above procedure yields one MinRSS sample of size n . The procedure can be replicated independently r times to yield a final sample in MinRSS method of size $n = nr$.

4.13 Robust Ranked Set Sampling (LRSS)

For the development of an effective LRSS^[35], n random samples, each of size n , are randomly drawn from the target population, where n is usually chosen to be small in order to minimize the ranking errors. The elements in each sample are then ranked according to the variable of interest using judgmental methods such as cost-free methods, assuming that the judgmental ranking is as reliable as the actual ranking. The coefficient k is defined as the greatest integer less than or equal to αn , where $0 < \alpha < 0.5$. The element in the first $\lfloor n/2 \rfloor$ ranked samples is chosen at the $k+1$ st rank, the element in the last $\lfloor n/2 \rfloor$ ranked samples is chosen at the $n-k$ th rank, and if n is an odd number, the median-ranked element is chosen at the $(n+1)/2$ th rank in the remaining sample for the completion of the ERSS cycle.

4.14 Percentile Ranked Set Sampling (PRSS)

The Percentile Ranked Set Sampling (PRSS) technique was^[36] an extension of ranked set sampling based on percentiles. In this technique, n random samples of size n are selected from the population and ranked on the basis of the variable of interest using judgmental or cost-free methods of ranking. Two percentiles p and q are selected such that $0 \leq p \leq 1$ and $p+q=1$. When n is even, the unit with the rank closest to $p(n+1)$ is selected from each of the first $n/2$ sets of samples, and the unit with the rank closest to $q(n+1)$ is selected from each of the remaining $n/2$ sets of samples. When n is odd, the unit with the rank closest to $p(n+1)$ is selected from each of the first $(n-1)/2$ sets of samples, the unit with the rank closest to $q(n+1)$ is selected from each of the remaining $(n-1)/2$ sets of samples, and the median unit is selected from the remaining set of samples. One complete cycle produces a sample of size n , and repeating the cycle m times produces a final PRSS sample of size mn , where the nearest integer is used if $p(n+1)$ or $q(n+1)$ is non-integer. When this procedure for percentile ranked set sampling is applied to each stratum of a stratified population, the procedure is known as Stratified Percentile Ranked Set Sampling (SPRSS), which combines the advantages of stratification and ranked set sampling to produce more representative and efficient samples for the population under study.

4.15 Balanced Ranked Set Sampling (BRSS):

The BRSS scheme, which was initially presented by McIntyre (1952) as part of the traditional RSS method, was developed to guarantee equal allocation for the ranks. In the BRSS method, H simple random samples, each of size H , are first obtained from the population in each cycle, and the units in each sample are ranked according to an auxiliary variable or a study variable through judgment or any inexpensive method for ranking. From the first ranked sample, the smallest ranked unit is selected for measurement, from the second ranked sample, the second smallest unit is selected, and so on until the H th ranked sample from which the largest ranked unit is selected. This process represents one BRSS cycle, which produces one observation from each rank. To ensure balance, the whole RSS process is replicated m times independently, which ensures that each rank occurs m times in the final sample. Thus, the BRSS method produces a balanced sample of size $n = Hm$, which includes equal representation from all the ranks, improving efficiency in estimation compared to the SRS method while maintaining symmetry in the allocation of the ranks.

4.16 Unbalance Ranked Set Sampling (UBRSS):

Unbalanced Ranked Set Sampling (URSS) is applied when equal allocation to strata is not feasible due to unequal variability, non-availability of data, etc. In Unbalanced RSS,

the population is divided into H strata, with unequal sample size $n_1, n_2, \dots, n_H$ allocated to these strata. Ranked set sampling is applied sequentially to these strata, but only to those strata which still require more samples to reach their allocated sample size $n_1, n_2, \dots, n_H$. For each cycle of ranked set sampling, a judgment or cost-effective method is applied to rank the elements within a stratum from which one element is selected to measure. The process is terminated when all strata have reached their allocated sample size.

