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Performance properties of disturbance variance estimation using the precautionary loss function

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Abstract

In general, the symmetrical loss functions are frequently employed in evaluating risk. Such a loss function gives equal weight to positive and negative estimation errors of the same magnitude. Therefore, in a situation when underestimation is more serious than overestimation and vice versa, it may be most suitable to use an asymmetric loss function. The precautionary loss function is a kind of asymmetric loss function, useful in situations when underestimation may lead to serious consequences. In this article, the risk function for usual and restricted error variance is derived under the precautionary loss function in a linear regression model.

Keywords: Ordinary least squares estimator (OLS), restricted least square estimator (RLS), mean squared error (MSE), precautionary loss function

1. Introduction

In statistical inference, the use of loss functions is quite valuable in minimizing the risk of the estimator and can affect estimators' properties and rankings. In general, the symmetrical loss functions are frequently employed in evaluating risk *i.e.*, Squared error loss function, used by various researchers [Ohtani (1987, 2001) ^[12, 15], Matta and Casella (1990) ^[10], James, & Stein (1992) ^[7]. In some situations, where positive and negative errors have differing effects then employment of asymmetric loss function has been most justified for *e.g.* LINEX loss function. Norstrom (1996) ^[11] introduced an alternative asymmetric precautionary loss function, and also presented a general class of precautionary loss functions as a special case. These loss functions approach infinitely near the origin to prevent underestimation, thus giving conservative estimators, especially when underestimation may lead to serious consequence (Ali et al. 2012) ^[1]. Precautionary loss has captured enormous attention and several authors have used this loss function in different contexts, specifically in Bayesian context Huda & Aref, (2016) and Selvam and Geetha (2023) ^[6, 15].

In the linear regression model sample information used to estimate unknown parameters. In some situations, prior information in the form of linear restrictions is available to estimate unknown parameters. By combining this information with sample information several improved estimators for regression coefficient and for variance were proposed. [see *e.g.* Srivastava and Rao (1992) and Dube, M., & Singh, V. (1995)]. Dube and Chandra (2007) ^[3, 4, 14] examined the effect of Stein rule estimator of the disturbance variance when disturbances are normally distributed under the mean squared error criterion. Chandra and Tyagi (2015) ^[2] obtained the approximate expression for the risks of the Stein rule and the usual estimator for regression error variance under this function. Dube *et al.* (2015) ^[5] examined the performance properties of the disturbance variance estimator under the LINEX loss function. Recently Kumar (2020), (2022) ^[8, 9] analyzed the Distribution and studied the performance properties of pre-test estimator of disturbance variance in linear regression model. Here, we will investigate properties of the usual estimator of disturbance variance estimator and restricted error variance under precautionary loss function.

The plan of the paper is as follows: Section 2 describes the model and the estimators. Section 3 provides the risk function of the estimators of disturbance variance under the precautionary

loss function. The risk comparison of the estimators of disturbance variance, results and conclusion is discussed for different parameters in Section 4. In appendix, a brief derivation of the risk function of estimators under the precautionary loss function is given.

2. The model and the estimators

Consider the standard linear regression model.

$$Y = X\beta + \varepsilon \quad (2.1)$$

where Y is a vector that includes information on the response variable, X is a matrix of observations on the explanatory variables, β is a vector of unknown parameters related to the regressors, and ε is a vector of disturbances. It is assumed that the components of ε are independently and identically distributed, each having a normal distribution with a variance of σ^2 and a mean of zero.

$E(\varepsilon) = 0, E(\varepsilon\varepsilon') = \sigma^2 I_n$, where σ^2 is finite and unknown.

The classical estimation of the linear regression model makes use of the least squares estimation procedure for the estimation of the coefficients in the model. When estimating, least squares approaches produce the best linear unbiased estimators, meaning that the resulting estimator is easily understandable and has the least variance around the true parametric value.

