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Bivariate volatility modelling of the impact of crude oil prices on Nigeria exchange rate

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Abstract

This study conducts a bivariate volatility analysis of crude oil prices and Nigeria's exchange rate using data from 2010 to 2024 (180 observations). It employs descriptive statistics, cross-correlation, Granger causality, Johansen cointegration tests, and both VAR and DBEKK MGARCH frameworks to capture dynamic interactions and volatility persistence. The VAR results show a positive effect of crude oil prices on the exchange rate (0.1574, $p = 0.0368$) and mean reversion in the exchange rate (-0.158 , $p = 0.0344$), with limited short-run causality at lag 1. DBEKK MGARCH estimates reveal pronounced volatility clustering with asymmetric persistence across series. Crude oil price volatility exhibits strong baseline and ARCH/GARCH dynamics, with $A1(1,1) = 0.674$ and $B1(1,1) = 0.529$, while cross-channel instantaneous spillovers are weak ($M(1,2) = 0.287$, not statistically significant, $p = 0.872$). The unconditional baseline reflects substantial volatility ($M(1,1) = 12.986$, $p = 0.0216$) and a persistent second channel with $B1(2,2) = 1.010$ and $A1(2,2) = 0.286$, indicating very strong persistence and long-lasting volatility in that channel. Exchange rate volatility is highly responsive to shocks, with $A1(2,2) = 0.286$ and $B1(2,2) = 1.010$, and $M(2,2) = -2.474$ ($p < 0.001$), suggesting a damping but dominant persistence pattern in the second variance component. The cross-channel influence remains weak under diagonal BEKK, as evidenced by a non-significant $M(1,2)$ and relatively small cross-covariance terms. Overall, a substantial long-run correlation exists between the volatility processes, linking oil-price and currency risks (strong co-movement in volatility). Implications emphasize continuous volatility monitoring and hedging to mitigate persistent risk, and policies to stabilize currency markets and enhance resilience to oil-price shocks through hedging, reserves management, and macroprudential tools.

Keywords: Crude oil price, exchange rate, VAR, DBEKK-MGARCH, volatility modelling

1. Introduction

Crude oil is one of the most valuable and widely traded commodities in the world. It is a complex mixture of hydrocarbons that is refined into various petroleum products, including gasoline, diesel fuel, jet fuel, and lubricants. The first commercial oil well was drilled in 1859 by Edwin Drake in Titusville, Pennsylvania, marking the beginning of the modern oil industry (Yergin, 1991)^[42]. Crude oil prices have been a significant factor influencing exchange rates in African countries as the continent is home to several major oil-producing countries, including Nigeria, Angola, and Algeria. A study by Benjamin *et al.*, (2020)^[43] using the Johansen cointegration and the vector error correction model to access the dynamic relationship between oil price fluctuations and the real exchange rates in sub-Saharan African countries, demonstrates the significant power of oil prices to predict the movement of real exchange rates in Nigeria, Angola, the republic of Congo, Equatorial Guinea and Gabon. Nigeria as the largest oil producing country in Africa has had its own share of the impact of crude oil prices on its foreign exchange since the discovery of oil in Oloibiri, Nigeria on the 15th January, 1956 and its first oil field producing about 5,100 barrels per day (bpd) in 1958. Exchange rate is the price of one currency in terms of another, usually expressed as the number of units one currency can be exchanged for one unit of another currency, according to the international monetary fund (IMF). Fluctuations in global crude oil prices significantly impact Nigeria's economy, including its exchange rates.

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Understanding the dynamics between crude oil prices and exchange rates is crucial for policymakers, investors, and stakeholders to make informed decisions.

2. Materials and Methods

This study adopts a descriptive research design, leveraging a quantitative approach aimed at examining the relationship between crude oil prices and exchange rates in Nigeria. This study will adopt the econometrics models, Vector Autoregressive (VAR) model and Dynamic Baba-Engle-Kraft-Kroner (DBEKK) Multivariate Generalized Autoregressive Heteroskedasticity (MGARCH) model to examine this relationship. The VAR model will analyze the dynamic interdependencies over time, while the DBEKK model will examine volatility effects on crude oil prices and exchange rate.

