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Mathematical modeling perspectives on hard skills and soft skills in entrepreneurial ecosystems for sustainable national development

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Abstract

In this study, a nonlinear mathematical model of soft skills and hard skills interactions was developed to study the dynamics of an entrepreneurial ecosystem in a business environments which comprises two populations: as soft skills utilization and hard skills utilization. The formulated model was analyzed to obtain the existence of equilibrium: bio-entrepreneurial ecosystem and bionomic, dynamical behavior of equilibrium points with conditions for stability and instability of the system. The exact solutions of the model also supports the dynamical behavior of the system and the simulations. The study revealed that the Soft Skills for National Development monotonically and asymptotically penetrates deep to sustain the economic growth of the nation with a steady and continuous pattern, that cross diffusion terms do not inhibit the acquisition and progression of the dynamics of Soft Skills and Hard Skills for sustainable National Development, the relationship showed that soft skills is the predator always at the leading top while hard skills is the prey-implies that soft skills dominates entrepreneurial ecosystem and the hard skills is the building blocks for sustainable national development and hard skills monotonically oscillates and cannot sustain national development unless when used in synergy with the soft skills to support industrial output, technological development and workforce competence.

Keywords: Mathematical modeling, hard skills, soft skills, entrepreneurial ecosystems, sustainable national development

1. Introduction

Over the years, entrepreneurial skills have been with unbalanced approach on its utilization both individually and organizationally in the world at large. Thus, lack of appropriate usage of hard skills and soft skills for sustainable national development is the major issue of entrepreneurial ecosystem. However, the ecological environment for skills development and usage have been mishandled thereby leading to some issues of concern as lacks exhaustion of resources and conservation of biodiversity in the skills acquisition which are of kin interest to right thinking individuals and entrepreneurs locally and globally. Nowadays, the public, stakeholders and scholars from the confined to universal spaces that human activities in business yield major environmental and ecological stresses with irreversible loss of skills, business, destruction of entrepreneurial ecosystem or climate upheavals as a result of inadequate utilization of both hard skills and soft skills for sustainable development of individuals or companies. For sustainable development of nations, these entrepreneurial environmental challenges must be preserved, conserved, managed and utilized the appropriate skills for all-round personality development of human beings. The emphasis is on the need to acquire, utilize, manage and sustain both soft skills and hard skills respectively. This however shows the dynamics and complex interactions between man, production activities and natural resources as to promote their coexistence and common evolutionary processes. Thus, the soft skills are personal attributes like communication, teamwork, adaptability, and problem-solving

that shape how someone works and interacts crucially for career success alongside technical hard skills. The basic examples include effective communication, critical thinking, leadership, emotional intelligence, time management, creativity, conflict resolution, and a positive attitude, which help in collaboration, navigating challenges, and improving workplace efficiency. Others are Teamwork/Collaboration, Problem-Solving, Adaptability/Flexibility and Work Ethic/Dependability. Soft skills are highly valued by employers because they enhance collaboration, productivity, and overall success in the workplace, making you a well-rounded and effective professional.

While the Hard skills are measurable and teachable abilities specific to a job like coding (Python, Java), data analysis, graphic design (Adobe Suite), foreign languages, digital marketing, bookkeeping, SEO, project management software, and technical writing, acquired through education or training to perform specific tasks. They are concrete and quantifiable, unlike soft skills (communication, teamwork). The Technology & IT skills- Programming Languages (Python, SQL, Java, C++), Cloud Computing (AWS, Azure), Cyber security, Database Management, Network Security, Mobile Development and User Interface (UI) / User Experience (UX) Design. Data & Analytics includes Data Analysis & Mining, Statistical Analysis, Financial Modeling & Analysis and Data Visualization. Marketing & Design:- Graphic Design (Adobe Creative Suite), Digital Marketing, SEO/SEM, Content Marketing/Copywriting and Social Media Marketing. Business & Finance:- Project Management (Software proficiency), Bookkeeping & Accounting, Grant Writing and Reporting & Technical Writing. Languages & Communication: Foreign Language Proficiency (Spanish, Mandarin, French etc.), Editing & Proofreading, Transcription. Other Examples Computer Literacy (Microsoft Office Suite, Word, Excel), Research Skills (Literature reviews), Curriculum Design and Photography/Videography which in turn makes individual become entrepreneurs in the organizational ecosystems in the near future. However, entrepreneurial ecosystem is an environment of investors, entrepreneurs, organizations, institutions, plants, animals, individuals with business ideas and other organisms interacting within themselves and their non-living components for structural setting up of a business. It is also the atmosphere of business, network of investors with factors and conditions that supports or hinders organizational startups and entrepreneurs in a region.

The sustainable development goals (SDGs) are 17 interconnected goals adopted the UN in 2015 as to achieve a better nation and more sustainable world at large by 2030. These goals run from 1 to 17 and out of which this study supports individuals engaging in decent work & economic growth, industry, innovation, infrastructure, sustainable cities & communities, and partnerships for the goals which is in line with the SDGs 8, 9, 11 & 17. In this view, efficient utilization and sustainable management of both soft and hard skills, and the control measures of such systems depends on understanding the mechanisms of their evolution over time where mathematical modeling in terms of non-linear ordinary differential equations (ODEs) can play a significant role. As a matter of fact, mathematical modeling is broadly used to discuss differential types of real phenomena which can lead to design and formulate better predictive, preventive, management and control strategies. For example, [35] carried out a study on creativity practices and innovation for technological advancement of small businesses in Agro-Allied Sector in Nigeria using mathematical modeling.