Using the principle of least squares, the following estimator is used.

$$b = (X'X)^{-1}X'Y \quad (2.2)$$

Consider a scenario in which the linear regression model has constraints on the parameters based on prior knowledge about one or more of the parameters. This prior information could be used to improve estimators' quality. If this earlier data is accurate, then the model's linear constraints on the coefficients as

$$R\beta = r \quad (2.3)$$

where R is a $q \times p$, ($q < p$) matrix of known elements with rank q , q being the number of constraints applied to the coefficients and r is $q \times 1$ vector of known elements.

The restricted least squares estimator of β using (2.1) and (2.3) is given by

$$b_r = b + (X'X)^{-1}R'(R(X'X)^{-1}R')^{-1}(r - Rb) \quad (2.4)$$

The usual and restricted estimator of error variance is one entirely based on the sample information, and the other utilizes both sample and prior information. The following estimators are constructed.

$$s^2 = \frac{e'e}{m}, \quad s_R^2 = \frac{e_R'e_R}{m'}$$

Here e and e_R the residual vector obtained using ordinary least squares and the restricted least squares estimator, respectively, and m and m' are the arbitrary scalars characterizing the estimators given by

$$e = Y - Xb, \quad e_R = Y - Xb_r, \\ m = v + \theta, \quad m' = v + q + \theta, \quad \text{where } v = n - p \text{ is a degree of freedom.} \quad (2.5)$$

The bias and MSE for s^2

$$\text{Bias}(s^2) = -\sigma^2 \frac{\theta}{v+\theta} \text{ and } \text{MSE}(s^2) = \frac{\sigma^4}{m^2}(\theta^2 + 2v)$$

Similarly, for s_R^2 given by

$$\text{Bias}(s_R^2) = -\sigma^2 \frac{\theta}{v+q+\theta} \text{ and } \text{MSE}(s_R^2) = \frac{\sigma^4}{(v+q+\theta)^2}(\theta^2 + 2(v+q))$$

3. Risk of variance estimator under precautionary loss function

The precautionary loss function is expressed by $L(\hat{\alpha}, \alpha) = \frac{(\alpha - \hat{\alpha})^2}{\hat{\alpha}}$, where α is population parameter is estimated by $\hat{\alpha}$. To estimate σ^2 by s^2 and s_R^2 under this function, the following expression will be solved

$$R[L(s^2, \sigma^2)] = E\left[\frac{(\sigma^2 - s^2)^2}{s^2}\right] = E[s^2] + \sigma^4 E\left[\frac{1}{s^2}\right] - 2\sigma^2 \quad (3.1)$$

$$R[L(s_R^2, \sigma^2)] = E \left[\frac{(\sigma^2 - s_R^2)^2}{s_R^2} \right] = E[s_R^2] + \sigma^4 E \left[\frac{1}{s_R^2} \right] - 2\sigma^2 \quad (3.2)$$

Theorem 1. If the linear regression model assumes normally distributed errors, the risk function of the estimator s^2 under the precautionary loss function is as follows:

$$R[L(s^2, \sigma^2)] = \sigma^2 \left[\frac{v}{m} + \frac{m}{(v-2)} - 2 \right] \quad (3.3)$$

Proof: See Appendix.

Theorem 2. If the linear regression model assumes normally distributed errors, the risk function of the estimator s_R^2 under the precautionary loss function is as follows:

$$R[L(s_R^2, \sigma^2)] = \sigma^2 \left[\frac{(v+q+2\lambda)}{m'} + m' \sum_{i=0}^{\infty} \frac{e^{-\lambda} \lambda^i}{i!} \frac{1}{v+q+2(i-1)} - 2 \right] \quad (3.4)$$

Proof: See Appendix.