2.1 Data collection

This study employs a secondary source of data. The monthly average time series data are collected from the central bank of Nigeria (CBN, www.cbn.gov.ng) spanning from 2010 - 2024

2.2 Model specification

The VAR introduced by (Christopher Sims, 1980) and DBEKK-MGARCH developed by Baba, Engle, Kraft and Kroner in 1990 are adopted in this study.

The representation of the model specification is as follows

Consider the bivariate time series (X_t, Y_t) , where crude oil prices = X_t and Exchange rate = Y_t ;

2.3 Model I: Vector Autoregressive VAR (p) model

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} C_X \\ C_Y \end{bmatrix} + \sum_{i=1}^p \begin{bmatrix} a_{11}^{(i)} & a_{12}^{(i)} \\ a_{21}^{(i)} & a_{22}^{(i)} \end{bmatrix} \begin{bmatrix} X_{t-i} \\ Y_{t-i} \end{bmatrix} + \begin{bmatrix} \mu_{X_t} \\ \mu_{Y_t} \end{bmatrix} \quad (2.1)$$

Where:

$$\begin{bmatrix} \mu_{X_t} \\ \mu_{Y_t} \end{bmatrix} \approx N(0, \Sigma_t)$$

- X_t : Crude oil prices at time t
- Y_t : Exchange rates at time t
- C_X, C_Y : Constant intercepts
- $a_{ij}^{(i)}$: Coefficients capturing the effect of lagged variables
- μ_{Y_t}, μ_{X_t} : Error terms
- p : Number of lags

2.4 Model II: Dynamic Baba - Engle - Kraft - Kroner (DBEKK) model.

$$H_t = \Omega_t + \sum_{i=1}^q A_{i,t} \epsilon_{t-i} \epsilon_{t-i}^T A_{i,t}^T + \sum_{j=1}^p B_{j,t} H_{t-j} B_{j,t}^T \quad (2.2)$$

Where:

H_t is the conditional covariance matrix at time t ,

Ω_t is a time varying positive definite matrix capturing the baseline variance,

$A_{i,t}$ and $B_{j,t}$ are time varying matrices representing the dynamic effect associated with past shocks and past covariances, ϵ_t are the residual errors

3. Results and Discussion

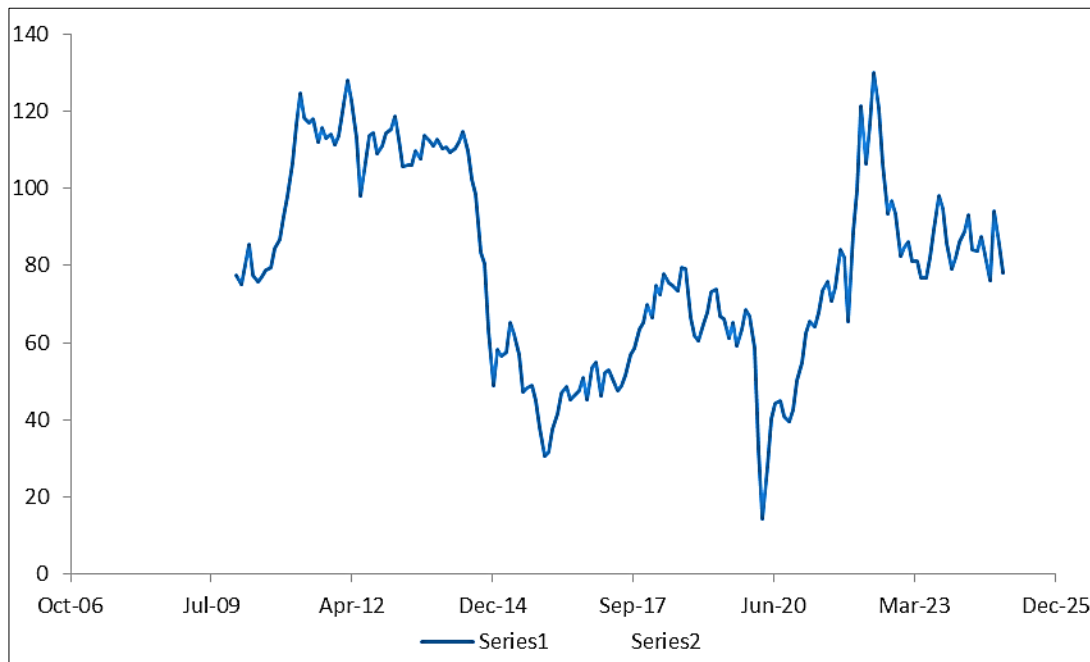


Fig 1: Time plot of Monthly Prices of Crude Oil (Unit per Barrel)