Among other researchers [25] developed a mathematical modeling to elucidate entrepreneurship outperforming innovation for industrial performance. [4] Used hierarchical decision model to assess entrepreneurship ecosystems. [5] Identified the relationships among the indicators of sustainable entrepreneurial ecosystems in agricultural startups while [6] critically reviewed entrepreneurial ecosystems research towards a future research agenda. For more exploration of this study the readers are encouraged to [7, 8, 9, 13, 14, 15, 16, 17, 18, 23, 2, 1, 32, 31, 30, 19, 20, 21, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45] and more on the applications of mathematical modeling are [3, 4, 5, 6, 9, 15, 17].

Recently, many researchers and scientists mathematical models for innovative teaching and sustainable development since mathematical modeling is bridging real life problems into mathematical formulae from theoretical applications and numerical analysis. Thus, [29] presented a mathematical modelling for teachers as a practical guide to applicable mathematics education. [24] and [3] studied mathematical modeling perspective on world problems in mathematics education. [12] Revealed mathematical mindsets in unleashing students' potential through creative math, inspiring messages and innovative teaching [26] and [34] proposed mathematical model to develop soft skills for sustainability. [28] explored education for sustainable development in mathematics education, [27] elucidated reimagining authentic mathematical tasks for non-STEM majors, [11] discussed math modeling for getting started & getting solutions and [46] explained how teaching mathematical modelling is a framework to support teachers' choice of resources. [22] Wrote on mathematics for ecology and environmental sciences; while [10] also explained system dynamics on modelling and simulation in business. [22] Proposed an entrepreneurial ecosystems for economic, technological and societal impacts. In the same vein, [47] considered inside the spiky bits with a critical review and conceptualization of entrepreneurial ecosystems, [33] reflected on entrepreneurial ecosystems and public policy in action with a critique of the latest industrial policy blockbuster.

Based on the above elucidated literature, this study proposed and developed a mathematical model on hard skills and soft skills in entrepreneurial ecosystems for sustainable national development with a system of nonlinear differential equation. The study discussed the steady points, dynamical behavior of the points and obtained stability and instability of the system of the governing model for study.

2. Model formulation

This study considered an entrepreneurial ecosystems consisting of two populations:- soft skills and hard skills. We study a predator-prey system in a two work environment, one is predator and the other one prey. Each work environment is supposed to be homogenous. The soft skills work environment is the predator while the other is prey. We supposed that both migrate between the two work places randomly as cross diffusion terms, the system is of functions of several soft skills and hard skills which gives the densities in terms of partial differential equations, hard skills is associated with noise that gives the term in sine function with phase shift, there are unused hard skills and virtually all the soft skills are in use, green noise and white noise found in equation (1&2) describes entrepreneurial ecological progress. The noise in equation (2) will have an influence over that of equation (1).

$S(H) = \frac{\delta H}{\beta + H}$ Where δ and β are positive constants. δ denotes the acquisition rate of the skills and β is the capacity constant of handling a skill. In population dynamics, a functional response of predator to prey density refers to the change in the density of

prey attached per unit time per predator as the prey density changes. $S(H)$ denotes the predator response function. Based on the above functional response assumed is called the Michaelis-Menten or Holling type-II and it satisfies the assumptions made and will be used for the model. Hence, it is assumed that in each work place skill population follows logistic model. The model becomes:

$$\frac{\partial S}{\partial t} = a_1 S \left(1 - \frac{S}{KE}\right) + D_{12} S \nabla^2 H + Q\gamma \epsilon S - US + A_1 \sin(\omega t + \varphi) \quad 1$$

$$\frac{\partial H}{\partial t} = a_2 H \left(1 - \frac{H}{P}\right) - \frac{\delta SH}{\beta + H} + D_{21} H \nabla^2 S - \mu H + A_2 \sin \omega t \quad 2$$

Here, the population densities of both skills are $H(t)$ and $S(t)$, are in three-dimensional space coordinates depicts the dynamics (movement) in the acquisition of the skills for human development respectively, while other parameters description are in the table below. In this Soft-Hard skills model, all the parameters are non-dimensional form without the units. This periodic force is considered to be positive for the reason that pollutants produced by different entrepreneurs have a significant role in shaping the dynamical entrepreneurial behavior of ecosystems. This also makes H and S vector as function of (x, t) in three dimensional space; where X is the position with the components x_1, x_2, x_3 and time t . That is to say $H(x, t)$ and $S(x, t)$

3. Model analysis

The nonlinear system of equations (1-2) are qualitatively analyzed as to ascertain the equilibrium points and the dynamical behavior and test for stability at positivity of equilibrium points.

3.1 Positivity analysis

Since (1) - (2) monitors density population of skill acquisition, all the state variables and parameters of the model are non-negative.

Lemma 1. If $S(t) \geq 0$, $H(t) \geq 0$ then the solution of $S(t)$ and $H(t)$ of the model equat (1) and (2) are positive.

Proof: From (1) $\frac{\partial S}{\partial t} = a_1 S \left(1 - \frac{S}{KE}\right) + D_{12} S \nabla^2 H + Q\gamma \epsilon S - US + A_1 \sin(\omega t + \varphi)$

$$\text{Now, } \frac{\partial S}{\partial t} - a_1 S - Q\gamma \epsilon S + US \geq 0 \Rightarrow \frac{\partial S}{\partial t} + (U - a_1 - Q\gamma \epsilon)S \geq 0$$

Let $P = U - a_1 - Q\gamma \epsilon$ and the integrating factor, $IF = \int e^{Pdt} = e^{Pt} = e^{(U-a_1-Q\gamma \epsilon)t}$.

