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Weighted distribution method for constructing flexible probability models with applications

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Abstract

In this article, an innovative generalization of certain well-known probability distributions is proposed using the weighted distribution approach. Several important statistical properties of the proposed model are derived and discussed in detail. The applicability and effectiveness of the model are illustrated through real data analysis, particularly in biomedical contexts, and its performance is compared with that of other established distributions.

Keywords: Weighted distributions, random variable, Sauleh distribution, data analysis

1. Introduction

Statistical distributions play a fundamental role in data science by offering a structured framework for understanding the behavior of random variables. A probability distribution characterizes the possible values a variable may assume and facilitates the computation of probabilities associated with different events. As a result, statistical modeling and real-world data analysis rely extensively on probability distributions. Nevertheless, data arising from diverse domains such as environmental studies, finance, biomedical sciences, and other applied disciplines often deviate from classical distributional assumptions. This inadequacy has led to the development of new and more flexible distributional models capable of capturing varying degrees of skewness and kurtosis, thereby enhancing goodness-of-fit and improving the accuracy of empirical data modeling.

Many widely used classical probability distributions are often inadequate for accurately modeling and predicting data arising in applied fields such as biology, environmental sciences, engineering, medical sciences, finance, and economics. Consequently, numerous generalized families of distributions have been developed as extensions of traditional models. These newly proposed families have been rigorously investigated and demonstrated to provide greater flexibility and improved adaptability in practical applications. Such extended distributional frameworks have been successfully utilized to model diverse types of data across various scientific disciplines. A common and effective strategy in distribution theory involves augmenting existing families with additional parameters. The introduction of extra parameters typically enhances model flexibility, enabling better accommodation of complex data structures and improving overall model performance.

The concept of weighted distributions (WD) provides an effective framework for addressing challenges in data interpretation and model construction. The idea was initially introduced by Fisher ^[1] to account for ascertainment bias in observed data, and later systematically formalized by Rao ^[2], who developed a structured approach for modeling situations where conventional probability distributions fail to adequately represent observations occurring with unequal selection probabilities. Weighted distributions have since found wide applications in diverse areas such as ecology, biomedicine, branching processes, and reliability analysis. This methodology provides an alternative mechanism for generating modified distributions by incorporating a weight function into the baseline model. When samples may arise from both the original and the weighted versions of a distribution, the weighted distribution framework

facilitates model fitting even when the weight function is unknown. A notable special case occurs when the weight function is proportional to the unit length, leading to the well-known length-biased distribution. A brief review, perspective, and characterization of Weighted distributions studied by Saghir. *et al.*, [3]. Area-biased Poisson exponential distribution with applications explained by Fazal [4]. Mathew [5] detailed some length-biased distribution and its characteristics. Over time, numerous researchers have explored various forms of weighted probability models and demonstrated their usefulness across different applied contexts. The Sauleh distribution is a composite model derived from a combination of the Gamma and Exponential distributions. It is a newly proposed one-parameter lifetime distribution introduced by Aijaz Ahmad *et al.* [6], and is designed for modeling lifetime data with enhanced flexibility compared to classical single-parameter models.

2. Weighted Distributions (WD)

Let X be a non-negative random variable with pdf $f(x)$, then the pdf of a weighted random variable X_w is presented as,

$$f_w(x) = \frac{w(x)f(x)}{E(w(x))}, x > 0 \quad (3)$$

where $w(x)$ be the non - negative weight function and $E(w(x)) = \int w(x)f(x)dx < \infty$.

For various selections of weight function $w(x)$ weighted models are of many forms when $w(x) = x^c$, the resulting distribution is termed as WD.

The nomenclature of a newly derived distribution need not necessarily reflect its parent distribution, particularly when the resulting model does not preserve the fundamental characteristics of the original one. In many instances, the newly constructed distribution exhibits structural and statistical properties that are substantially different from those of its parent model. This transformation may be likened to the formation of water from oxygen and hydrogen, where the resulting compound possesses properties entirely distinct from those of its constituent elements.

3. General case: KrishSupra-P distribution

Consider the initial distribution as Sauleh distribution with probability density function (pdf)

$$f(x; \theta) = \frac{\theta^4}{\theta^4 + 2\theta + 6} (\theta + x^2 + x^3) e^{-\theta x}; x > 0, \theta > 0 \quad (1)$$

and the cumulative distribution function (cdf)

$$F(x; \theta) = 1 - \left(1 + \frac{\theta^2 x (\theta^2 x^2 + \theta x + 3x + 2\theta + 6)}{\theta^4 + 2\theta + 6} \right) e^{-\theta x}; x > 0, \theta > 0 \quad (2)$$