Figure 1 depicts the fluctuations in crude oil prices per barrel over the period of 2010 to 2024. Notably, there are periods of sharp increases, sharp decreases, and some recovery,

indicating an overall volatile trend with multiple peaks and troughs, reflecting the unpredictable nature of oil prices affected by global events and market dynamics.

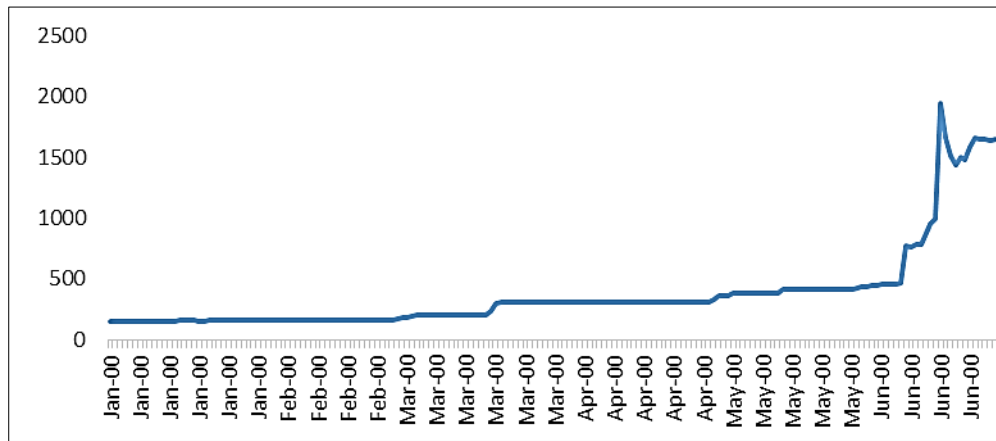


Fig 2: Time plot of Monthly Averages of Exchange Rate (2010 - 2024)

Figure 2 displays the monthly average exchange rate in Nigeria from 2010 to 2024. Initially remaining relatively stable, followed by a significant and rapid increase around 2022-2023, indicating a period of sharp currency fluctuation or economic change during that time.

Table 1: Descriptive Statistics of Crude oil Prices and Exchange Rate

	Crude oil prices	Exchange rate
Mean	79.68683333	376.0187222
Standard Error	1.920974827	27.23415723
Median	78.395	305.73
Mode	45.25	197
Standard Deviation	25.77258178	365.3845613
Sample Variance	664.2259715	133505.8776
Kurtosis	-0.92876715	6.880470348
Skewness	-0.01020288	2.728688569
Range	115.82	1795.35
Minimum	14.28	150.08
Maximum	130.1	1945.43
Sum	14343.63	67683.37
Count	180	180

From Table 1 the crude oil prices have a mean of approximately 79.69 with a standard deviation of about 25.77, indicating moderate variability around the average. The median price is 78.40, close to the mean, suggesting a symmetric distribution, supported by a near-zero skewness. For the exchange rate, the mean is approximately 376.02 with a much larger standard deviation of 365.38, reflecting high variability and volatility. Its median (305.73) is lower than the mean, and the positive skewness (2.73) shows right-skewed

distribution with extreme high values, confirmed by a high kurtosis (6.88), indicating heavy tails and potential outliers.

3.1 Unit root Test at level (Augmented Dickey- Fuller Test)

Table 2: Unit root test at level

Crude Oil Price		Exchange rate
Dickey-Fuller	-2.0561	0.9495
Lag order	5	5
P-value	0.5523	0.99

After the Augmented Dickey-fuller Test, on the raw data, for crude oil price: The test statistic (-2.0561) has a p-value (0.5528) greater than 0.05, indicating fail to reject the null hypothesis of non-stationarity. Similarly, for the exchange rate the test statistic (0.94956) has a p-value (0.99), also far above 0.05. This suggests the crude oil prices and exchange rate series are non-stationary.