Multiply both sides by IF gives: $e^{(U-a_1-Q\gamma \epsilon)t} \frac{\partial S}{\partial t} + e^{(U-a_1-Q\gamma \epsilon)t} (U - a_1 - Q\gamma \epsilon)S \geq 0$

$$\Rightarrow e^{(U-a_1-Q\gamma \epsilon)t} S \geq A \Rightarrow S(t) \geq Ae^{-(U-a_1-Q\gamma \epsilon)t} \quad (3)$$

Where A is an integer constant of integration. Applying the conditions when $t = 0$, $S(t) = S(0)$

$\therefore S(0) \geq A$. Substituting the value of A into (3) yields $S(t) \geq S(0)e^{-(U-a_1-Q\gamma \epsilon)t}$. Since $S(t) \geq 0$ and $U - a_1 - Q\gamma \epsilon \geq 0$, it follows that $S(t) \geq 0$ for all $t \geq 0$. Similarly, from equation (2)

$$\frac{\partial H}{\partial t} = a_2 H \left(1 - \frac{H}{P}\right) - \frac{\delta SH}{\beta + H} + D_{21} H \nabla^2 S - \mu H + A_2 \sin \omega t \Rightarrow \frac{\partial H}{\partial t} + \mu H - a_2 H \geq 0 \Rightarrow \frac{\partial H}{\partial t} + (\mu - a_2)H \geq 0 \text{ and the integrating factor } IF = \int e^{(\mu-a_2)dt} = e^{(\mu-a_2)t}.$$

$$\text{Multiply both sides by IF gives: } e^{(\mu-a_2)t} \frac{\partial H}{\partial t} + e^{(\mu-a_2)t} (\mu - a_2)H \geq 0 \Rightarrow \frac{\partial}{\partial t} (e^{(\mu-a_2)t} H) \geq 0 \\ \Rightarrow (e^{(\mu-a_2)t} H) \geq B \Rightarrow H \geq Be^{-(\mu-a_2)t} \quad (4)$$

Where B is an integer constant of integration. Applying the conditions when $t = 0$, $H(t) = H(0)$

$\therefore H(0) \geq B$. Substituting the value of B into (4) yields $H(t) \geq H(0)e^{-(\mu-a_2)t}$. Since $H(t) \geq 0$ and $\mu - a_2 \geq 0$, it follows that $H(t) \geq 0$ for all $t \geq 0$. Therefore it is true that $S(t)$ and $H(t)$

Are positive for all $t \geq 0$,

3.2. Existence of Equilibrium

The steady states of the system is obtained by setting $\frac{\partial S}{\partial t} = 0$ and $\frac{\partial H}{\partial t} = 0$

and solving algebraic equations gives two possible steady states as $\rho(0, 0)$ and $\rho(S^*, H^*)$. The second solution is the positive solutions;

$$a_1 S \left(1 - \frac{S}{KE}\right) + D_{12} S \nabla^2 H + Q\gamma \epsilon S - US + A_1 \sin(\omega t + \varphi) = 0 \quad (5)$$

$$a_2 H \left(1 - \frac{H}{P}\right) - \frac{\delta SH}{\beta + H} + D_{21} H \nabla^2 S - \mu H + A_2 \sin \omega t = 0 \quad (6)$$

From (5), coefficient of cross diffusion is zero $D_{12} = 0, H = 0$ at equilibrium and the higher order terms of S^* is ignored and setting $H^* = 0$ so that

$$a_1 S^* + Q\gamma\epsilon S^* + A_1 \sin(\omega t + \varphi) = 0 \Rightarrow (a_1 + Q\gamma\epsilon - US)S^* = -A_1 \sin(\omega t + \varphi)$$

$$S^* = -\frac{A_1 \sin(\omega t + \varphi)}{(a_1 + Q\gamma\epsilon - US)} \quad (7)$$

Similarly, (6), coefficient of cross diffusion is zero $D_{21} = 0, S = 0$ at equilibrium and the higher order terms of H^* is ignored and setting $S^* = 0$ so that

$$(a_2 - \mu)H^* + A_2 \sin \omega t = 0 \Rightarrow (a_2 - \mu)H^* = -A_2 \sin \omega t$$

$$H^* = -\frac{A_2 \sin \omega t}{(a_2 - \mu)} \quad (8)$$

From the above solutions, one of the equilibria solution is the trivial solution with non-acquisition of both skills; $\rho(0, 0) = (0, 0)$ in a company where both skills are not yet existing. The other equilibrium solution revealed co-existence of both skills interacting for sustainable development of a nation as $\rho(S^*, H^*) = \left(-\frac{A_1 \sin(\omega t + \varphi)}{(a_1 + Q\gamma\epsilon - U)}, -\frac{A_2 \sin \omega t}{(a_2 - \mu)}\right)$. In the feasible equilibrium, it will interest you to understand that soft skills or hard skills acquisition population is independent of their growth or death rates in the company setting.

3.3. Dynamical behavior of equilibria

In this study, the dynamical behavior of equilibria can be calculated by computing vibrational matrices corresponding to each equilibrium. Note that the trivial equilibrium $\rho(0, 0)$ is unstable. By applying the Routh-Hurwitz criteria all the eigenvalues of the vibrational matrix corresponding to ρ^* have negative real parts and ρ^* is locally asymptotically stable in the 2-D plane. This means that a small circle with centre ρ^* can be located such that any solution $(S(t), H(t))$ of system model equations which is inside the circle at some time $t = t_1$ will still remain inside the circle for all $t \geq t_1$ and will tend to (S^*, H^*) as $t \rightarrow \infty$.