The weighted version of the Sauleh distribution by letting $w(x) = x^c$ and the new distribution is termed as the KrishSupra-P Distribution (KSPD). The pdf of the WD is provided by

$$f_w(x) = \frac{x^c f(x)}{E(x^c)} \quad (4)$$

Where,

$$E(x^c) = \int_0^{\infty} x^c f(x, \theta) dx$$

$$E(x^c) = \frac{\theta^4 \Gamma(c+1) + \theta \Gamma(c+3) + \Gamma(c+4)}{\theta^c (\theta^4 + 2\theta + 6)} \quad (5)$$

Substituting the eqs. (1) and (5) in eq. (4), the KSPD's pdf as

$$f_w(x) = \frac{x^c \theta^{c+4}}{\theta^4 \Gamma(c+1) + \theta \Gamma(c+3) + \Gamma(c+4)} (\theta + x^2 + x^3) e^{-\theta x} \quad (6)$$

And the cdf of KSPD is attained as

$$F_w(x) = \int_0^x f_w(x) dx$$

$$F_w(x) = \frac{1}{\theta^4 \Gamma(c+1) + \theta \Gamma(c+3) + \Gamma(c+4)} \left(\theta^4 \gamma(c+1, \theta x) + \theta \gamma(c+3, \theta x) + \gamma(c+4, \theta x) \right) \quad (7)$$

Here $\gamma(c+1, \theta x)$, $\gamma(c+3, \theta x)$ & $\gamma(c+4, \theta x)$ represents the Incomplete Gamma function.

The survival function of KSPD is, $S(x) = 1 - F_w(x)$

When $c = 1$, the pdf can observe from (6),

$$f_w(x) = \frac{x^c \theta^{c+4}}{\theta^4 \Gamma(c+1) + \theta \Gamma(c+3) + \Gamma(c+4)} (\theta + x^2 + x^3) e^{-\theta x}$$

$$S(x) = 1 - \frac{1}{\theta^4 \Gamma(c+1) + \theta \Gamma(c+3) + \Gamma(c+4)} \left(\theta^4 \gamma(c+1, \theta x) + \theta \gamma(c+3, \theta x) + \gamma(c+4, \theta x) \right). \quad (8)$$

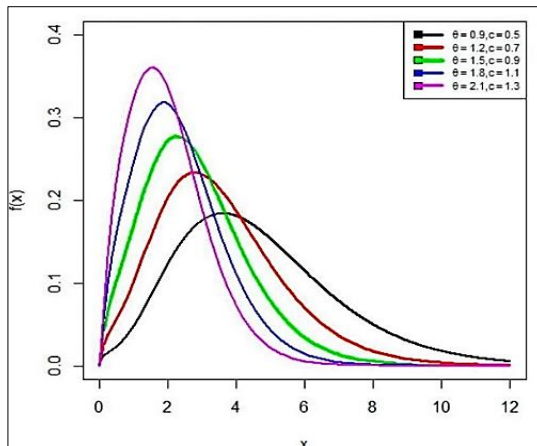


Fig 1: pdf of KSPD

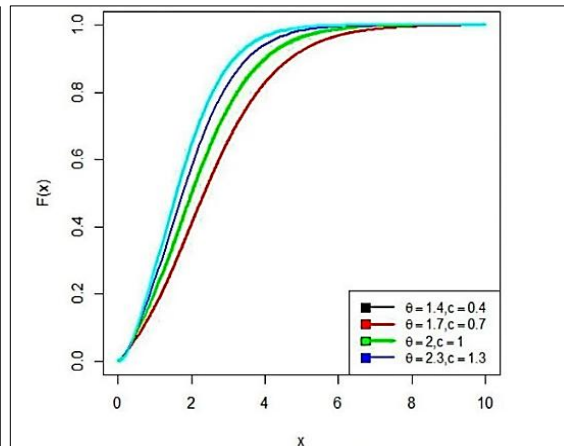


Fig 2: cdf of KSPD

From Figures 1 & 2 the nature of pdf & cdf of the KSPD is observed with different α & θ .

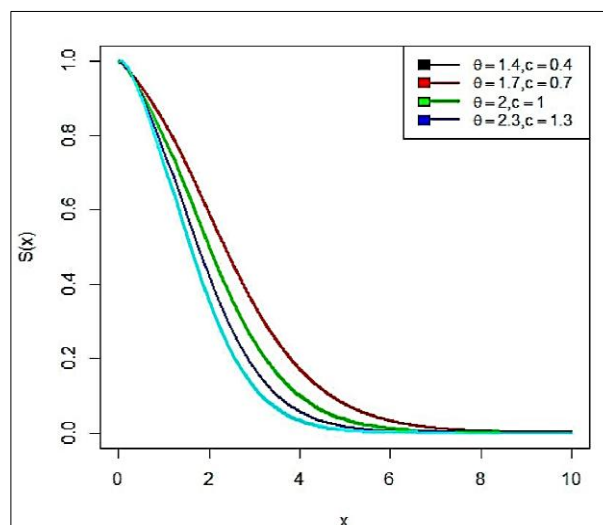


Fig 3: Survival Function of KrishSupra – P Distribution

Figure 3 implies the Survival Function of KrishSupra – P Distribution decreases while X increases w.r.to different α & θ . The Hazard function is expressed by

$$h(x) = \frac{f_w(x)}{1 - F_w(x)} = \frac{x^c \theta^{c+4} (\theta + x^2 + x^3) e^{-\theta x}}{(\theta^4 \Gamma(c+1) + \theta \Gamma(c+3) + \Gamma(c+4)) - (\theta^4 \gamma(c+1, \theta x) + \theta \gamma(c+3, \theta x) + \gamma(c+4, \theta x))} \quad (9)$$