Table 3: Unit root test after first difference

Crude Oil Price		Exchange rate
Dickey-Fuller	-5.6091	-6.2013
Lag order	5	5
P-value	0.01	0.01

After taken the first difference which was achieved by subtracting each value from its previous value, the ADF test results above indicate that both the crude oil price and exchange rate series are stationary after differencing, with p-values of 0.01 for both series.

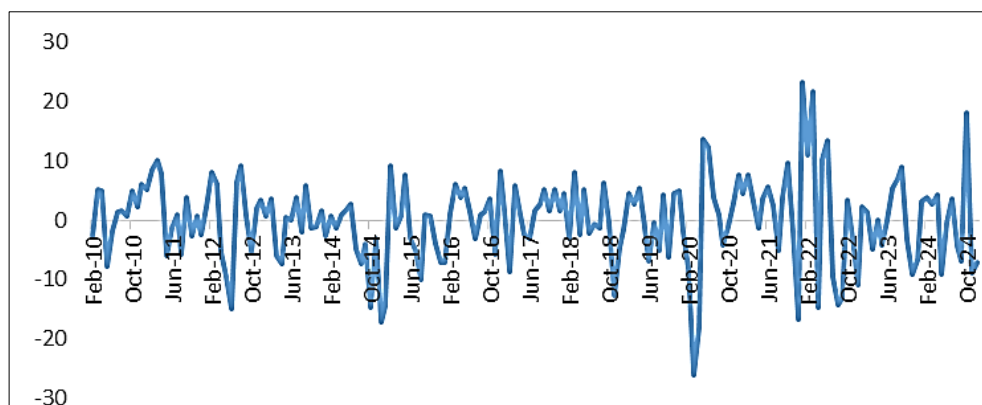


Fig 3: Time plot of crude oil prices at stationary

Figure 3 shows crude oil prices from approximately 2010 to 2024, exhibiting significant volatility with frequent

fluctuations, including sharp drops and spikes over time, refer to as volatility clustering.

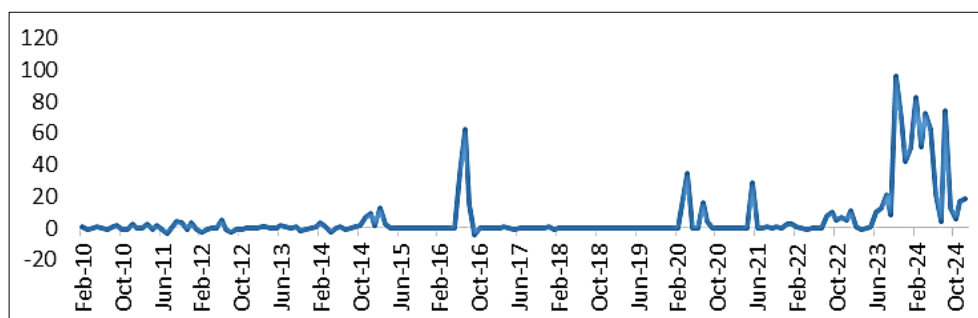


Fig 4: Time plot of exchange rate at stationary

The plot depicts the stationary series of the exchange rate from February 2010 to Decembbber 2024. It shows periods of relative stability with minor fluctuations, followed by several

sharp spikes indicating significant volatility at various points, notably around late 2015, early 2016, and mid-2023.