3.3.1 Stability of steady states

Considering the fact that stability of deals with the equilibrium values, the study still maintains that cross diffusion is zero at equilibrium points. Suppose the two functions $u = u(s, h) = \frac{\partial s}{\partial t}$ and $v = v(s, h) = \frac{\partial h}{\partial t}$ from the steady state $(S^*, H^*) = (0, 0)$ of the study model (1) and (2). Thus the Jacobian of the system gives-

$$J_{(\bar{s}, \bar{h})} = \begin{pmatrix} \frac{\partial u}{\partial s} & \frac{\partial u}{\partial h} \\ \frac{\partial v}{\partial s} & \frac{\partial v}{\partial h} \end{pmatrix} = \begin{pmatrix} a_1 - \frac{2a_1 s}{KE} + Q\gamma\epsilon - U & 0 \\ -\frac{\delta H}{\beta + H} & a_2 - \frac{2a_2 H}{P} - \frac{\delta S(\beta + H) - \delta SH}{(\beta + H)^2} - \mu \end{pmatrix}$$

For the steady states $(\bar{s}, \bar{h}) = (0, 0)$ gives,

$$J_{(0,0)} = \begin{pmatrix} a_1 + Q\gamma\epsilon - U & 0 \\ 0 & a_2 - \mu \end{pmatrix} \quad (9)$$

Hence the characteristics equation of the matrix with eigenvalue ω yields,

$$|J - \omega I| = \begin{vmatrix} a_1 + Q\gamma\epsilon - U - \omega & 0 \\ 0 & a_2 - \mu - \omega \end{vmatrix}$$

So that $a_1 + Q\gamma\epsilon - U - \omega = 0$ or $a_2 - \mu - \omega = 0$

$$\therefore \omega_1 = a_1 + Q\gamma\epsilon - U, \omega_2 = a_2 - \mu \quad (10)$$

However, the eigenvalue ω of the matrix determine the stability of the states. This study poses two conditions for stability depending on the value of ω : 1. If the eigenvalue $\omega < 0$, then the steady state is stable, 2. If the eigenvalue $\omega > 0$, then the system is unstable. $\omega_1 = a_1 + Q\gamma\epsilon - U > 0$, $\omega_2 = a_2 - \mu > 0$. Therefore the system is unstable because $\omega_1 > 0$ and $\omega_2 > 0$

4. Exact solution

$$\text{From (1)} \quad \frac{\partial S}{\partial t} = a_1 S \left(1 - \frac{S}{KE}\right) + D_{12} S \nabla^2 H + Q\gamma\epsilon S - US + A_1 \sin(\omega t + \varphi)$$

$$\text{Now, } \frac{\partial S}{\partial t} - a_1 S - Q\gamma\epsilon S + US = A_1 \sin(\omega t + \varphi) \Rightarrow \frac{\partial S}{\partial t} - a_1 S - Q\gamma\epsilon S + US \geq 0 \Rightarrow \frac{\partial S}{\partial t} + (U - a_1 - Q\gamma\epsilon)S \geq 0$$

Let $P = U - a_1 - Q\gamma\epsilon$ and the integrating factor, $IF = \int e^{Pdt} = e^{Pt} = e^{(U-a_1-Q\gamma\epsilon)t}$

$$\frac{d}{dt}(Se^{Pt}) = A_1 \sin(\omega t + \varphi) (e^{Pt})$$

$$Se^{Pt} = A_1 \int \sin(\omega t + \varphi) e^{Pt} dt$$

$$\text{Put: } \int e^{Pt} \sin(\omega t + \varphi) dt = G \Rightarrow Se^{Pt} = A_1 G$$

Integrating by parts $\int u dv = vu - \int v du$; Put $u = e^{Pt} \Rightarrow du = P e^{Pt}$

$$dv = \sin(\omega t + \varphi) \Rightarrow v = -\frac{\cos(\omega t + \varphi)}{\omega}$$

$$\begin{aligned} G &= -\frac{e^{Pt} \cos(\omega t + \varphi)}{\omega} + \int \frac{P}{\omega} \cos(\omega t + \varphi) e^{Pt} dt \\ &= -\frac{e^{Pt} \cos(\omega t + \varphi)}{\omega} + \frac{P e^{Pt}}{\omega^2} \sin(\omega t + \varphi) + \frac{P}{\omega} \int \frac{P}{\omega} \sin(\omega t + \varphi) e^{Pt} dt \end{aligned}$$

$$G = -\frac{e^{Pt} \cos(\omega t + \varphi)}{\omega} + \frac{P e^{Pt}}{\omega^2} \sin(\omega t + \varphi) + \frac{GP^2}{\omega^2}$$

$$G \left(1 - \frac{P^2}{\omega^2} \right) = -\frac{e^{Pt} \cos(\omega t + \varphi)}{\omega} + \frac{P e^{Pt}}{\omega^2} \sin(\omega t + \varphi)$$

$$G = -\frac{P e^{Pt} \cos(\omega t + \varphi)}{\omega^2 + P^2} - \frac{P e^{Pt}}{\omega^2 + P^2} \sin(\omega t + \varphi) + K$$

$$Se^{Pt} = \frac{A_1 P e^{Pt} \cos(\omega t + \varphi)}{\omega^2 + P^2} + \frac{P e^{Pt}}{\omega^2 + P^2} \sin(\omega t + \varphi) + K$$

$$S(t) = -\frac{A_1 P \cos(\omega t + \varphi)}{\omega^2 + P^2} + \frac{P}{\omega^2 + P^2} \sin(\omega t + \varphi) + K e^{Pt}$$

Initial condition $S(0) = C_1$ (constant), from equation the above

$$-\frac{A_1 P \cos(\omega t + \varphi)}{\omega^2 + P^2} + \frac{P}{\omega^2 + P^2} \sin(\omega t + \varphi) + K = C_1$$

$$K = \frac{A_1 P \cos(\omega t + \varphi)}{\omega^2 + P^2} - \frac{P}{\omega^2 + P^2} \sin(\omega t + \varphi) + C_1$$

$$S(t) = -\frac{A_1 P \cos(\omega t + \varphi)}{\omega^2 + P^2} + \frac{P}{\omega^2 + P^2} \sin(\omega t + \varphi) + e^{Pt} \left(\frac{A_1 P \cos(\omega t + \varphi)}{\omega^2 + P^2} - \frac{P \sin(\omega t + \varphi)}{\omega^2 + P^2} + C_1 \right) \quad (11)$$

