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Generalized separation via γ -sets in L-fuzzy topological spaces

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Abstract

In this paper, we introduce and define the concept of γ -separated sets by using γ -open sets in L-fuzzy topological spaces. It is observed that every separated set is also γ -separated, which shows that γ -separation is a generalization of the usual notion of separation. Furthermore, we investigate various characterizations of γ -separated sets and examine their fundamental properties in detail. Several important results and relationships with other types of fuzzy separation concepts are also discussed, which help to better understand the structure and behavior of γ -separated sets in L-fuzzy topological spaces.

Keywords: γ -closed sets, γ -open sets, γ -separated sets, L-fuzzy topological spaces

1. Introduction

Let S be the set of all non-negative integers. Connectedness is a fundamental concept in fuzzy topology. In 1975, Bruce Hutton^[10] constructed the L-fuzzy unit interval $I(L)$ and investigated its fuzzy topological properties, where $I = [0, 1]$. He proved that when L is orthocomplemented, the fuzzy open sets of $I(L)$ correspond one-to-one with the usual open sets of I , and this correspondence preserves unions and intersections. As a consequence, $I(L)$ possesses many fuzzy topological properties, including connectedness. However, he observed that $I(L)$ fails to be connected when L is a chain. Later, Rodabaugh^[14] showed that $I(L)$, the L-fuzzy open unit interval $(0, 1)(L)$, and the L-fuzzy real line $R(L)$ are connected if $0 \in L^b$ (a condition satisfied by chains), where $L^b = \{\alpha \in L: \alpha < \beta \text{ and } \alpha < \gamma \Rightarrow \alpha < \beta \wedge \gamma\}$. This condition is satisfied by chains. His observation can be summarized as follows: a greater degree of connectivity should be associated with a lesser degree of dis-connectivity. Furthermore, Zheng Chong-You^[17] studied several properties of the L-fuzzy unit interval and introduced the concept of L-fuzzy path connectedness. He examined the relationships among different types of fuzzy connectedness, such as O-connectedness, Q-connectedness, 1-connectedness, and standard L-fuzzy path connectedness.

Ajmal and Kohli^[3] introduced the concepts of fuzzy connectedness and provided their characterizations. They observed that fuzzy connectedness remains invariant under fuzzy continuous mappings, highlighting its stability as a topological property. In^[4], it was further demonstrated that fuzzy C_2 -connectedness and fuzzy C_4 -connectedness are not necessarily preserved under the product of spaces. This result shows that certain types of fuzzy connectedness behave differently when forming product spaces. Turanli^[15] presented several counterexamples concerning the product of fuzzy connected spaces. Moreover, he introduced the notion of fuzzy super C_s -connectedness and investigated its relationship with fuzzy C_s -connectedness, thereby extending the existing theory.

Liu and Pu^[13] introduced the concept of connectedness in fuzzy topological spaces in 1980. Their work laid the foundation for further research in this area. Subsequently, G. J. Wang^[16] extended the definition proposed by Liu and Pu to the broader framework of L-fuzzy topological spaces, thereby generalizing the concept. Many researchers have since investigated different forms of connectedness^[1, 2, 8] in L-fuzzy topological spaces, particularly in the sense

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defined by Chang ^[6]. These studies have contributed significantly to the development of fuzzy topology. Using pre-closed sets, Bai ^[5] introduced the notion of P-connectedness, which provided a new perspective on connectedness in L-fuzzy spaces. These concepts further enriched the theory by offering refined classifications of connectedness.

Recently, El-Atik ^[7] investigated the notion of *b-connectedness* and explored its applications in general topology. His work contributed to a deeper understanding of alternative forms of connectedness and their significance in topological structures. Hanafy ^[9] introduced the concept of a γ -open set and defined γ -connectedness in fuzzy topological spaces. He also examined several properties of γ -open sets and studied fuzzy γ -continuity in such spaces. These investigations helped clarify the behavior of γ -related structures and enriched the theory of fuzzy topology by providing new tools for analysis.