Table 4: Estimated VAR model For Crude oil prices (X_t) and Exchange Rate (Y_t)

Model & Predictor	Estimate	Std. Error	t-value	p-value	Note
Crude oil prices model					
Intercept	-0.0073	0.5305	-0.014	0.9890	Not sigf.
Crude oil prices (X_t)	0.1574	0.0748	2.104	0.0368	Significant
Exchange Rate (Y_t)	0.0021	0.0065	0.322	0.7475	Not sigf.
Residual std. Error	7.04	-	-	-	On 175 df
R^2	0.02511	-	-	-	Model fit
Adjusted R^2	0.01397	-	-	-	Mosel fit
F-statistic	2.254	-	-	0.108	Sigf. L
Exchange Rate model					
Intercept	9.8071	5.9820	1.639	0.1029	Not sigf.
Crude oil price (X_t)	-1.1791	0.8438	-1.397	0.1641	Not sigf.
Exchange rate(Y_t)	-0.15835	0.0742	-2.132	0.0344	Significant
Residual std. error	79.38	-	-	-	On 175 df
R^2	0.03526	-	-	-	Model fit
Adjusted R^2	0.02423	-	-	-	Model fit
F-statistic	3.198	-	-	0.0432	Sigf. L
Covariance matrix of residuals					
$X_t Y_t$ X_t 49.5634 0.5597 Y_t 0.5597 6301.1685					
Correlation matrix of residuals					
$X_t Y_t$ X_t 1.0000 0.0010 Y_t 0.0001 1.0000					

Note: $p < 0.05$ indicates statistical significance

The VAR model with 179 observations shows the system is stable, with roots around 0.15, indicating shocks decay over time. Crude oil prices have moderate persistence; coefficient ~ 0.16 , $p=0.0368$, but their lag does not significantly depend on the exchange rate. The exchange rate shows mean reversion; coefficient ~ -0.159 , $p=0.0344$, but is unaffected by past oil prices. Residuals are nearly uncorrelated, suggesting

minimal contemporaneous shocks. Overall, neither variable significantly influences the other at lag 1, nor the system tends to return to equilibrium over time.

3.2 Estimation of Volatility Model for Crude oil prices and Exchange rate

Table 5: Transformed Variance Coefficients (BEKK-Diagonal Parameters)

Parameter	Value	Std. Error	z. statistic	Prob.	Interpretation
M(1,1)	12.98605	5.650841	2.298074	0.0216	Positive and relatively large; implies a baseline level of unconditional variance in the first volatility component. Indicates notable volatility magnitude in that channel.
M(1,2)	0.286528	1.784022	0.160608	0.8724	Not statistically different from zero; cross-effect in the M (variance) matrix is weak for the (1,2) path. Little cross-channel instantaneous variance transmission observed in this parameterization.
M(2,2)	-2.473825	0.302795	-8.169979	0.0000	Negative and highly significant; in BEKK, a negative M(2,2) can imply a damping or stabilization effect in the second variance component, though interpretation depends on the full matrix structure. Ensure model identification; this term dominates the conditional variance in its path.

From table 5, the diagonal M terms relevant to the baseline variance show a strong positive effect for the first variance component. The cross-term is not supported, and the second

diagonal term is strongly negative, suggesting a stabilizing dynamic in the channel within the system.

Table 6: ARCH (A1) Coefficients

Parameter	Value	Std. Error	z. statistic	Prob.	Interpretation
A1(1,1)	0.673945	0.103510	6.510904	0.0000	Strong positive ARCH effect in the first variance equation; shocks to the mean (residuals) have a substantial impact on future volatility of the first channel.
A1(2,2)	0.285565	0.013967	20.44531	0.0000	Very strong ARCH effect in the second variance equation; recent shocks considerably amplify future volatility in this channel.

From table 6, both residual shocks have strong immediate effects on future volatility, with the second channel exhibiting

an even larger and highly significant ARCH effect, implying persistent volatility clustering after shocks.

Table 7: GARCH (B1) Coefficients

Parameter	Value	Std. Error	z. statistic	Prob.	Interpretation
B1(1,1)	0.528984	0.137438	3.848883	0.0001	Moderate to strong persistence in the first volatility process; past conditional variance persists into the future, sustaining volatility over time.
B1(2,2)	1.009669	0.000896	1127.300	0.0000	Extremely large and highly significant persistence in the second variance process; volatility is highly persistent, with shocks decaying very slowly in this channel.

From table 7, the first channel shows moderate persistence, while the second channel exhibits extremely persistent volatility, indicating long memory-like behavior in the second variance equation. This suggests prolonged periods of high or low volatility in that channel.

4. Conclusion

The results indicate a long-term relationship between the crude oil prices and exchange rates, but short-term causality is limited. Volatility due to shocks persists, highlighting the influence of external factors and policies on their dynamic interaction over time.