Similarly, from euq (2)

$$\frac{\partial H}{\partial t} - a_2 H + \mu H = A_2 \sin \omega t \Rightarrow \frac{\partial S}{\partial t} + \mu H - a_2 H \geq 0; \frac{\partial S}{\partial t} + (\mu - a_2) H \geq 0$$

Let $Q = (\mu - a_2)$ and the integrating factor, $IF = \int e^{Qdt} = e^{Qt} = e^{(\mu - a_2)t}$

$$\frac{d}{dt}(He^{Qt}) = A_1 \sin \omega t (e^{Qt}) \Rightarrow He^{Qt} = A_1 \int \sin \omega t e^{Qt} dt$$

$$\text{Put: } \int e^{Qt} \sin \omega t dt = W \Rightarrow He^{Qt} = A_1 W$$

Integrating by parts, $\int u dv = vu - \int v du$: Put $u = e^{Qt} \Rightarrow du = Q e^{Qt}$

$$dv = \sin \omega t \Rightarrow v = -\frac{\cos \omega t}{\omega}$$

$$W = -\frac{e^{Qt} \cos \omega t}{\omega} + \int \frac{Q}{\omega} \cos \omega t e^{Qt} dt$$

$$= -\frac{e^{Qt} \cos \omega t}{\omega} + \frac{Q e^{Qt}}{\omega^2} \sin(\omega t + \varphi) + \frac{Q}{\omega} \int \frac{Q}{\omega} \sin \omega t e^{Qt} dt$$

$$W = -\frac{e^{Qt} \cos \omega t}{\omega} + \frac{Q e^{Qt}}{\omega^2} \sin \omega t + \frac{WQ}{\omega^2}$$

$$W \left(1 - \frac{Q^2}{\omega^2} \right) = -\frac{e^{Qt} \cos \omega t}{\omega} + \frac{Q e^{Qt}}{\omega^2} \sin \omega t$$

$$W = -\frac{Q e^{Qt} \cos \omega t}{\omega^2 + Q^2} + \frac{Q e^{Qt}}{\omega^2 + Q^2} \sin \omega t + K$$

$$H e^{Qt} = \frac{A_1 Q e^{Qt} \cos \omega t}{\omega^2 + Q^2} + \frac{Q e^{Qt}}{\omega^2 + Q^2} \sin \omega t + K$$

$$H(t) = -\frac{A_1 Q \cos \omega t}{\omega^2 + Q^2} + \frac{Q}{\omega^2 + Q^2} \sin \omega t + K e^{Qt}$$

Initial condition $H(0) = C_2$ (constant), from equation the above

$$-\frac{A_1 Q \cos \omega t}{\omega^2 + Q^2} + \frac{Q}{\omega^2 + Q^2} \sin \omega t + K = C_2$$

$$K = \frac{A_1 Q \cos \omega t}{\omega^2 + Q^2} - \frac{Q}{\omega^2 + Q^2} \sin \omega t + C_2$$

$$H(t) = -\frac{A_1 Q \cos \omega t}{\omega^2 + Q^2} + \frac{Q}{\omega^2 + Q^2} \sin \omega t + e^{Qt} \left(\frac{A_1 Q \cos \omega t}{\omega^2 + Q^2} - \frac{Q \sin \omega t}{\omega^2 + Q^2} + C_2 \right) \quad (12)$$

From all indications, the solutions of equations (1) and (2) given in (11) and (12) showed that there soft and hard skills acquisition, progression and growth for continuous enlargement of any domain or company for individual or national development. Equations (11) and (12) revealed the effect of national economy growth or inflation in the utilization of both skills thereby giving a fluctuating pattern.

Table 1: Values of the model parameters, description and units

Variables	Description	Value	Units	Source
S	State variable -Soft skills		People	Assumed
H	State variable -Hard skills		People	Assumed
T	Time independent variable	6, 12, 24, 36	Months	Assumed
$\frac{\partial}{\partial t}$	Partial derivatives with respect to time			Assumed
a_1	Rate of Enrollment rate into soft skills	0.670	People	Assumed
a_2	Rate of Enrollment rate into soft skills	0.450	People	
D_{12}	cross diffusion for soft skills to hard skills	1, 2, 3, 4....	Dimensionless	
D_{21}	cross diffusion for hard skills to soft skills	1, 2, 3, 4....	Dimensionless	
k	Rate of entering into soft skills and hard skills dynamics	1, 2, 3, 4	Dimensionless	
Q	Profit maximization for soft skills	0.5	Dimensionless	
γ	Competition coefficient for soft skills	0.8	Dimensionless	
ϵ	Force applied for soft skills	2		Assumed
U	Unutilized soft skills	1, 2, 3	Dimensionless	
∇^2	Second order Laplacian operator in 3-dimension for the soft and hard skills			Assumed
KE	Number of soft soft skills acquired and utilized per time	1, 2, 3, 4,		
P	Number of hard skills utilized per time	1, 2, 3...		
ω	Angular frequency	$\frac{2\pi}{12}$	Radians/sec	Assumed
φ	Phase shift	$\pm \frac{\pi}{4}$	Radians	Assumed
δ	maximum demand of hard skills that can be used per soft skills per time, .	1, 2,	dimensionless	
β	Saturation value of hard skills to achieve one half the maximum rate δ	2, 3, 4, 5....	dimensionless	
μ	Unused hard skills	2, 3, 4, 5, ...	dimensionless	Estimated
A	Amplitude of the noise	1		
$A_1 \sin(\omega t + \varphi)$	Noise and periodic force term for soft skills		Radians	
$A_2 \sin \omega t$	Noise and periodic force term for hard skills			
$G(x - kt)$	Travelling wave equations in the positive x-direction			
∇	Laplacian operator in 3-dimensions			
$\nabla^2 H$	$9(x_i - kt)$	Use the values of x, k, t		Differential calculus
$\nabla^2 S$	$9(x_i - kt)$	Use the values of x, k, t		
x	Position or places to acquire soft and hard skills is a function of $x = x_1, x_2, x_3$	1, 2, 3, ...	Metres	Assumed