It is easy to compute optimum value of θ denoted by θ_{opt} from (3.4) which is given by

$$\theta_{opt} = \sqrt{v+q+2\lambda} \left(\frac{1}{\sum_{i=0}^{\infty} \frac{e^{-\lambda} \lambda^i}{i!} \frac{1}{v+q+2(i-1)}} \right) - (v+q)$$

$$R(s_R^2)_{\theta_{opt}} = 2\sigma^2 \left[\sqrt{v+q+2\lambda} \sum_{i=0}^{\infty} \frac{e^{-\lambda} \lambda^i}{i!} \frac{1}{v+q+2(i-1)} - 1 \right]$$

It can be easily verified that for $q = 0$, $\lambda = 0$ the risk of the restricted estimator reduces to the usual estimator of error variance which is given in (3.3) and optimum value for usual estimator is

$$\theta_{opt} = \sqrt{v(v-2)} - v \text{ and risk is } R(s^2)_{\theta_{opt}} = 2\sigma^2 \left[\sqrt{\frac{v}{v-2}} - 1 \right]$$

4. Numerically computation, results and conclusion

The performance of estimators assessed under different conditions numerically for different parameters λ, θ, q and v . The numerical results reveal the restricted estimator s_R^2 has lower risk compared to usual estimator s^2 under the precautionary loss function from Table 1-3. The usual estimator s^2 and the restricted estimator s_R^2 of error variance are unbiased estimators for $\theta = 0$ and have minimum MSE for $\theta = 2$, therefore, the performance of these estimators is also analyzed for $\theta = 0, \theta = 2$ and for optimum value (θ_{opt}) under precautionary loss function. The behavior of the risk of s_R^2 when $\sigma^2 = 1$, is summarized in figures 1-10, while the number of restrictions (q) and the degrees of freedom (v) vary. Figures 1 and 2 provide the risk of s_R^2 for $\theta = 0$, while Figures 3 and 4 correspond to $\theta = 2$, and Figures 5 and 6 illustrate the risks associated with the optimum value (θ_{opt}). From these figures, it is evident that the risks at $\theta = 0$ and $\theta = 2$ increase as the non-centrally parameter λ increases. The risk at the optimal value of θ , on the other hand, decreases as λ increases. The optimum choice offers higher stability, but fixed choices of θ are more susceptible to variations in λ . An additional trend in these figures is that risk decreases as the number of restrictions increases, which indicates the efficiency gained by the incorporation of additional information.

Moreover, as the degrees of freedom become larger, the risk functions for different values of q nearly coincide, suggesting that the effect of restrictions becomes less pronounced as the sample size or information increases. Figures 7 and 8 further demonstrate the risks for $\theta = 0, \theta = 2$, and θ_{opt} when $q = 1$ and $q = 5$ with $v = 10$. Similarly, Figures 9 and 10 shows the same patterns when $v = 50$. It is evident from these results that the risk is always minimal at the optimal value, θ_{opt} , and that it stabilizes as λ increases. When comparing $\theta = 0$ and $\theta = 2$, the point for alteration is approximately at $\lambda = 1$. There is a smaller risk at $\theta = 0$ below this value and at $\theta = 2$ higher. Overall, these findings suggest that the optimal value, θ_{opt} , consistently offers the lowest and most stable risk across all situations, and the choice of θ should be based on the magnitude of the non-centrality parameter λ . This shows that the optimal choice of θ is more reliable than fixed selections of θ .