In this paper, we define the concept of γ -separated sets in L-fuzzy topological spaces using γ -open sets. This approach provides a generalized framework for studying separation in fuzzy topology. It is observed that every separated set is also γ -separated, showing that γ -separation is a broader notion. We investigate various characterizations of γ -separated sets and establish several of their important properties. These results contribute to a deeper understanding of separation axioms in L-fuzzy topological spaces.

2. Preliminaries

In this section, we present some basic concepts and preliminary notions that will be useful for developing our main results. Liu ^[12] introduced the notion of separated sets in the framework of L-fuzzy topological spaces. These sets are commonly referred to as separated L-fuzzy sets. The concept plays an important role in understanding the structural properties of L-fuzzy topological spaces. In the following, we recall and discuss some important definitions introduced by Liu ^[12], which form the foundation for our subsequent study.

Definition 2.1. ^[12] Let (X, τ) be an L-fuzzy topological space and $A, B \in L^X$. Then A and B are called separated if $A \wedge cl(B) = 0$ and $B \wedge cl(A) = 0$.

Definition 2.2. ^[12] Let (X, τ) be an L-fuzzy topological space and $A \in L^X$. Then A is said to be connected if A cannot be represented as join of two separated non-null L-fuzzy sets.

Definition 2.3. ^[9] Let (X, τ) be an L-fuzzy topological space and $A \in L^X$. Then A is said to be γ -open

$$\text{if } A \leq cl(int(A)) \vee int(cl(A))$$

and γ -closed

$$\text{if } cl(int(A)) \vee int(cl(A)) \leq A.$$

Definition 2.4. ^[9] Let (X, τ) be an L-fuzzy topological space and $A \in L^X$.

The fuzzy γ -interior of A (in short γ -int (A)) is defined as γ -int (A) = $\bigvee \{H: H \text{ is } \gamma\text{-open L-fuzzy set}, H \leq A\}$. The γ -interior of A is the join of all γ -open subsets of L^X contained in A .

The γ -interior of $A \in L^X$ is just largest γ -open subset of L^X contained in A . Note that L-fuzzy set A is γ -open iff γ -int (A) = A .

Definition 2.5. ^[9] Let (X, τ) be an L-fuzzy topological space and $A \in L^X$.

The fuzzy γ -closure of A (in short γ -cl (A)) is defined as γ -cl (A) = $\bigwedge \{G: G \text{ is } \gamma\text{-closed L-fuzzy set}, A \leq G\}$. The γ -closure of A is the meet of all γ -closed subsets of L^X containing A .

The γ -closure of $A \in L^X$ is just smallest γ -closed subset of L^X containing A . Note that L-fuzzy set A is γ -closed iff γ -cl (A) = A .

Main Results

We are now in a position to introduce the concept of γ -separated L-fuzzy sets within the setting of L-fuzzy topological spaces. In this section, we formally define γ -separated L-fuzzy sets and explore their fundamental features. Furthermore, we prove that every separated L-fuzzy set is also γ -separated, thereby establishing a significant relationship between these two notions. In addition, we investigate several characterizations and study various important properties of γ -separated sets, which contribute to a deeper understanding of their structural behavior in L-fuzzy topological spaces.

Definition 3.1. Let (X, τ) be an L-fuzzy topological space and $A, B \in L^X$. Then A, B are called γ -separated if $A \wedge \gamma\text{-cl}(B) = 0$ and $B \wedge \gamma\text{-cl}(A) = 0$.

Theorem 3.1. Every separated L-fuzzy set in L-fuzzy topological space (X, τ) is γ -separated.

Proof. Let A and B be separated L-fuzzy sets in L-fts (X, τ) . By Definition 2.1, we have $A \wedge cl(B) = 0$ and $B \wedge cl(A) = 0$. By using the fact $\gamma\text{-cl}(A) \leq cl(A)$ for every $A \in L^X$, we get $A \wedge \gamma\text{-cl}(B) = 0$ and $B \wedge \gamma\text{-cl}(A) = 0$. Hence by Definition 2.6, A and B are γ -separated L-fuzzy sets in L-fts (X, τ) .