3.1 Analysis of data

Table 1 represents the weight loss of 77 randomly selected patients (age group 12 -15, Secondary School Students, of Thrissur District of Kerala– (June 2021)) after being infected with COVID-19.

Table 1: The weight loss of patients (Kilograms)

1.07	1.08	1.08	1.08	1.09	1.00	0.99	1.02	1.05	1.07	0.96
0.59	0.72	0.74	0.92	0.93	0.10	0.33	0.44	0.56	0.59	1.12
1.22	1.22	1.24	1.30	1.34	1.13	1.15	1.16	1.2	1.21	2.31
1.6	1.63	1.68	1.71	1.72	1.39	1.44	1.46	1.53	1.59	3.61
0.77	2.93	3.27	3.42	3.47	4.02	4.32	4.58	5.55	2.54	1.76
2.13	2.15	2.16	2.22	2.30	1.83	1.95	1.96	1.97	2.02	2.02
2.78	1.83	1.95	1.96	1.97	2.4	2.45	2.51	2.53	2.54	

To compare the KSPD with Sauleh distributions, Exponential distributions, as well as LDs, we consider the parameters *AICC* (“Akaike Information Criterion Corrected”), *AIC* (“Akaike Information Criterion”), *BIC* (“Bayesian Information Criterion”), and $-2\log L$. The distribution with lower values of $-2\log L$, *AICC*, *BIC*, & *AIC* is the better one. For determining the criteria *AIC*, *AICC*, *BIC*, and $-2\log L$ formulas mentioned below are applied.

$$AIC = 2k - 2\log L,$$

$$BIC = k \log n - 2\log L$$

$$AICC = AIC + \frac{2k(k+1)}{n-k-1}$$

Here, n represents the size of the sample, k indicates the number of parameters inside the statistical model, and $-2\log L$ presents “the maximized value of the log-likelihood function in the model.

Table 2: Comparison- different distributions fitted

Distributions	MLE	S.E	$-2\log L$	AIC	BIC	AICC
KSPD	$\hat{\theta} = 2.6090$ $\hat{c} = 1.8501$	$\hat{\theta} = 0.3019$ $\hat{c} = 0.50912$	191.4293	195.3943	199.9900	195.5960
Sauleh	$\hat{\theta} = 1.5014$	$\hat{\theta} = 0.0697$	214.7476	216.7476	219.0243	216.8047
Exponential	$\hat{\theta} = 0.5697$	$\hat{\theta} = 0.0596$	224.8929	226.8929	229.1696	226.9500
Lindley	$\hat{\theta} = 0.8697$	$\hat{\theta} = 0.0697$	213.049	215.049	217.3276	215.1080

Table 2 implies the KSPD has the lesser values of $-2\log L$, *BIC*, *AICC*, and *AIC* as compared to exponential, Sauleh, and Lindley distributions. Hence, it shows the KSPD leads to a better fit than Sauleh, exponential as well and Lindley distributions.

3.2 Conclusion

This study introduces a novel probability distribution, KSPD, and examines its key statistical properties. Application to a real data set demonstrates that KSPD outperforms the Exponential, Sauleh, and Lindley distributions in terms of model fit.

4. Area biased distribution: PrasNikh–N distribution

The ‘2-parameter’ Akash distribution (TAD) is a freshly proposed life-time distribution introduced by Shanker & Shukla^[7] of which the Akash distribution with one parameter is a specific case. Shanker^[8] explained Akash distribution and applications. the pdf (probability density function) of TAD is,

$$f(x; \theta, \alpha) = \frac{\theta^3}{\alpha \theta^2 + 2} (\alpha + x^2) e^{-\theta x} ; x > 0, \theta > 0, \alpha > 0 \quad (10)$$

The cdf (Cumulative distribution function) of TAD is,

$$F(x; \theta, \alpha) = 1 - \left(1 + \frac{\theta x (\theta x + 2)}{\alpha \theta^2 + 2} \right) e^{-\theta x} ; x > 0, \theta > 0, \alpha > 0 \quad (11)$$