Table 1: Risk of s_R^2 for different values of σ^2, λ, q and $v = 10$

q	$\sigma^2 = 0.1$				$\sigma^2 = 0.5$			$\sigma^2 = 1$		
	λ	$\theta = 0$	$\theta = 2$	θ_{opt}	$\theta = 0$	$\theta = 2$	θ_{opt}	$\theta = 0$	$\theta = 2$	θ_{opt}
0	0	0.025	0.033	0.024	0.125	0.167	0.118	0.250	0.333	0.236
1	0	0.022	0.029	0.021	0.111	0.145	0.106	0.222	0.291	0.211
	0.1	0.022	0.028	0.021	0.109	0.140	0.106	0.219	0.280	0.211
	0.25	0.021	0.027	0.021	0.107	0.133	0.105	0.214	0.266	0.211
	0.5	0.021	0.025	0.021	0.105	0.123	0.105	0.210	0.246	0.210
	1	0.021	0.022	0.021	0.106	0.109	0.103	0.212	0.217	0.207
	2	0.025	0.020	0.020	0.124	0.100	0.099	0.249	0.200	0.197
	5	0.052	0.034	0.016	0.261	0.170	0.082	0.522	0.340	0.163
	10	0.122	0.086	0.012	0.608	0.428	0.060	1.217	0.856	0.120
	20	0.287	0.220	0.008	1.434	1.099	0.038	2.869	2.198	0.076
	50	0.819	0.666	0.004	4.097	3.330	0.018	8.194	6.660	0.036
5	0	0.015	0.019	0.015	0.077	0.095	0.074	0.154	0.190	0.148
	0.1	0.015	0.018	0.015	0.076	0.092	0.074	0.152	0.185	0.148
	0.25	0.015	0.018	0.015	0.075	0.089	0.074	0.150	0.177	0.148
	0.5	0.015	0.017	0.015	0.074	0.083	0.074	0.148	0.167	0.148
	1	0.015	0.015	0.015	0.075	0.076	0.073	0.150	0.152	0.147
	2	0.017	0.014	0.014	0.086	0.072	0.071	0.172	0.144	0.142
	5	0.034	0.024	0.013	0.172	0.119	0.063	0.344	0.238	0.125
	10	0.081	0.059	0.010	0.403	0.297	0.050	0.805	0.594	0.099
	20	0.196	0.157	0.007	0.979	0.783	0.034	1.958	1.566	0.068
	50	0.580	0.492	0.003	2.901	2.459	0.017	5.802	4.918	0.034
10	0	0.011	0.013	0.011	0.056	0.066	0.054	0.111	0.131	0.108
	0.1	0.011	0.013	0.011	0.055	0.064	0.054	0.110	0.128	0.108
	0.25	0.011	0.012	0.011	0.054	0.062	0.054	0.109	0.124	0.108
	0.5	0.011	0.012	0.011	0.054	0.059	0.054	0.108	0.118	0.108
	1	0.011	0.011	0.011	0.055	0.055	0.054	0.109	0.110	0.107
	2	0.012	0.011	0.011	0.062	0.053	0.053	0.123	0.107	0.105
	5	0.023	0.017	0.010	0.116	0.085	0.048	0.233	0.169	0.096
	10	0.054	0.041	0.008	0.271	0.207	0.041	0.541	0.414	0.081
	20	0.135	0.112	0.006	0.677	0.558	0.030	1.353	1.116	0.059
	50	0.417	0.364	0.003	2.086	1.822	0.016	4.172	3.644	0.032

Table 2: Risk of s_R^2 for different values of $\sigma^2, \theta, \lambda, q$ and $v = 20$

q	$\sigma^2 = 0.1$				$\sigma^2 = 0.5$			$\sigma^2 = 1$		
	λ	$\theta = 0$	$\theta = 2$	θ_{opt}	$\theta = 0$	$\theta = 2$	θ_{opt}	$\theta = 0$	$\theta = 2$	θ_{opt}
0	0	0.011	0.013	0.011	0.056	0.066	0.054	0.111	0.131	0.108
1	0	0.011	0.012	0.010	0.053	0.062	0.051	0.105	0.124	0.103
	0.1	0.010	0.012	0.010	0.052	0.060	0.051	0.104	0.121	0.103
	0.25	0.010	0.012	0.010	0.052	0.059	0.051	0.103	0.117	0.103
	0.5	0.010	0.011	0.010	0.051	0.056	0.051	0.102	0.112	0.102
	1	0.010	0.010	0.010	0.052	0.052	0.051	0.104	0.105	0.102
	2	0.012	0.010	0.010	0.058	0.051	0.050	0.117	0.101	0.100
	5	0.022	0.016	0.009	0.109	0.080	0.046	0.218	0.160	0.092
	10	0.051	0.039	0.008	0.253	0.194	0.039	0.505	0.388	0.078
	20	0.127	0.105	0.006	0.635	0.526	0.029	1.269	1.051	0.058
	50	0.394	0.346	0.003	1.970	1.728	0.016	3.941	3.457	0.031
5	0	0.009	0.010	0.009	0.043	0.050	0.043	0.087	0.100	0.085
	0.1	0.009	0.010	0.009	0.043	0.049	0.043	0.086	0.098	0.085
	0.25	0.009	0.010	0.009	0.043	0.048	0.043	0.086	0.095	0.085
	0.5	0.009	0.009	0.009	0.043	0.046	0.043	0.085	0.092	0.085
	1	0.009	0.009	0.008	0.043	0.043	0.042	0.086	0.087	0.085
	2	0.010	0.008	0.008	0.048	0.042	0.042	0.096	0.085	0.084
	5	0.017	0.013	0.008	0.086	0.065	0.039	0.171	0.129	0.078
	10	0.039	0.031	0.007	0.197	0.154	0.034	0.394	0.308	0.068
	20	0.101	0.084	0.005	0.503	0.422	0.026	1.005	0.845	0.052
	50	0.321	0.285	0.003	1.603	1.426	0.015	3.206	2.852	0.030
10	0	0.007	0.008	0.007	0.036	0.040	0.035	0.071	0.080	0.070
	0.1	0.007	0.008	0.007	0.035	0.040	0.035	0.071	0.079	0.070

