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Division of generalized fuzzy numbers

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Abstract

In this paper, we discuss the division of generalized trapezoidal fuzzy numbers with example, which shows that the existing operations make some approximation, thereby causing loss of information.

Keywords: Fuzzy set theory, generalized fuzzy numbers, function principle

1. Introduction

The Generalized Fuzzy Number (GFN) is realized to be more flexible than the normalized fuzzy number since it considers the degrees of confidence of experts' opinions. In 1985 ^[1], Chen introduced the concepts of GFNs and also formulated different arithmetic operations on GFNs by proposing *function principle*. Based on Chen's (1985) ^[1] arithmetic operations, a few researchers, of late, have provided important theoretical foundations on GFNs and their applications (e.g., Chen and Chen 2003 ^[3], Chen and Hsieh 1999 ^[2], Chen and Wang 2009 ^[8], Chen *et al.* 2007 ^[7], Hsieh and Chen 1999 ^[3], Islam and Roy 2006 ^[5], Mahapatra and Roy 2006 ^[6], Wei and Chen 2009) ^[9].

Although *function principle* (Chen 1985) ^[1] was employed to develop arithmetic operations on GFNs, eventually some shortcomings of these operations have been observed. From mathematical point of view computation of different arithmetic operations using *function principle* is basically a point-wise operation. Due to this reason, it has been observed that Chen's arithmetic operations on generalized trapezoidal (and also triangular) fuzzy numbers causes loss of information, leading to inexact results. In view of this, the aim of this paper to discuss the division of generalized trapezoidal fuzzy numbers with example, which shows that the existing operations make some approximation, thereby causing loss of information.

Definition 1.1 Positive Trapezoidal Fuzzy Number

A generalized trapezoidal fuzzy number denoted as $\tilde{A} = (m_1, m_2; \beta, \gamma; w)$ is said to be positive if $m_1 - \beta \geq 0$ and $m_1 - \beta, m_1, m_2$ and $m_2 + \gamma$ are not identical.

2. The fuzzy division operation using function principle (Chen 1985) ^[1]

Let us consider two positive generalized trapezoidal fuzzy numbers

$$\tilde{A}_1 = (m_1, m_2; \beta_1, \gamma_1; w_1) \text{ and } \tilde{A}_2 = (m_3, m_4; \beta_2, \gamma_2; w_2).$$

Then the operational laws for these two fuzzy numbers are presented as follows

The division, $\tilde{A}_1$ is divided by $\tilde{A}_2$:

(Assuming $m_1, m_2, m_3, m_4, m_1 - \beta_1, m_2 + \gamma_1, m_3 - \beta_2, m_4 + \gamma_2$ are all nonzero real numbers)

$$\tilde{A}_1 / \tilde{A}_2 = \left(\frac{m_1}{m_4}, \frac{m_2}{m_3}; \frac{m_4\beta_1 + m_1\gamma_2}{m_4(m_4 + \gamma_2)}, \frac{m_2\beta_2 + m_3\gamma_1}{m_3(m_3 - \beta_2)}; \min(w_1, w_2) \right)$$

Setting $m_1 = m_2$ and $m_3 = m_4$ in the above operations, the corresponding results for generalized triangular fuzzy numbers (GTFNs) may be obtained. However, in course of the

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study some shortcomings of Chen's operations have been observed which are better reflected through GTFNs compared to generalized trapezoidal fuzzy numbers. Therefore, in the following section the shortcomings of Chen's method have been illustrated by utilizing example of GTFNs.