5. Simulation

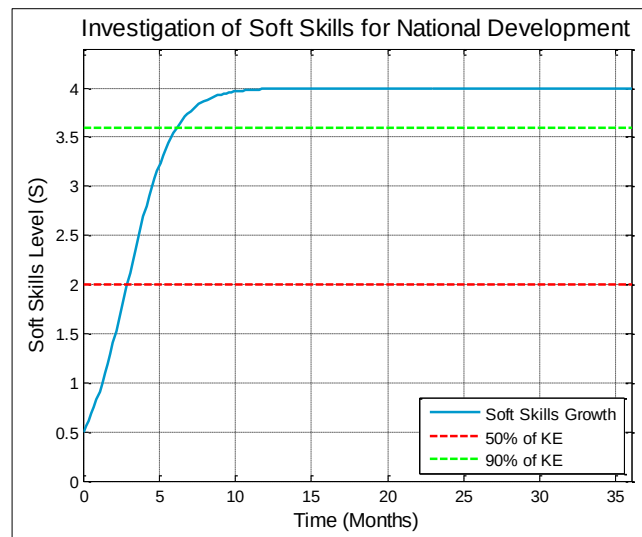


Fig 1: Showing the place of Soft Skills for National Development

The graph illustrates how soft skills grow over time and their contribution to national development. Initially, soft skills start at a moderate level and gradually increase, following a logistic growth pattern, which reflects the natural progression of skill acquisition in a population. As time progresses, the growth slows down as it approaches the maximum capacity, representing the saturation point where most individuals have acquired the available soft skills.

The reference lines at 50% and 90% of the maximum soft skills highlight important milestones in development. Reaching 50% of the capacity indicates a significant portion of the population is developing essential soft skills, which is critical for boosting productivity, innovation, and effective participation in national development. The 90% line represents near-full utilization of soft skills, showing the entrepreneurs and potential for maximum impact on national growth if skill acquisition is fully optimized. The plot emphasizes that investing in and nurturing soft skills over time can play a crucial role in enhancing national development. It suggests that policy interventions, education programs and training initiatives that accelerate soft skills acquisition can significantly influence the pace and extent of national progress.

The graph shows a smooth and steady increase in the growth of soft skills over time when there are no external disturbances or random influences. At the beginning, the curve rises quickly, representing the early stages of national investment in human development—where individuals begin to acquire communication, leadership, and problem-solving abilities. As training and experience accumulate, growth continues but gradually slows down, showing that soft skills approach a sustainable and balanced level within the population. The absence of noise means the growth process is stable and predictable, reflecting a system where development efforts are consistent and uninterrupted by sudden changes in policy or external shocks. This steady pattern demonstrates that with continuous support for education, mentorship, and innovation, soft skills can mature naturally to become a permanent and reliable foundation for national development, productivity, and economic progress.

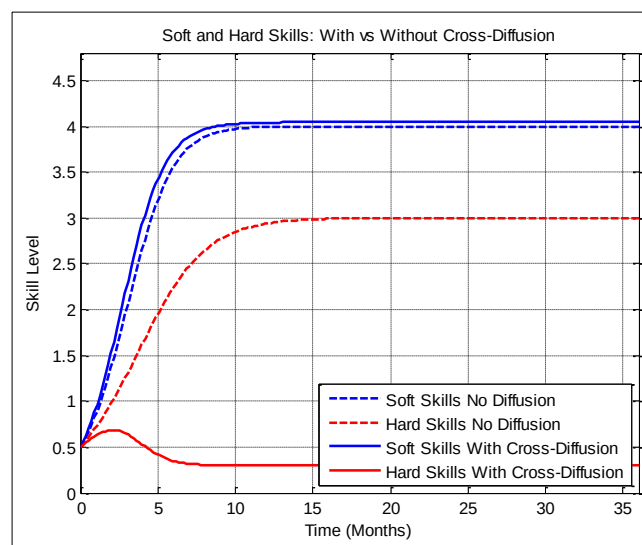


Fig 2: Showing the dynamics of Soft Skills and Hard Skills for National Development with and without cross diffusion terms

In this setup, we focus entirely on how soft skills and hard skills grow over time, without considering spatial movement or cross-diffusion effects. Soft skills follow a logistic growth pattern, meaning they increase rapidly at first and then slow down as they approach a maximum capacity. This maximum represents the highest level of soft skills that can be acquired and effectively utilized for national development. The growth of soft skills is independent and unaffected by hard skills in this model, which simplifies the dynamics and highlights their contribution over time.

Hard skills also grow according to a logistic pattern but are slightly moderated by the presence of soft skills through a Holling type-II interaction. This means that as soft skills increase, they can reduce the effective growth rate of hard skills, reflecting how soft skills can influence the efficiency or utilization of hard skills.