4.17 Truncation-based RSS

In TBRSS^[37] method, m number of simple random samples, each consisting m units, are obtained from the population under consideration. The units in each sample are ranked according to the variable of interest through judgmental methods such as visual inspections. A trimming coefficient k is determined as the integer part of αm , where $0 < \alpha < 0.5$. From the first k samples, the smallest ranked unit is selected, while from the last k samples, the largest ranked unit is selected. From the remaining $m-2k$ samples, the $i+k$ th ranked unit is selected from the i th sample, where $i = 1, 2, \dots, m-2k$. This process forms one TBRSS sample of size m . This process can be replicated independently n times to obtain a TBRSS sample of size mn .

4.18 Robust extreme RSS

In order to build an effective RERSS plan^[11], n random samples, each of size n , are drawn from the population, where n is usually chosen to be small in order to minimize the errors in the ranking procedure. It is assumed that the accuracy of judgment-based ranking is the same as the accuracy of actual ranking. Each sample is ranked according to the variable of interest through the use of visual inspection or any cost-free ranking procedure. Then, the coefficient k is defined as the greatest integer less than or equal to $n\alpha$, where $0 < \alpha < 0.5$. For each of the first $\lfloor n/2 \rfloor$ ranked samples, the item at the $k+1$ st rank is chosen, while for each of the last $\lfloor n/2 \rfloor$ ranked samples, the item at the $n-k$ th rank is chosen. If n is odd, the median item at the $(n+1)/2$ th rank is chosen for the remaining sample, in order to complete the RERSS procedure. This procedure may be repeated independently m times in order to produce the final RERSS sample of size nm .

4.19 Generalized Ranked Set Sampling (GRSS)

GRSS^[38] approach can be viewed as a generalization of the traditional RSS method, which uses more flexible selection methods while maintaining the traditional ranked set sampling concept. To carry out a GRSS design, a number of sampling cycles must be performed, in which samples from the population can be obtained through any sampling method that produces a sampling structure similar to that obtained through RSS. The units obtained through sampling must be arranged in rows according to judgmental ranking, auxiliary information, or any cost-free method that can be employed for the variable of interest. However, unlike traditional RSS, the units obtained through sampling do not need to be exact order statistics. The only condition that must be satisfied for GRSS is that the average distribution functions of the units obtained through sampling must equal the distribution function for the population. This ensures that the population is represented in an unbiased manner. After a GRSS sampling cycle has been performed, which produces a fixed number of units, the sampling process can be repeated independently to produce the desired sample size.

4.20 Simple Z Ranked Set Sampling (SZRSS)

The SZRSS^[39] method first selects m^2 units from the target population at random, which are then divided equally among m groups, with each group having m units. Each group of

units is then ranked according to the variable of interest through visual inspection without performing any actual measurement. If the size of the group m is odd, the $(i + 1)^{th}$ smallest ranked units from each of the first $(m - 1)/2$ groups, the median units from the middle group, and the $(i + 1)^{th}$ smallest ranked units from each of the last $(m - 1)/2$ groups are selected for measurement. This makes the selection process symmetrical. If the group size m is even, the $(i + 1)^{th}$ smallest ranked units from each of the first $m/2$ groups and the $(i + 1)^{th}$ smallest ranked units from each of the last $m/2$ groups are selected for measurement. This selection process forms one SZRSS cycle of size m . To select a larger sample size, the process may be repeated independently k times to obtain a sample size $n = mk$.

4.21 Weighted Ranked Set Sampling (WRSS)

In WRSS method [29], a ranked set sample is first identified using a classical RSS method. In the classical RSS method, k random samples of size k are selected from a target population, and elements within these samples are ranked based on a variable of interest using judgmental methods or cost-free methods. One element from a ranked set is selected based on rank position, which yields k ranked elements from a single cycle. Unlike a classical RSS method, WRSS uses unequal weights for ranked elements to assign less weight to elements with greater variability, extreme ranks and greater weights to elements with less variability, central ranks. These weighted elements are then combined to produce a population mean as a weighted average with weights optimized to produce maximum efficiency for positively skewed distributions. However, if a larger sample size is to be considered, this process may be repeated independently to obtain the required sample size. WRSS, therefore, enhances precision in estimation by considering suitable weights instead of increased allocations, thus improving the cost efficiency of RSS.