Recommendations

Based on the findings, the following recommendations are proposed:

Policy Diversification: Nigeria should diversify its economy to reduce overreliance on oil revenues, which directly influence the exchange rate and economic stability. Promoting sectors like agriculture, manufacturing, and technology can mitigate the adverse effects of oil price volatility.

- Enhanced Stabilization Mechanisms:** Establish sovereign wealth funds or stabilization funds aimed at smoothing economic and currency fluctuations during periods of extreme oil price shocks, as indicated by the heavy tails and volatility persistence.
- Market and Risk Management:** Nigerian policymakers and investors should utilize derivatives, hedging, and financial instruments to manage oil price risk and exchange rate volatility effectively.
- Macroprudential Regulation:** Strengthen monitoring of currency market shocks and implement measures to contain excessive volatility considering the heavy tails observed in the GARCH analysis such as intervention strategies and flexible exchange rate policies.
- Focus on Long-term Strategies:** Given the cointegration findings, policies should focus on stabilizing the long-term relationship between oil prices and exchange rates, perhaps through structural reforms, improved transparency, and better information dissemination.

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References

- Adewale EA, Musari OA. Nexus between oil prices and exchange rate in Nigeria. *Lafia Journal of Economics and Management Sciences*. 2021;6(1):1-12.
- Akpan E, Riman H, Offiong A. Asymmetric effect of oil price shocks on exchange rate volatility and domestic investment in Nigeria. 2013;3:513-532.
- Aliyu O, Abubakar I. Impact of crude oil price on Nigeria's exchange rate and foreign reserves: 2009-2021. 2024.<https://www.researchgate.net/publication/377088839>
- Attahir BA. Oil price and exchange rate nexus in Nigeria: are there asymmetries? *Central Bank of Nigeria Journal of Applied Statistics*. 2019;10(1):1-18.
- Babajide AA. Oil price bubbles and exchange rate movement in Nigeria. *Nexus International University Journal*. 2022;8(3):103-114.
- Bollerslev T. Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*. 1986;31(3):307-327.
- BP. Statistical review of world energy 2022. London: BP plc; 2022.
- Carnero MA, Eratalay MH. Estimating VAR-MGARCH in multiple steps. *Studies in Nonlinear Dynamics and Econometrics*. 2014;18(3):339-365.
- Cassel G. The world monetary problem. Geneva: League of Nations; 1921.
- Cumby RE, Obstfeld M. A note on the effectiveness of the forward market in the foreign exchange market. *Journal of Finance*. 1981;36(3):813-829.
- David RO, Zangdap KL, Nnamani CN, Ishaq AI, Obalowu J. The impact of crude oil price shocks on the Nigerian exchange rate: a time series analysis using ARIMAX models. *Royal Statistical Society Nigeria Local Group*. 2025.
- Deborah KA, Rowland TO, Adegbelemi BO, Andy TO, Aworinde BO. Oil price and exchange rate nexus in