The weighted random variable X_w is with pdf,

$$f_w(x) = \frac{w(x)f(x)}{E(w(x))}, \quad x > 0.$$

Since the weight function $w(x) = x^2$, the new distribution is called Area Biased Distribution with pdf,

$$f_a(x) = \frac{x^2 f(x)}{E(x^2)} \quad (12)$$

$$\text{here } E(x^2) = \int_0^{\infty} x^2 f(x, \theta, \alpha) dx$$

$$E(x^2) = \frac{2\alpha\theta^2 + 24}{\theta^2(\alpha\theta^2 + 2)} \quad (13)$$

In equation (12), apply the equations (10) and (13) we get the pdf of new distribution which is termed as PrasNikh–N Distribution (P2ND) as,

$$f_a(x) = \frac{x^2 \theta^5}{2\alpha\theta^2 + 24} (\alpha + x^2) e^{-\theta x} \quad (14)$$

And the cdf of P2ND as,

$$F_a(x) = \int_0^x f_a(x) dx = \frac{1}{2\alpha\theta^2 + 24} (\alpha\theta^5 \int_0^x x^2 e^{-\theta x} dx + \theta^5 \int_0^x x^4 e^{-\theta x} dx) \quad (15)$$

Following equation simplification (16), we obtain the cdf of P2ND as

$$F_a(x) = \frac{1}{2\alpha\theta^2 + 24} (\alpha\theta^2 \gamma(3, \theta x) + \gamma(5, \theta x)) \quad (16)$$

Here $\gamma(3, \theta x)$ & $\gamma(5, \theta x)$ represents the Incomplete Gamma function

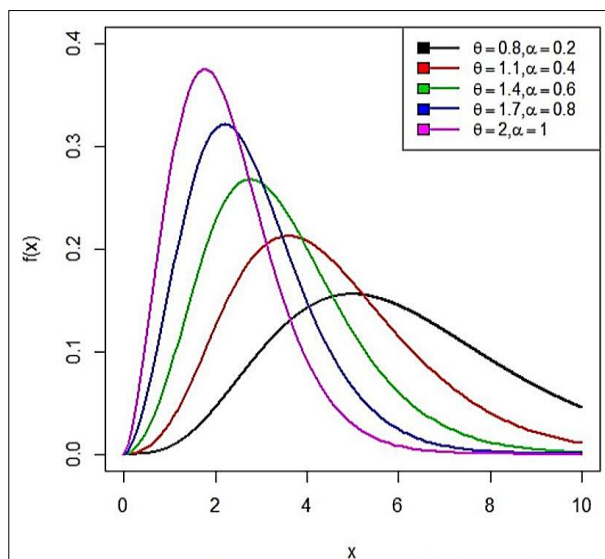


Fig 4: pdf of P2ND

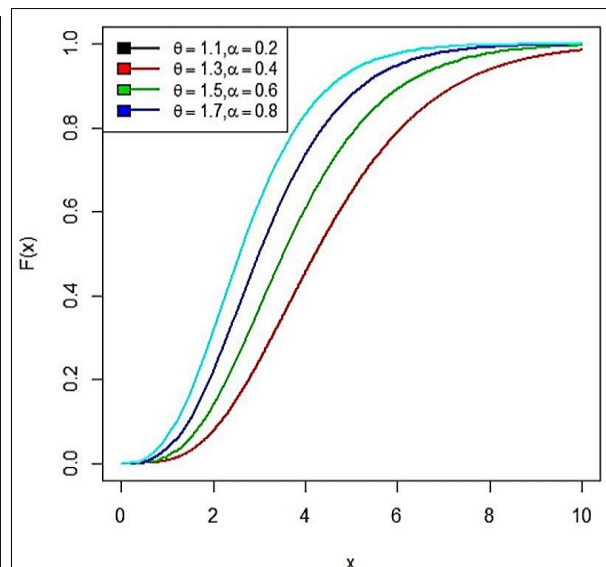


Fig 5: CDF of P2ND

Figure 4 and Figure 5 imply the nature of the pdf and cdf of new P2ND for the different values of its parameters. From Figure 1 the symmetric nature of the distribution is clear.

The Reliability function of P2ND,

$$S(x) = 1 - F_a(x) = 1 - \frac{1}{2\alpha\theta^2 + 24} (\alpha\theta^2\gamma(3, \theta x) + \gamma(5, \theta x))$$

The hazard function of P2ND is

$$h(x) = \frac{f_a(x)}{1 - F_a(x)}$$

$$= \frac{x^2\theta^5(\alpha + x^2)e^{-\theta x}}{(2\alpha\theta^2 + 24) - (\alpha\theta^2\gamma(3, \theta x) + \gamma(5, \theta x))}$$