4.22 Partial Stratified Ranked Set Sampling (PSRSS)

In case of Partially Stratified Ranked Set Sampling (PSRSS) design, a coefficient c is defined as $c = \beta n$, where $0 \leq \beta \leq 0.5$. Using this value, the number of units are selected using simple random sampling. In this case, $2c$ units are selected from the population using the simple random sampling method. The remaining $n - 2c$ units are collected using a stratified ranked set sampling method. In this case, the population is divided into H mutually exclusive and exhaustive strata. Using judgmental or cost-free ranking, independent ranked set samples are collected from each stratum. The number of samples collected from each stratum is proportional to the size of each stratum. The total number of samples collected using this method is $n - 2c$. The above procedure is referred to as one cycle of PSRSS. The above procedure can be repeated independently r times to obtain a sample size of rn .

4.23 Neutrosophic Median Ranked Set Sampling (NMRSS)

In Neutrosophic Median Ranked Set Sampling [30], the main idea is to select sample units that can represent the population with imprecise, interval, or indeterminate values for the study variable as well as the auxiliary variable. To do this, we first select m independent random samples, each of size m , from the population, with each sample unit represented as a neutrosophic number. Then, we apply neutrosophic ranking for each sample, with the auxiliary variable as the basis for ranking, according to neutrosophic ranking criteria. If m is odd, we select the median units from each sample for actual measurement. If m is even, we select the $m/2$ smallest units from the first $m/2$ samples and the $(m + 2)/2$ smallest units from the remaining samples. This process constitutes one sampling cycle. We repeat this process r times to get a neutrosophic sample of size $n = mr$.

Table 1: Analyze of Ranked Set Sampling methods

S.No.	Method	Abbreviation	Author(s)	Year	Need & Method of Evaluation
1.	Ranked Set Sampling	RSS	McIntyre	1952	Proposed to obtain unbiased and more efficient estimators than SRS when ranking is easy and measurement is costly; performance assessed through unbiasedness, variance, and relative efficiency of the sample mean.
2.	Extreme Ranked Set Sampling	ERSS	Samawi <i>et al.</i>	1996	Designed to improve estimation by focusing on extreme observations in skewed populations; evaluated through variance reduction and efficiency comparison with RSS.
3.	Median Ranked Set Sampling	MRSS	Muttlak	1998	Motivated by the need to reduce sensitivity to skewness and outliers; assessed using bias and mean squared error of estimators.
4.	Double Ranked Set Sampling	DRSS	Al-Saleh & Al-Kadiri	2000	Developed to improve ranking accuracy by applying the ranking process twice; evaluated through relative efficiency over RSS.
5.	Simple Z Ranked Set Sampling	SZRSS	Barabesi and El-Sharaawi	2001	Motivated by symmetry requirements and zero skewness conditions; assessed via bias and variance under symmetric distributions.
6.	Weighted Modified RSS	WMRSS	Muttlak	2003	Aimed at enhancing robustness and efficiency through rank modification; examined using MSE and efficiency comparisons.
7.	Quartile Ranked Set Sampling	QRSS	Muttlak	2003	Motivated by the need for a balance between robustness and efficiency using quartiles; assessed using bias, variance, and efficiency measures.
8.	Generalized Ranked Set Sampling	GRSS	Bai & Chen	2003	Formulated as a general theoretical framework covering many RSS variants; evaluated using asymptotic variance and Fisher information.
9.	L-Ranked Set Sampling	LRSS	Al-Nasser	2007	Proposed to provide robustness against ranking errors in visual sampling; evaluated through robustness properties and MSE analysis.
10.	Robust Extreme RSS	RERSS	Al-Nasser & Mustafa	2009	Intended to stabilize extreme RSS under high variability; evaluated through robustness and efficiency under skewed data.
11.	Partial Ranked Set Sampling	PRSS	Haq <i>et al.</i>	2013	Motivated by high ranking cost and practical constraints; performance evaluated through cost-efficiency trade-offs and MSE.