	0.25	0.007	0.008	0.007	0.035	0.039	0.035	0.071	0.077	0.070
	0.5	0.007	0.007	0.007	0.035	0.037	0.035	0.070	0.075	0.070
	1	0.007	0.007	0.007	0.035	0.036	0.035	0.071	0.071	0.070
	2	0.008	0.007	0.007	0.039	0.035	0.035	0.078	0.070	0.069
	5	0.013	0.010	0.007	0.067	0.052	0.033	0.134	0.104	0.066
	10	0.030	0.024	0.006	0.151	0.120	0.030	0.303	0.241	0.059
	20	0.078	0.067	0.005	0.391	0.333	0.024	0.782	0.666	0.047
	50	0.257	0.232	0.003	1.285	1.158	0.014	2.571	2.316	0.028

Table 3: Risk of s_R^2 for different values of $\sigma^2, \theta, \lambda, q$ and $v = 50$

q	$\sigma^2 = 0.1$				$\sigma^2 = 0.5$			$\sigma^2 = 1$		
	λ	$\theta = 0$	$\theta = 2$	θ_{opt}	$\theta = 0$	$\theta = 2$	θ_{opt}	$\theta = 0$	$\theta = 2$	θ_{opt}
0	0	0.004	0.004	0.004	0.021	0.022	0.021	0.042	0.045	0.041
1	0	0.004	0.004	0.004	0.020	0.022	0.020	0.041	0.044	0.040
	0.1	0.004	0.004	0.004	0.020	0.022	0.020	0.041	0.043	0.040
	0.25	0.004	0.004	0.004	0.020	0.021	0.020	0.041	0.043	0.040
	0.5	0.004	0.004	0.004	0.020	0.021	0.020	0.040	0.042	0.040
	1	0.004	0.004	0.004	0.020	0.020	0.020	0.041	0.041	0.040
	2	0.004	0.004	0.004	0.022	0.020	0.020	0.043	0.041	0.040
	5	0.007	0.005	0.004	0.033	0.027	0.020	0.065	0.054	0.039
	10	0.014	0.011	0.004	0.069	0.057	0.019	0.137	0.114	0.037
	20	0.036	0.032	0.003	0.182	0.159	0.016	0.363	0.319	0.033
	50	0.131	0.121	0.002	0.653	0.604	0.011	1.306	1.208	0.023
5	0	0.004	0.004	0.004	0.019	0.020	0.019	0.038	0.040	0.037
	0.1	0.004	0.004	0.004	0.019	0.020	0.019	0.038	0.040	0.037
	0.25	0.004	0.004	0.004	0.019	0.020	0.019	0.037	0.039	0.037
	0.5	0.004	0.004	0.004	0.019	0.019	0.019	0.037	0.039	0.037
	1	0.004	0.004	0.004	0.019	0.019	0.019	0.038	0.038	0.037
	2	0.004	0.004	0.004	0.020	0.019	0.019	0.040	0.037	0.037
	5	0.006	0.005	0.004	0.030	0.025	0.018	0.059	0.050	0.037
	10	0.012	0.010	0.003	0.061	0.051	0.017	0.123	0.102	0.035
	20	0.032	0.029	0.003	0.162	0.143	0.015	0.324	0.285	0.031
	50	0.118	0.110	0.002	0.590	0.548	0.011	1.181	1.095	0.022
10	0	0.003	0.004	0.003	0.017	0.018	0.017	0.034	0.037	0.034
	0.1	0.003	0.004	0.003	0.017	0.018	0.017	0.034	0.036	0.034
	0.25	0.003	0.004	0.003	0.017	0.018	0.017	0.034	0.036	0.034
	0.5	0.003	0.004	0.003	0.017	0.018	0.017	0.034	0.035	0.034
	1	0.003	0.003	0.003	0.017	0.017	0.017	0.034	0.034	0.034
	2	0.004	0.003	0.003	0.018	0.017	0.017	0.036	0.034	0.034
	5	0.005	0.004	0.003	0.026	0.022	0.017	0.053	0.045	0.034
	10	0.011	0.009	0.003	0.054	0.045	0.016	0.108	0.090	0.032
	20	0.028	0.025	0.003	0.142	0.125	0.014	0.284	0.251	0.029
	50	0.105	0.098	0.002	0.525	0.488	0.010	1.049	0.976	0.021