Theorem 3.2. Let (X, τ) be L-fuzzy topological space and $A, B \in L^X$ such that $A \neq 0$ and $B \neq 0$. Then following statements hold;

1. If A and B are γ -separated L-fuzzy sets and A_1, B_1 are non-null L-fuzzy sets in L-fits (X, τ) such that $A_1 \leq A$ and $B_1 \leq B$, then A_1, B_1 are also γ -separated.
2. If $A \wedge B = 0$ such that each of A and B are both γ -closed (γ -open), then A and B are γ -separated.
3. Let A and B be both γ -closed (γ -open). If $H = A \wedge B'$ and $G = B \wedge A'$, then H and G are γ -separated.

Proof: (I) Given $A_1 \leq A$ and $B_1 \leq B$ then $\gamma - cl(A_1) \leq \gamma - cl(A)$ and $\gamma - cl(B_1) \leq \gamma - cl(B)$. Since, A and B are γ -separated L-fuzzy sets, so $A \wedge \gamma - cl(B) = 0$ and $B \wedge \gamma - cl(A) = 0$, then

$$A \wedge \gamma - cl(B) = 0 \text{ and } B \wedge \gamma - cl(A) = 0$$

$$\Rightarrow A_1 \wedge \gamma - cl(B) = 0 \text{ and } B_1 \wedge \gamma - cl(A) = 0$$

$$\Rightarrow A_1 \wedge \gamma - cl(B_1) = 0 \text{ and } B_1 \wedge \gamma - cl(A_1) = 0$$

Hence, A_1, B_1 are also γ -separated.

(II) Since A and B are γ -closed so $A = \gamma - cl(A), B = \gamma - cl(B)$ and $A \wedge B = 0$, then $\gamma - cl(A) \wedge B = 0, \gamma - cl(B) \wedge A = 0$. Hence, A and B are γ -separated.

Since A and B are γ -open so A' and B' are γ -closed so $A' = \gamma - cl(A'), B' = \gamma - cl(B')$ and $A' \wedge B' = 0$. then $\gamma - cl(A) \wedge B = 0, \gamma - cl(B) \wedge A = 0$. Therefore, A' and B' are γ -separated. Clearly,

$$A \leq B' \Rightarrow \gamma - cl(A) \leq \gamma - cl(B') \Rightarrow \gamma - cl(A) \leq B' \Rightarrow \gamma - cl(A) \wedge B = 0.$$

Similarly, we get $A \wedge \gamma - cl(B) = 0$. Hence, A and B are γ -separated.

(III) If A and B are γ -open L-fuzzy sets then A' and B' are γ -closed L-fuzzy sets so $A' = \gamma - cl(A'), B' = \gamma - cl(B')$. Since, $H = A \wedge B'$ and $G = B \wedge A'$ then $H \leq A$ and $G \leq B$, also

$$H = A \wedge B' \text{ and } G = B \wedge A'$$

$$\Rightarrow \gamma - cl(H) \leq \gamma - cl(B') = B' \text{ and } \gamma - cl(G) \leq \gamma - cl(A') = A'$$

$$\Rightarrow \gamma - cl(H) \wedge B = 0 \text{ and } \gamma - cl(G) \wedge A = 0$$

$$\Rightarrow \gamma - cl(H) \wedge G = 0 \text{ and } H \wedge \gamma - cl(G) = 0.$$

Hence, G and H are γ -separated.

If A and B are γ -closed L-fuzzy sets so $A = \gamma - cl(A), B = \gamma - cl(B)$. Since, $H = A \wedge B'$ and $G = B \wedge A'$ then $H \leq B'$ and $G \leq A'$, also

$$H = A \text{ and } G = B$$

$$\Rightarrow \gamma - cl(H) \leq \gamma - cl(A) = A \text{ and } \gamma - cl(G) \leq \gamma - cl(B) = B$$

$$\Rightarrow \gamma - cl(H) \wedge A' = 0 \text{ and } \gamma - cl(G) \wedge B' = 0$$

$$\Rightarrow \gamma - cl(H) \wedge G = 0 \text{ and } H \wedge \gamma - cl(G) = 0.$$

Hence, G and H are γ -separated.