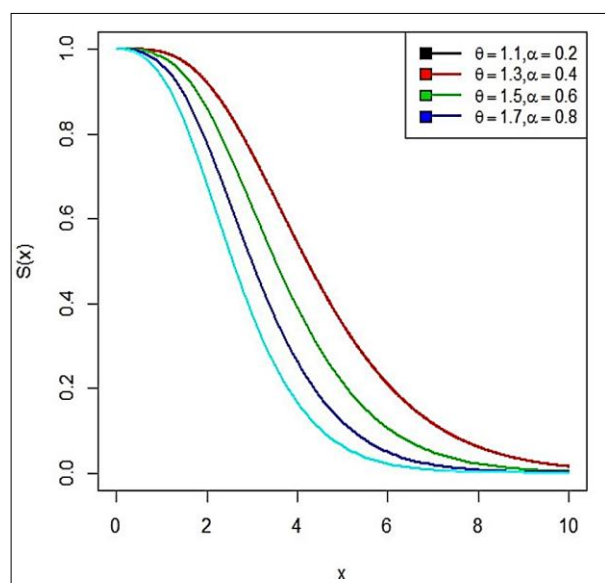


Fig 6: Reliability Function of P2ND

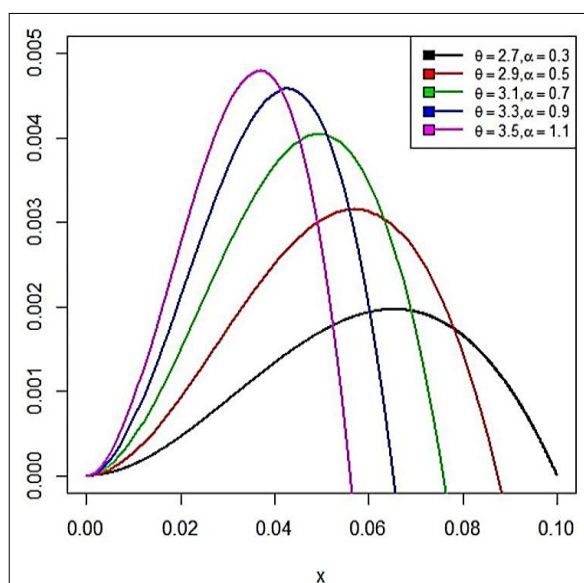


Fig 7: Hazard Function of P2ND

Figure 6 & 7 implies the nature of reliability function and hazard function of P2ND for the different values of parameters.

4.1 Application

Table 3 presents real medical lifetime data comprising the weight loss (kg) observed after the first cycle of chemotherapy for 60 randomly selected patients from a hospital in Thrissur district, Kerala. All patients were diagnosed with gastrointestinal (GI) cancer, which includes malignancies of the digestive tract such as the small and large intestine, stomach, colon, pancreas, anus, rectum, biliary system, and liver.

Table 3: Weight loss (Kgs)

4.390	4.975	5.380	4.395	4.645	4.755	3.350	5.075
4.390	3.855	4.050	4.395	4.645	3.765	3.750	3.985
0.320	1.260	2.025	0.490	0.620	1.150	1.210	1.410
2.910	3.430	3.535	3.190	3.265	3.350	3.350	3.500
2.035	2.690	2.910	2.160	2.210	2.370	2.530	2.800
4.050	3.750	3.985	4.245	4.325	4.380	3.765	3.855
3.350	4.975	5.380	3.430	3.500	3.535	4.930	5.075
3.265	3.190	3.350	2.910				

To compare the performance of P2ND with Lindley, TAD, Akash & exponential distributions, consider the standard general criteria & notations. The lesser values of BIC , AIC , $-2\log L$, and $AICC$ imply the better distribution to which they correspond.

Table 4: Analysis of fitted distributions

Distributions	MLE	S.E	-2logL	AIC	BIC	AICC
P2ND	$\hat{\alpha} = 1.6977$ $\hat{\theta} = 1.4369$	$\hat{\alpha} = 1.6893$ $\hat{\theta} = 0.1293$	143.7944	147.7944	151.1927	148.1987
TAD	$\hat{\alpha} = 0.02967$ $\hat{\theta} = 0.8936$	$\hat{\alpha} = 0.1596$ $\hat{\theta} = 0.0894$	147.2922	151.2922	154.67	151.6165
Akash	$\hat{\theta} = 0.8010$	$\hat{\theta} = 0.0697$	152.6894	154.6894	156.3783	154.7946
Exponential	$\hat{\theta} = 0.2931$	$\hat{\theta} = 0.0495$	170.9553	172.9556	174.9245	172.9661
Lindley	$\hat{\theta} = 0.4952$	$\hat{\theta} = 0.0610$	159.9501	161.9501	163.919	161.9606

Table 4 implies that P2ND has lesser *BIC*, *AIC*, *-2logL*, and *AICC* values by comparing to the other distribution and can be concluded that the new distribution P2ND shows a better fit.

4.2 Conclusion

A generalized form of the TAD distribution is proposed and termed P2ND. The properties of P2ND are thoroughly examined, and its practical relevance is illustrated using medical data. Studying the characteristics of such biomedical data is essential for reliable modeling and inference. The results indicate that P2ND provides a better fit compared to the TAD, Akash, Exponential, and Lindley distributions.