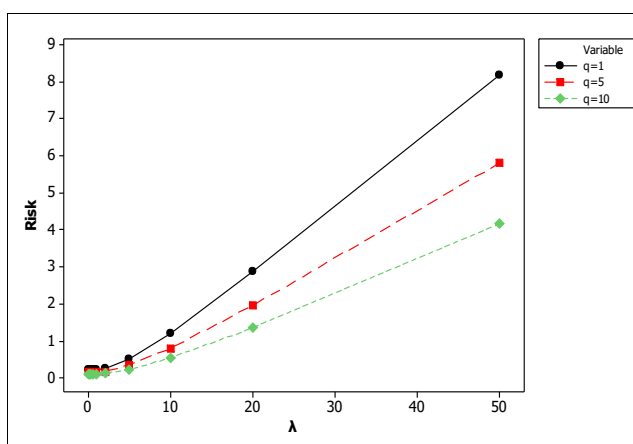
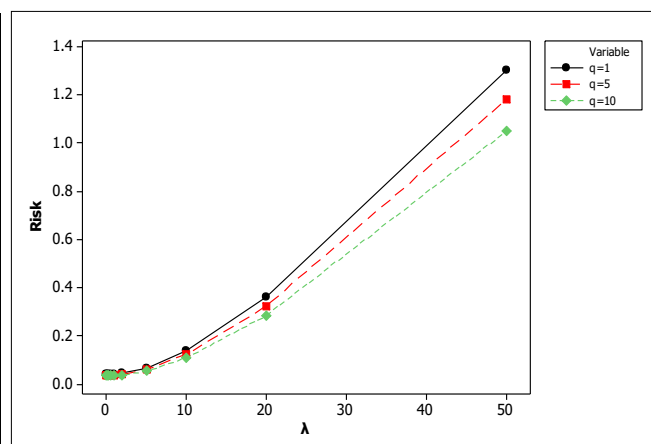
**Fig 1:** $v=10$ **Fig 2:** $v=50$

Fig 1: and Fig 2: show the risk of s_R^2 for different value $\lambda, q, \theta = 0$ and $\sigma^2 = 1$