3. Operations on GFNs

Let $\tilde{A}_1 = (m_1, m_2; \beta_1, \gamma_1; w_1)$ and $\tilde{A}_2 =$

$(m_3, m_4; \beta_2, \gamma_2; w_2)$ are two GFNs with the following membership functions

$$\mu_{\tilde{A}_1}(x) = \begin{cases} 0 & \text{if } -\infty < x \leq m_1 - \beta_1 \\ w_1 f_a(x) & \text{if } m_1 - \beta_1 \leq x \leq m_1 \\ w_1 & \text{if } m_1 \leq x \leq m_2 \\ w_1 h_a(x) & \text{if } m_2 \leq x \leq m_2 + \gamma_1 \\ 0 & \text{if } m_2 + \gamma_1 \leq x < \infty \end{cases}$$

$$\mu_{\tilde{A}_2}(y) = \begin{cases} 0 & \text{if } -\infty < y \leq m_3 - \beta_2 \\ w_2 f_b(y) & \text{if } m_3 - \beta_2 \leq y \leq m_3 \\ w_2 & \text{if } m_3 \leq y \leq m_4 \\ w_2 h_b(y) & \text{if } m_4 \leq y \leq m_4 + \gamma_2 \\ 0 & \text{if } m_4 + \gamma_2 \leq y < \infty \end{cases}$$

Here $f_a(x)$ and $f_b(y)$ both are strictly increasing functions, where $f_a(x): [m_1 - \beta_1, m_1] \rightarrow [0, 1]$ and $f_b(y): [m_3 - \beta_2, m_3] \rightarrow [0, 1]$. Again $h_a(x)$ and $h_b(y)$ both are strictly decreasing functions, where $h_a(x): [m_2, m_2 + \gamma_1] \rightarrow [0, 1]$ and $h_b(y): [m_4, m_4 + \gamma_2] \rightarrow [0, 1]$.

Let $\tilde{C} = \tilde{A}_1(*)\tilde{A}_2$ (where $(*)$ is an extended fuzzy binary operation to combine $\tilde{A}_1$ and $\tilde{A}_2$), then by utilizing *extension principle* it can be said that $\tilde{C}$ will be a GFN.

Theorem 1.1

Product of two positive generalized trapezoidal fuzzy numbers $\tilde{A}_2 = (m_3, m_4; \beta_2, \gamma_2; w_2)$ generates a generalized trapezoidal fuzzy number of the form $\tilde{C} = \tilde{A}_1(\times)\tilde{A}_2 = (\underline{m}_{\tilde{C}}, \overline{m}_{\tilde{C}}; \beta_{\tilde{C}}, \gamma_{\tilde{C}})$, where $w_{\tilde{C}} = \min(w_1, w_2)$. For two different assumptions the values of $\underline{m}_{\tilde{C}}, \overline{m}_{\tilde{C}}, \beta_{\tilde{C}}$ and $\gamma_{\tilde{C}}$ are defined as follows:

Assumption 1: Spreads are small compared to the mean values

$$\begin{aligned} \underline{m}_{\tilde{C}} &\approx m_1 m_3 - m_1 \beta_2 - m_3 \beta_1 + w_{\tilde{C}}(m_1 \beta_2 / w_2 + m_3 \beta_1 / w_1) \\ \overline{m}_{\tilde{C}} &\approx m_2 m_4 + m_2 \gamma_2 + m_4 \gamma_1 - w_{\tilde{C}}(m_2 \gamma_2 / w_2 + m_4 \gamma_1 / w_1) \\ \beta_{\tilde{C}} &\approx w_{\tilde{C}}(m_1 \beta_2 / w_2 + m_3 \beta_1 / w_1) \\ \gamma_{\tilde{C}} &\approx w_{\tilde{C}}(m_2 \gamma_2 / w_2 + m_4 \gamma_1 / w_1) \end{aligned}$$

Assumption 2: Spreads are not small compared to the mean values

$$\begin{aligned} \underline{m}_{\tilde{C}} &\approx m_1 m_3 - m_1 \beta_2 - m_3 \beta_1 + w_{\tilde{C}}(m_1 \beta_2 / w_2 + m_3 \beta_1 / w_1) \\ \overline{m}_{\tilde{C}} &\approx (m_2 m_4 + m_2 \gamma_2) + m_4 \gamma_1 - w_{\tilde{C}}(m_2 \gamma_2 / w_2 + m_4 \gamma_1 / w_1) \\ \beta_{\tilde{C}} &\approx w_{\tilde{C}}(m_1 \beta_2 / w_2 + m_3 \beta_1 / w_1) - \beta_1 \beta_2 \\ \gamma_{\tilde{C}} &\approx w_{\tilde{C}}(m_2 \gamma_2 / w_2 + m_4 \gamma_1 / w_1) + \gamma_1 \gamma_2 \end{aligned}$$

4. Division of GFNs

Let $\tilde{A} = (m_1, m_2; \beta_1, \gamma_1; w_1)$ be a positive generalized trapezoidal fuzzy number with membership function $\mu_{\tilde{A}}(x)$.