5. Length-biased distribution: ChandBhas-P Distribution (CBPD)

The two-parameter Sujatha distribution is a recently proposed lifetime model introduced by Tesfay and Shanker ^[9], wherein the one-parameter Akash and Sujatha distributions emerge as special cases. In a subsequent work ^[10], Tesfay and Shanker further developed a two-parameter Sujatha model that incorporates both the size-biased Lindley and the Sujatha distributions.

The probability density function (pdf) of the new two-parameter Sujatha distribution is,

$$f(x; \theta, \alpha) = \frac{\theta^3}{\theta^2 + \alpha\theta + 2} \left(1 + \alpha x + x^2\right) e^{-\theta x}; \quad x > 0, \theta > 0, \alpha \geq 0 \quad (17)$$

and the cumulative density function (cdf) of the new two-parameter Sujatha distribution is presented by

$$F(x; \theta, \alpha) = 1 - \left(1 + \frac{\theta x(\theta x + \alpha\theta + 2)}{\theta^2 + \alpha\theta + 2}\right) e^{-\theta x}; \quad x > 0, \theta > 0, \alpha > 0 \quad (18)$$

If $f(x)$, the pdf of a non-negative random Variable X , then the pdf of weighted X_w is,

$$f_w(x) = \frac{w(x)f(x)}{E(w(x))}, \quad x > 0.$$

Where $w(x)$ be the non-negative weight function

The distribution that results from our consideration of the "weight function" at $w(x) = x$ is termed the "length-biased distribution". The New Two -Parameter Sujatha distribution is termed as ChandBhas-P Distribution (CBPD) which is a length-biased transformation with pdf,

$$f_1(x) = \frac{x \cdot f(x)}{E((x))} \quad (19)$$

Where $E(x) = \int_0^{\infty} x f(x; \theta, \alpha) dx$

$$E(x) = \frac{\theta^2 + 2\alpha\theta + 6}{\theta(\theta^2 + \alpha\theta + 2)} \quad (20)$$

Thereby the pdf of the CBPD by putting eqs. (17) & (20) into eq. (19).

$$f_l(x) = \frac{x\theta^4}{\theta^2 + 2\alpha\theta + 6} \left(1 + \alpha x + x^2\right) e^{-\theta x} \quad (21)$$

And the cdf of CBP Distribution is,

$$F_l(x) = \int_0^x f_l(x) dx$$

$$= \frac{1}{\theta^2 + 2\alpha\theta + 6} \left(\theta^4 \int_0^x x e^{-\theta x} dx + \alpha \theta^4 \int_0^x x^2 e^{-\theta x} dx + \theta^4 \int_0^x x^3 e^{-\theta x} dx \right) \quad (22)$$

Put $\theta x = t \Rightarrow \theta dx = dt \Rightarrow dx = \frac{dt}{\theta}$, When $x \rightarrow 0, t \rightarrow 0$, Also $x = \frac{t}{\theta}$

By simplifying the eq. (6), the cdf of CBPD is,

$$F_l(x) = \frac{1}{\theta^2 + 2\alpha\theta + 6} \left(\theta^2 \gamma(2, \theta x) + \alpha \theta \gamma(3, \theta x) + \gamma(4, \theta x) \right) \quad (23)$$

Figure 2.1 & Figure 2.2 shows the nature of the pdf and cdf. The pdf curve became less skewed while α and θ decreases.