12.	Truncation-Based RSS	TBRSS	Al-Omari & Raqab	2013	Developed to limit the influence of extreme ranks by truncation; assessed using MSE of truncated estimators.
13.	Mixed Ranked Set Sampling	MxRSS	Haq <i>et al.</i>	2014	Developed to combine SRS and RSS within a single framework; evaluated by efficiency comparisons under limited ranking ability.
14.	MiniMax Ranked Set Sampling	MMRSS	Al-Nasser & Al-Omari	2018	Designed to improve estimator performance by alternating minimum and maximum ranks; evaluated using variance and efficiency measures.
15.	Neoteric Ranked Set Sampling	NRSS	Taconeli & Cabral	2019	Proposed to increase efficiency using a single large ranked set; assessed through simulation-based MSE and efficiency analysis.
16.	Hybrid Ranked Set Sampling	HRSS	Haq <i>et al.</i>	2016	Proposed to unify several RSS designs into a single framework; evaluated through efficiency across embedded special cases.
17.	Modified MiniMax RSS	MMMRSS	Hanandeh & Al-Nasser	2021	Developed to correct inefficiencies of MiniMax RSS in asymmetric distributions; assessed using simulation-based MSE.
18.	Neutrosophic Ranked Set Sampling	NRSS	G K Vishwakarma & Singh	2022	Motivated by the presence of indeterminacy and interval uncertainty; assessed using neutrosophic mean, variance, and interval width.
19.	Weighted Ranked Set Sampling	WRSS	Dinesh S Bhoj & Chandra	2024	Developed to reduce estimator variance by assigning optimal weights to ranks; evaluated using weighted MSE and efficiency under skewness.
20.	Neutrosophic Median Ranked Set Sampling	NMRSS	Kumari, Singh & Smarandache	2024	Proposed for robust median estimation under neutrosophic uncertainty; evaluated using neutrosophic MSE and estimator stability.

5. Directions for Future Studies

Future research in the application of RSS in the domain of SPC is expected to have a vast scope for development, as the interest in this domain has increased manifold over the past decade. Although significant developments have been made in the construction of RSS-based control charts, the focus in the vast majority of the literature is still concentrated on the monitoring of the process mean, while the study of the process dispersion, variability structure, and profile monitoring is relatively less explored in the context of RSS-based control charts. Moreover, the use of traditional sampling plans in the vast majority of the literature provides an opportunity for investigating the application of alternative sampling plans, which may have the potential for better performance in the context of RSS-based control charts. As the complexity of modern production and service systems is increasing, the application of RSS-based control charts in the context of multistage, interdependent, high-dimensional, or even real-time processes, possibly through the support of automation technologies, is expected to open new avenues for the strengthening of the theoretical foundations and the application of RSS-based control charts in the domain of quality improvement.

6. Research Gaps and Future Directions

Despite significant advancements in RSS, important research gaps remain. Most studies assume perfect ranking and focus mainly on univariate process monitoring within SPC, with limited attention to ranking errors in dynamic environments, multivariate characteristics, and high-dimensional data structures. The effect of imperfect ranking under non-normal or time-varying conditions is not comprehensively analyzed. Future research may evolve robust RSS models that account for ranking uncertainty, extend methods to multivariate and real-time monitoring systems, and integrate RSS with machine learning-based and adaptive sampling strategies to enhance performance in modern smart manufacturing environments.

7. Conclusion

The study demonstrates that, without increasing sample size, adding ranking information to the sampling framework increases process monitoring performance and estimation accuracy. The suggested method maintains practical viability while offering more effective process shift detection. Overall, the results demonstrate how well-structured sample designs

can improve decision-making and quality control in practical applications.

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