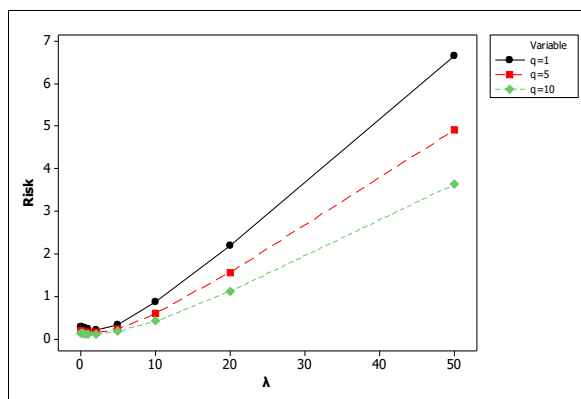
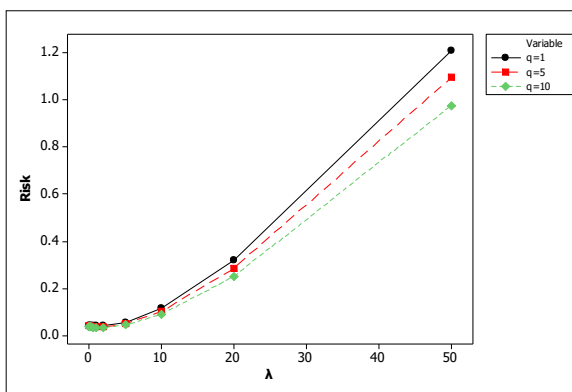
Fig 3: $v=10$,Fig 4: $v=50$

Fig 3: and Fig 4: show the risk of s_R^2 for different values of $\lambda, q, \theta = 2$ and $\sigma^2 = 1$

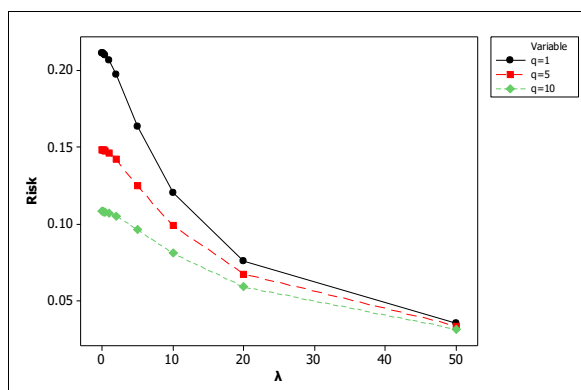
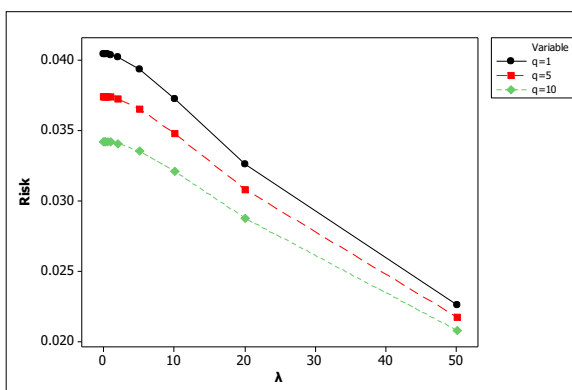
Fig 5: $v=10$,Fig 6: $v=50$

Fig 5: and Fig 6: show the risk of s_R^2 for different values of λ, q, θ_{opt} and $\sigma^2 = 1$

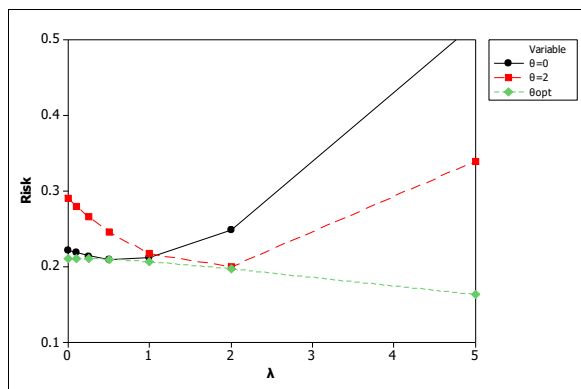
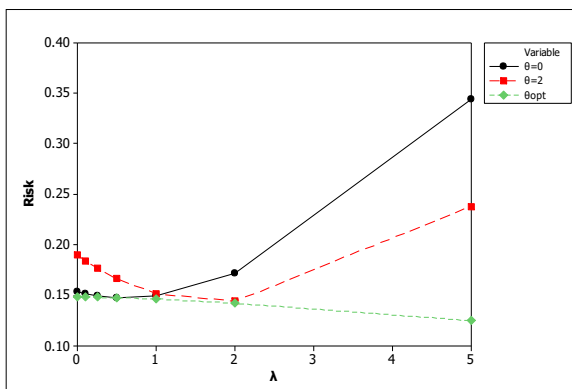
Fig 7: $q=1$ Fig 8: $q=5$