By removing diffusion and cross-diffusion, we eliminate any spatial effects, so the skills do not “spread” across regions or interact through space. The system is purely temporal, showing how both types of skills evolve over time in the national development context. Soft skills tend to dominate over time as they reach their maximum level, while hard skills stabilize at a lower level due to the moderating effect of soft skills. This approach provides a clear and interpretable view of the roles of soft and hard skills in driving national development, focusing on their intrinsic growth dynamics rather than spatial interactions. This implies that cross diffusion terms do not inhibit the acquisition and progression of soft skills and hard skills respectively.

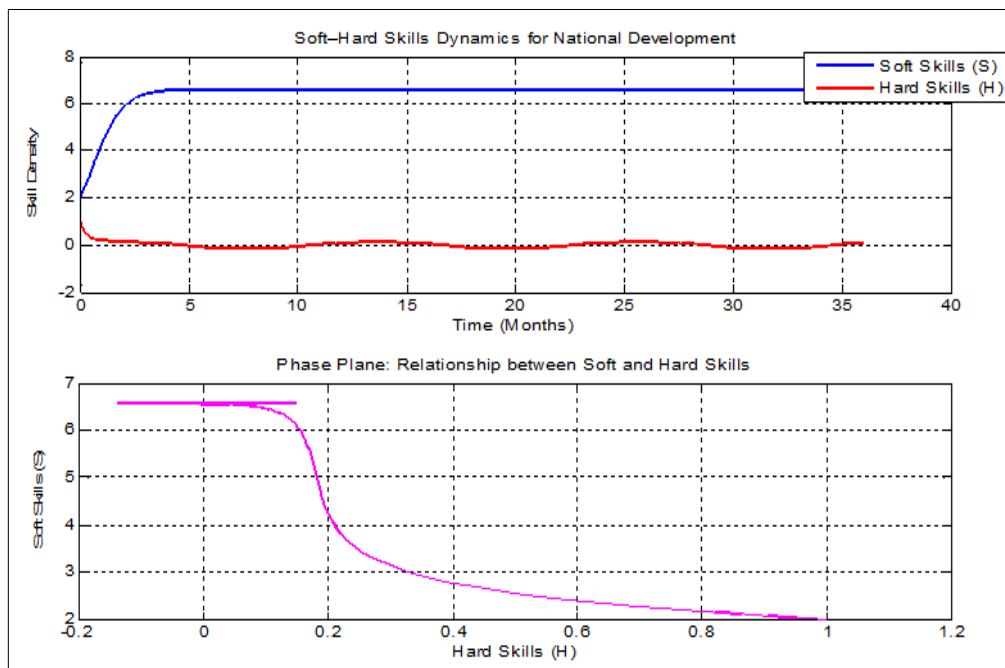


Fig 3: Showing the relationship between Soft Skills and Hard Skills for National Development

The plotted result captures a fascinating predator-prey dynamic between soft skills (S) and hard skills (H) within the entrepreneurial ecosystem of national development. Just as lions (predators) depend on gazelles (prey) for sustenance, soft skills feed on the availability and quality of hard skills to grow, adapt, and sustain productivity within an entrepreneurial system. In the first graph, both skills initially rise together, this is the growth phase of a balanced ecosystem. The “prey,” representing hard skills such as technical expertise, craftsmanship, and professional competencies, increases rapidly as individuals acquire formal training and specialized knowledge. The “predator,” soft skills like communication, adaptability, innovation, and teamwork, begin to thrive because they depend on the presence of those technical competencies to demonstrate effectiveness. However, as time progresses, the graph reveals an oscillatory pattern: when hard skills (the prey) become too abundant, soft skills (the predator) flourish, taking advantage of the structured environment to enhance creativity, leadership, and coordination. But as soft skills grow stronger, they begin to regulate the over-reliance on hard skills, reducing redundancy and driving more balanced productivity just as predators keeping prey populations in ecological balance.

The periodic oscillations observed in hard skills are caused by the external forcing term $A \sin(\omega t + \phi)$, symbolizing unpredictable “environmental factors” such as global economic trends, new technologies, or policy reforms. These act like climatic changes in nature and sometimes creating abundance (a rise in hard skills) and sometimes scarcity (a decline), forcing soft skills to adapt, reorganize, and sustain equilibrium in the system.

The phase-plane plot of soft skills versus hard skills provides a deeper ecological insight. The looping pattern reflects a cyclic interdependence — just as an ecosystem cycles between predator and prey dominance, the entrepreneurial landscape alternates between periods where technical ability (hard skills) leads innovation and times when creativity, adaptability, and communication (soft skills) steer transformation. In the long run, the graph shows that soft skills dominate the system, not by eliminating hard skills, but by maintaining control, balance, and adaptability. This aligns with the model’s assumption that soft skills are the “predators” agile, adaptive, and strategically intelligent, while hard skills are the “prey” — structured, tangible, and essential for survival but needing continuous refinement. In real-life terms, this mirrors how modern economies operate: Nations or industries that rely solely on technical know-how (hard skills) may initially grow, but without creativity, leadership, and collaboration (soft skills), progress soon stagnates.