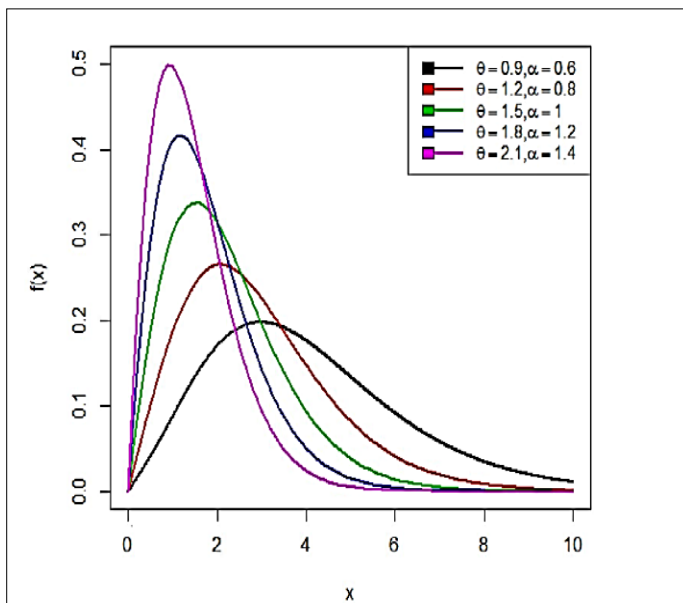


Fig 8: pdf of CBP Distribution

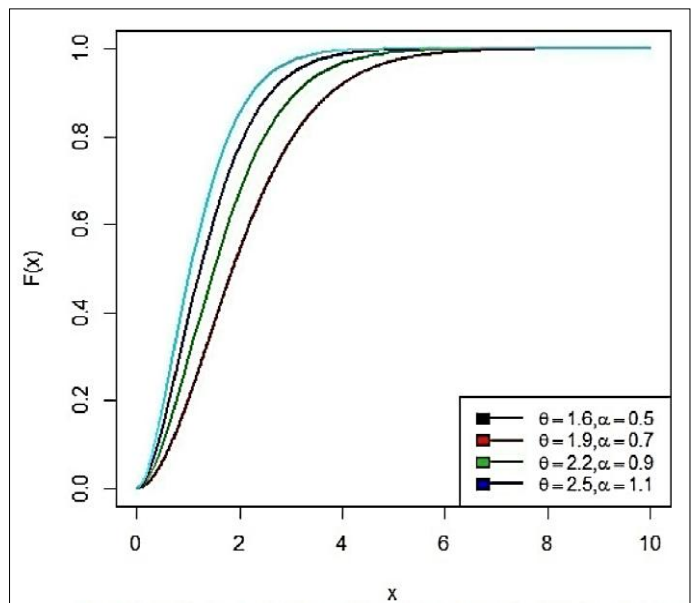


Fig 9: cdf of CBP Distribution

The Survival function is,

$$S(x) = 1 - F_l(x) = 1 - \frac{1}{\theta^2 + 2\alpha\theta + 6} \left(\theta^2 \gamma(2, \theta x) + \alpha \theta \gamma(3, \theta x) + \gamma(4, \theta x) \right)$$

The Hazard Rate of the distribution is,

$$h(x) = \frac{f_l(x)}{1 - F_l(x)} = \frac{x\theta^4(1 + \alpha x + x^2) e^{-\theta x}}{(\theta^2 + 2\alpha\theta + 6) - (\theta^2 \gamma(2, \theta x) + \alpha \theta \gamma(3, \theta x) + \gamma(4, \theta x))}$$

is clear from Figure 10. The nature of hazard rate is noticed from Figure 11.

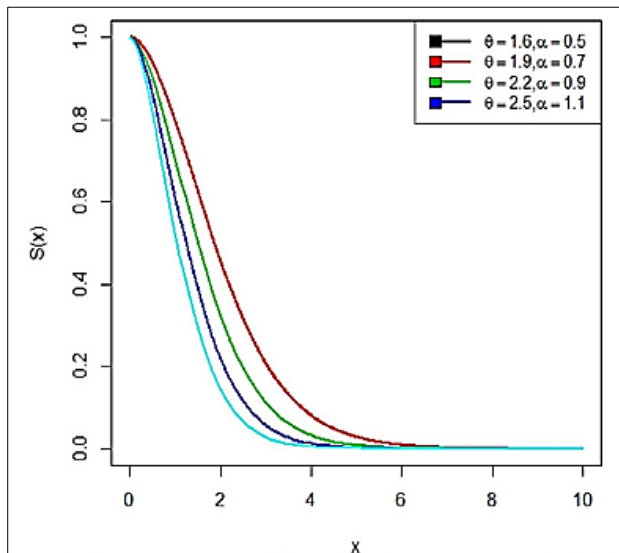


Fig 10: Survival function of CBPD

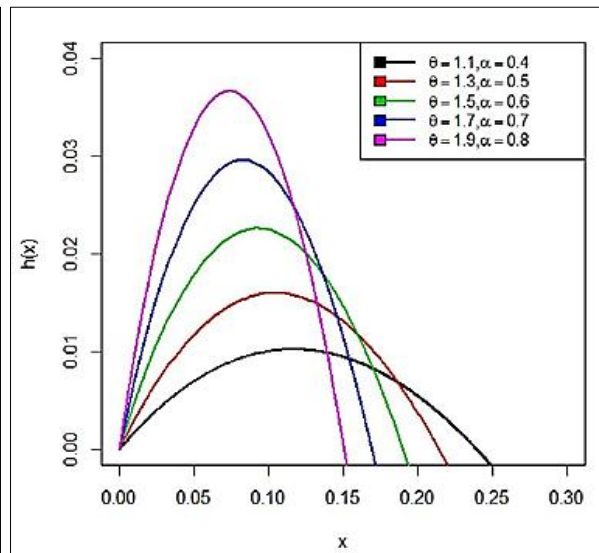


Fig 11: The nature of hazard rate of CBPD

The Figure 8 and Figure 9 shows the Survival function and Hazard rate of the new distribution. The decreasing nature of survival functions of CBPD with increase in x for different ' α ' & ' θ '

5.2 Applications

Data set 5.1: Table 5 represents the birth weight (Kg) of new born babies, randomly selected from a hospital at Thrissur, Kerala.