The membership function of the inverse of $\tilde{A}$, written as $\tilde{A}^{-1}$, is defined as $\mu_{\tilde{A}^{-1}}(x) = \mu_{\tilde{A}}(1/x) \forall x \neq 0$. Then using the concept of inverse of a fuzzy number (Dubois and Prade 1980) we can deduce the approximate formula of $\tilde{A}^{-1}$ as $\tilde{A}^{-1} \approx (1/m_2, 1/m_1; \gamma_1/m_2^2 \beta_1 / m_1^2; w_1)$.

Now the division of $\tilde{A}_1(/)\tilde{A}_2$ may be defined as $\tilde{A}_1(/)\tilde{A}_2 = \tilde{A}_1(\times)\tilde{A}_2^{-1}$ where $\tilde{A}_1 = (m_1, m_2; \beta_1, \gamma_1; w_1)$ and $\tilde{A}_2 = (m_3, m_4; \beta_2, \gamma_2; w_2)$ where are two positive generalized trapezoidal fuzzy numbers. By employing the product operation on $\tilde{A}_1$ and $\tilde{A}_2^{-1}$ as discussed in Theorem 1.1 the following approximate result may be obtained:

$$\tilde{C} = \tilde{A}_1(/)\tilde{A}_2 = (\underline{m}_{\tilde{C}}, \overline{m}_{\tilde{C}}; \beta_{\tilde{C}}, \gamma_{\tilde{C}}; w_{\tilde{C}})$$

where,

$$\begin{aligned} \underline{m}_{\tilde{C}} &\approx \frac{m_1}{m_4} - \left(\frac{m_1 \gamma_2}{m_4^2}\right) \left(1 - \frac{w_{\tilde{C}}}{w_2}\right) - \left(\frac{\beta_1}{m_4}\right) \left(1 - \frac{w_{\tilde{C}}}{w_1}\right) \\ \overline{m}_{\tilde{C}} &\approx \frac{m_2}{m_3} + \left(\frac{m_2 \beta_2}{m_3^2}\right) - \left(1 - \frac{w_{\tilde{C}}}{w_2}\right) + \left(\frac{\gamma_1}{m_3}\right) \left(1 - \frac{w_{\tilde{C}}}{w_1}\right) 1 - \beta_1 \\ \beta_{\tilde{C}} &\approx w_{\tilde{C}} \left(\frac{m_1 \gamma_2}{m_4^2 w_2} + \frac{\beta_2}{m_4 w_2}\right) \end{aligned}$$

$$\gamma_{\tilde{C}} \approx w_{\tilde{C}}(m_2 \beta_2 / m_3^2 w_2 + \gamma_1 / m_3 w_1) \text{ and } w_{\tilde{C}} = \min(w_1, w_2).$$

5. Numerical illustration

Example 1.1 Let $\tilde{A}_1 = (5; 1, 1; 1)$ and $\tilde{A}_2 = (2; 1, 2; 0.5)$ be two GTFNs. Find $\tilde{A}_1(/)\tilde{A}_2$.

Result: We obtain $\tilde{A}_1(/)\tilde{A}_2 \approx (2.25, 2.75; 2.75, 1.5; 0.5)$

6. Conclusion

In this paper, division of generalized trapezoidal and also triangular fuzzy numbers have been analyzed. For this purpose, the existing operations on GFNs have been studied and the drawbacks have been pointed out. In order to overcome the drawbacks, new arithmetic operators on generalized trapezoidal and triangular fuzzy numbers have been proposed based on *extension principle*. Furthermore, the proposed operators can deal with both the non-normalized and normalized fuzzy numbers and also eventually causes less loss of information than the existing ones.

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