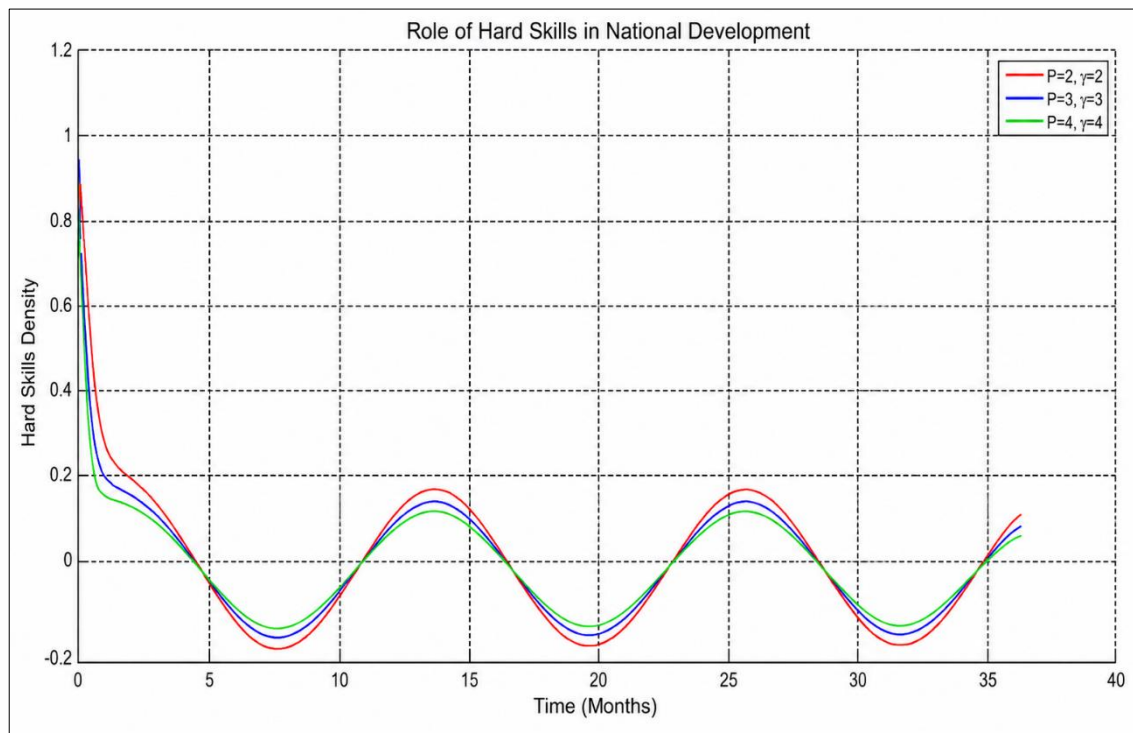


Fig 4: Showing the place Hard Skills for National Development

The plot vividly illustrates how hard skills (H) evolve over time under varying conditions of utilization and neglect, offering deep insight into their pivotal role in driving national development.

At lower utilization rates ($P = 2, \mu = 2$), hard skills initially rise due to early technical acquisition but soon decline sharply. This mirrors economies where technical knowledge exists but is poorly applied, underfunded, or disconnected from real-world innovation. In such systems, hard skills stagnate because industrial structures, policies, and institutions fail to absorb or sustain technical expertise — much like machinery left idle without maintenance.

As utilization improves ($P = 3, \mu = 3$), the hard skills curve stabilizes, reflecting a healthy balance between skill acquisition and application. Here, hard skills interact effectively with soft skills — leadership, adaptability, and communication — to sustain productivity. This stage represents an economy where technical capacity is consistently harnessed to support industrial output, technological development, and workforce competence.

At higher utilization and reduced decay ($P = 4, \mu = 4$), the hard skills trajectory shows strong and stable growth, symbolizing a mature, innovation-driven economy. In this environment, technical expertise is continuously renewed through research, training, and digital adaptation. The result is a resilient economy where technical know-how forms a reliable engine of progress, propelling entrepreneurship, manufacturing, and technological advancement.

Overall, the graph underscores that hard skills serve as the structural backbone of national productivity, forming the tangible basis for innovation and growth. Yet, their sustained impact depends on how well they are renewed, coordinated, and integrated with soft skills — the strategic “drivers” that transform technical ability into meaningful output.

In essence, hard skills build the nation’s industrial strength, but soft skills navigate its direction. Without effective leadership, creativity, and adaptability, even a highly skilled workforce risks stagnation. Thus, national development thrives not merely on the possession of hard skills, but on the intelligent, adaptive, and innovative use of those skills within a dynamic socio-economic system.

6. Conclusion

The simulations highlight a dynamic interplay between soft skills and hard skills in driving national development. Hard skills provide the technical foundation necessary for productivity and innovation, but their impact is significantly amplified when complemented by soft skills, which act as strategic “drivers” guiding adaptability, creativity, and effective application. The predator-prey analogy illustrates that soft skills (predators) regulate and optimize the use of hard skills (prey), ensuring balanced growth and sustained economic progress. National development, therefore, thrives when both skill sets evolve in synergy, with soft skills steering technical competencies toward meaningful and sustainable outcomes.

However, systems or organizations that develop and apply soft skills effectively manage, innovate, and lead technological expertise toward meaningful outcomes — ensuring long-term stability and sustainable development. Just as a balanced ecosystem thrives when both predator and prey coexist harmoniously, national development flourishes when soft and hard skills evolve in synergy. The dominance of soft skills in the simulation reflects the real-world truth that emotional intelligence, adaptability, and innovation now “hunt” for and shape the direction of technical capacity, making them the driving force of modern entrepreneurship and national progress. Based on the findings of this study, the following recommendations are made: that individuals, organizations and governments should strengthen soft skills programs in education and professional training to enhance communication, leadership, adaptability, and innovation, ensuring hard skills are applied effectively in real-world contexts and that individuals should optimize utilization of hard skills through continuous professional development, research

initiatives, and integration with soft skills, thereby maximizing their contribution to national productivity and sustainable development.

Declaration

The review paper has not been published previously or under publication consideration elsewhere.

Ethical Approval

This being a review paper and as such the clearance of ethical committee was not required.

Disclaimer (Artificial Intelligence)

Author hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of this manuscript.

Competing Interests Disclaimer

Authors have declared that they have no known competing financial interests OR non-financial interests OR personal relationships that could have appeared to influence the work reported in this paper.

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