Theorem 3.3. Let (X, τ) be L-fuzzy topological space and $A, B \in L^X$ such that

$A \neq 0$ and $B \neq 0$. Then A and B are γ -separated iff there exists γ -open L-fuzzy sets A_1, B_1 in L-fuzzy topological space (X, τ) such that $A \leq A_1, B \leq B_1$ and $A \wedge B_1 = 0, A_1 \wedge B = 0$.

Proof. Let A and B are γ -separated L-fuzzy sets in L-fuzzy topological space (X, τ) . Take $A_1 = (\gamma - cl(B))^c$ and $B_1 = (\gamma - cl(A))^c$ then A_1 and B_1 are γ -open L-fuzzy sets in L-fuzzy topological space (X, τ) such that $A \leq A_1, B \leq B_1$. We have,

$$A_1 = (\gamma - cl(B))^c \text{ and } B_1 = (\gamma - cl(A))^c$$

$$\Rightarrow A_1 \wedge \gamma - cl(B) = 0 \text{ and } \gamma - cl(A) \wedge B_1 = 0$$

$$\Rightarrow A_1 \wedge B = 0 \text{ and } A \wedge B_1 = 0.$$

Hence, $A_1 \wedge B = 0$ and $A \wedge B_1 = 0$.

Conversely, Let A_1, B_1 are γ -open L -fuzzy sets in L -fuzzy topological space (X, τ) such that $A \leq A_1, B \leq B_1$ and $A \wedge B_1 = 0, A_1 \wedge B = 0$. Then $\neg B_1 \leq A'$ and $A_1 \leq B'$. Since, A'_1, B'_1 are γ -closed L -fuzzy sets in L -fuzzy topological space (X, τ) then $\gamma-cl(A'_1) = A_1$ and $\gamma-cl(B'_1) = B_1$.

Now,

$$\gamma-cl(A) \leq \gamma-cl(A_1) \leq \gamma-cl(B') = B'$$

And

$$\gamma-cl(B) \leq \gamma-cl(B_1) \leq \gamma-cl(A') = A'.$$

Thus, $\gamma-cl(A) \wedge B = 0$ and $\gamma-cl(B) \wedge A = 0$.

Hence, A and B are γ -separated.

Theorem 3.4. Let (X, τ) be L -fuzzy topological space and $A, B, C \in L^X$. If B and C are γ -separated then $A \wedge B = 0$ and $A \wedge C = 0$ are γ -separated.

Proof. Let (X, τ) be L -fuzzy topological space and $A, B, C \in L^X$. If B and C are γ -separated then $B \wedge \gamma-cl(C) = 0$ and $\gamma-cl(B) \wedge C = 0$. Since, $A \wedge B \leq B$ and $A \wedge C \leq C$ implies $\gamma-cl(A \wedge B) \leq \gamma-cl(B)$ and $\gamma-cl(A \wedge C) \leq \gamma-cl(C)$.

We get,

$$\gamma-cl(A \wedge B) \wedge (A \wedge C) \leq \gamma-cl(B) \wedge C = 0$$

And

$$(A \wedge B) \wedge \gamma-cl(A \wedge C) \leq B \wedge \gamma-cl(C) = 0.$$

Hence, $A \wedge B$ and $A \wedge C$ are γ -separated.

Conclusion

In this paper, we have introduced and developed the concept of γ -separated sets in L -fuzzy topological spaces as a natural generalization of classical separation axioms. By employing γ -open sets, a broader and more flexible framework for separation has been established. We proved that every separated L -fuzzy set is γ -separated, which confirms that γ -separation properly extends the existing notion of separation. Various characterizations and fundamental properties of γ -separated sets were investigated, providing deeper insight into their structural behavior. Furthermore, the relationships between γ -separated sets and other fuzzy separation concepts were explored, enriching the theory of L -fuzzy topology. These results contribute significantly to the advancement of fuzzy topological studies and may serve as a foundation for further research in generalized separation structures and their applications.

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