- Nigeria: evidence from wavelet analysis. *Journal of Economics and Allied Research*. 2022;7(1):1-15.
13. Efuntade AO, Omolayo EO. Oil price-exchange rate interdependence: relevance of theory of exchange rate overshooting in Nigeria. *International Journal of Economics and Financial Management*. 2023;8(2):1-14.
 14. Ehikioya B, Omankhanlen A, *et al*. Oil price fluctuations and exchange rate in selected Sub-Saharan Africa countries: a vector error correction model approach. *International Journal of Energy Economics and Policy*. 2020;10(6):242-249.
 15. Engle RF. Co-integration and error correction: representation, estimation and testing. *Econometrica*. 1987;55(2):251-276.
 16. Engle RF. Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*. 1982;50(4):987-1007.
 17. Faruk BU. Relationship between volatility in domestic oil production, oil price and exchange rate in Nigeria: co-integration and Granger causality tests. *Bullion*. 2020;44(4):1-12.
 18. Frisch R, Roy C. Testing the linearity of the relation between the rate of interest and the rate of profit. *Economica*. 1947;14(55):61-76.
 19. Fisher I. The theory of interest: as determined by impatience to spend income and opportunity to invest it. New York: Macmillan; 1930.
 20. Fratzscher M, Schneider D, Van Robays I. Oil prices, exchange rates and asset prices. European Central Bank Working Paper. 2014;1689:1-40.
 21. Froot KA, Thaler RH. Anomalies: foreign exchange. *Journal of Economic Perspectives*. 1990;4(3):179-192.
 22. Girton L, Roper D. Theory and implications of currency substitution. *Journal of Money, Credit and Banking*. 1981;13:12-30.
 23. Granger CWJ. Investigating causal relations by econometric models and cross-spectral methods. *Econometrica*. 1969;37(3):424-438.
 24. Granger CWJ, Perron P. Solving the paradox of persistent shocks. *Econometrica*. 1983;51(3):587-612.
 25. Golub SS. Oil prices and exchange rates. *Economic Journal*. 1983;93(371):576-593.
 26. Hillary CE, Nwanneka JM, Eje GC, Anthony EA. Oil price and exchange rate nexus: a vector error correction approach on Nigeria. *International Journal of Energy Economics and Policy*. 2017;7(4):31-37.
 27. Igbinoia LE, Ogiemudia AO. Oil price and exchange rate volatility in Nigeria. *Oradea Journal of Business and Economics*. 2021;6(1):74-86.
 28. Iheanacho E. Dynamic relationship between crude oil price, exchange rate and stock market performance in Nigeria. *African Research Review*. 2016;10(4):224-240.
 29. Fasanya IO, Oyewole OJ, Raheem ID. Oil prices and exchange rate dynamics: how important is the role of asymmetry and structural breaks? *Journal of African Business*. 2021;22:1-20.
 30. James TH. Impact of oil price volatility on exchange rate in Nigeria. *International Journal of Research and Innovation in Social Science*. 2019;3(2):1-9.
 31. Jarque CM, Bera AK. A test for normality of observations and regression residuals. *Economics Letters*. 1980;6(3):255-259.
 32. Kamaldeen A, Musa AS, Sesan OA. The asymmetric effect of oil price on the exchange rate and stock price in Nigeria. *International Journal of Energy Economics and Policy*. 2021;11(4):202-208.
 33. Krugman P. Oil and the dollar. Cambridge (MA): National Bureau of Economic Research; 1983.
 34. MacDonald R. Concepts to calculate equilibrium exchange rates: an overview. Deutsche Bundesbank Discussion Paper. 2000;3:1-30.
 35. Obstfeld M, Rogoff K. Foundations of international macroeconomics. Cambridge (MA): MIT Press; 1996.
 36. Nasir NI, Sa'ad S, Rafindadi AS, Usman AB. Accounting for the effects of oil prices on exchange rate in Nigeria: empirical evidence from linear and non-linear ARDL models. *ESI Preprints*. 2023;5:204-215.
 37. Ogede JS, Adegboyega SB. Heterogeneous impact of oil price volatility on exchange rate in African countries: evidence from quantile regression approach. *Studies of Applied Economics*. 2021;39(8):1-16.
 38. Okeke S, Nwoha C, Duru A. Effect of crude oil price shock on inflation and exchange rate in Nigeria. *European Journal of Business and Innovation Research*. 2024;12:56-66.
 39. Organization of the Petroleum Exporting Countries (OPEC). Annual statistical bulletin. Vienna: OPEC; 2022.
 40. Perron P. The great crash, the oil price shock and the unit root hypothesis. *Econometrica*. 1989;57(6):1361-1401.
 41. Samir S, Salisu A, Kekere SI, Olajide IS. Oil price and exchange rate dynamics: empirical evidence. *International Journal of Energy Economics and Policy*. 2022;12(4):482-493.
 42. Yergin D, Hörmann G, Laudann R, Ukert T. Der Preis: Die Jagd nach Öl, Geld und Macht. Frankfurt: S. Fischer; 1991.
 43. Virani SS, Alonso A, Benjamin EJ, Bittencourt MS, Callaway CW, Carson AP, *et al*. Heart disease and stroke statistics—2020 update: a report from the American Heart Association. *Circulation*. 2020 Mar 3;141(9):e139-596.