Table 5: Birth weight (Kg) of new born babies

3.1091	3.515	3.385	3.145	3.215	2.495	4.275	4.280	3.380
2.910	1.910	3.640	2.770	2.190	2.420	4.870	3.100	2.800
2.610	1.580	3.360	2.840	3.140	2.530	3.640	1.520	3.210
4.380	4.170	3.110	3.280	3.860	4.040	2.870	3.230	4.220

Data set 5.2: Table 6 shows the weight (Kg) of 100 randomly selected children- of age range between 3 months to 4 years- from a few babysitting centres and play schools across Thrissur.

Table 6: Weight (kg) of 100 children (Age b/w 3months to 4 years)

7.50	7.00	7.75	7.50	7.25	7.00	6.50	6.75	10.00	5.50	5.00	8.75
8.75	6.75	12.25	12.50	6.50	14.00	14.50	6.50	15.50	12.50	7.50	7.25
10.00	8.75	7.25	8.00	7.50	9.00	7.00	8.75	7.00	7.50	6.50	8.00
12.00	6.75	5.00	7.50	6.50	7.25	5.25	7.50	5.50	8.75	7.25	6.50
7.25	14.50	6.75	7.50	7.00	6.75	12.25	8.75	7.00	7.50	5.25	7.00
7.50	9.00	7.00	8.75	9.00	6.75	7.50	10.00	9.00	7.00	7.50	12.50
6.75	12.25	12.50	6.50	16.00	14.50	6.50	16.00	5.50	6.75	5.50	6.50
8.75	12.25	12.50	6.50	16.00	14.50	6.75	14.50	7.50	7.00	6.50	6.75
7.00	7.75	7.50	7.25								

Table 7: Analysis of fitted distributions for data 1

"Distributions"	MLE	S.E	-2logL	AIC	BIC	AICC
CBP distribution	$\hat{\alpha} = 0.001$ $\hat{\theta} = 1.1139$	$\hat{\alpha} = 0.009$ $\hat{\theta} = 0.0495$	118.94	123.07	126.29	123.49
Two Parameter Sujatha	$\hat{\alpha} = 0.001$ $\hat{\theta} = 0.819$	$\hat{\alpha} = 0.011$ $\hat{\theta} = 0.0539$	125.78	129.82	133.01	130.21
New Two Parameter Sujatha	$\hat{\alpha} = 0.001$ $\hat{\theta} = 0.698$	$\hat{\alpha} = 0.020$ $\hat{\theta} = 0.2110$	133.96	137.97	141.09	138.32
Sujatha	$\hat{\theta} = 0.691$	$\hat{\theta} = 0.067$	134.19	136.20	137.790	136.29
Exponential	$\hat{\theta} = 0.3047$	$\hat{\theta} = 0.051$	157.59	159.56	161.137	159.68
Lindley	$\hat{\theta} = 0.50694$	$\hat{\theta} = 0.061509$	144.19	146.19	147.775	146.31

Table 8: Analysis of fitted distributions for data 2

Distributions	M L E	S E	-(2logL)	AIC	BIC	AICC
CBP distribution	$\hat{\alpha} = 0.001$ $\hat{\theta} = 0.469$	$\hat{\alpha} = 0.020$ $\hat{\theta} = 0.025$	223.79	227.80	231.29	228.11
Two Parameter Sujatha	$\hat{\alpha} = 0.001$ $\hat{\theta} = 0.347$	$\hat{\alpha} = 0.0001$ $\hat{\theta} = 0.019$	233.78	237.78	241.35	238.09
New Two Parameter Sujatha	$\hat{\alpha} = 0.001$ $\hat{\theta} = 0.350$	$\hat{\alpha} = 0.010$ $\hat{\theta} = 0.022$	234.20	238.20	241.69	238.49
Sujatha	$\hat{\theta} = 0.336$	$\hat{\theta} = 0.029$	236.58	238.58	240.36	238.67
Exponential	$\hat{\theta} = 0.121$	$\hat{\theta} = 0.0183$	273.58	275.58	277.36	275.67
Lindley	$\hat{\theta} = 0.2208$	$\hat{\theta} = 0.0237$	251.80	253.80	255.59	253.91

The distribution with the smallest values of AIC, AICC, BIC, and $-2\log L$ is regarded as the best-fitting model. From Tables 7 and 8, the CBP distribution exhibits lower AIC, AICC, BIC, and $-2\log L$ values compared to the New Two-Parameter Sujatha, Sujatha, Two-Parameter Sujatha, Lindley, and Exponential distributions. This confirms that the CBP distribution provides a superior fit to the given biomedical data. Hence, the effectiveness and significance of the proposed distribution are established.

6. Conclusion

In this study, a novel distribution, termed the ChandBhas-P (CBP) distribution, is introduced using the length-biased approach as a modification of the conventional distribution. The superiority of the proposed model is evaluated using two biomedical data sets. The results demonstrate that the CBP distribution provides a substantially better fit than the New Two-Parameter Sujatha, Sujatha, Two-Parameter Sujatha, Exponential, and Lindley distributions for such biological data.

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