Fig 7 and Fig 8: show risk of s_R^2 for different values of $\lambda, q, \theta, v = 10$ and $\sigma^2 = 1$

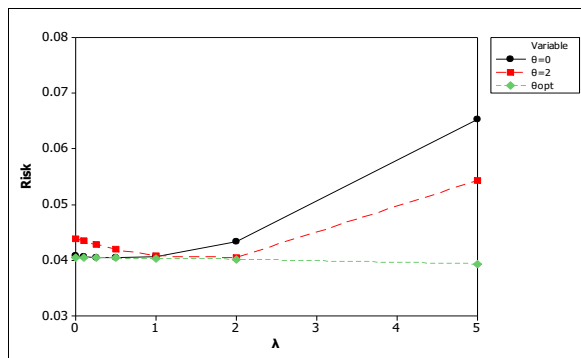
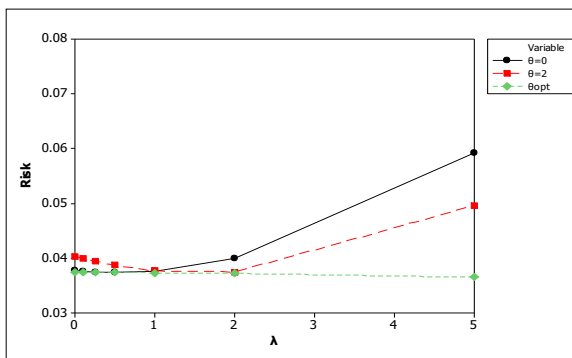
Fig 9: $q=1$ Fig 10: $q=5$

Fig 9 and Fig 10: show risk of s_R^2 for different values of $\lambda, q, \theta, v = 50$ and $\sigma^2 = 1$

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Appendix

Proof of Theorem 1

To estimate σ^2 by s^2 under this function, s^2 could be express in the following form

$s^2 = \frac{\varepsilon' \bar{P}_X \varepsilon}{m}$, where $\bar{P}_X = I - X(X'X)^{-1}X'$ is an idempotent matrix of $v = n - p$, so that

$$E[s^2] = \frac{1}{m} \sigma^2 v \quad (\text{A.1})$$

Now to find the value of $E\left[\frac{1}{s^2}\right]$, We have $s^2 = \frac{\varepsilon' \bar{P}_X \varepsilon}{m}$

$\frac{\varepsilon' \bar{P}_X \varepsilon}{\sigma^2} \sim \chi_v^2$ i.e., it follows χ_v^2 with v degree of freedom, therefore $\frac{ms^2}{\sigma^2} \sim \chi_v^2$ and $\frac{1}{m s^2} \sim \text{inv}(\chi^2)_v$

Now, for random variables which follow inverse chi-square distribution i.e., $X \sim \text{inv}(\chi^2)_v$

$$E\left[\frac{1}{s^2}\right] = \frac{m}{\sigma^2(v-2)} \quad (\text{A.2})$$

Using equations (A.1) and (A.2) in equation (3.1), we get an expression for the risk of s^2 in (3.3).

Proof of Theorem 2

Using the properties of quadratic forms, it can be easily deduced that

$$\frac{(v+q)}{\sigma^2} S_R^2 \sim \chi^2_{(v+q, \lambda)}$$

The probability density function of non-central χ^2 distribution with $v + q$ degree of freedom and non-centrality parameter λ is given by

$$f(\chi^2_{(v+q, \lambda)}) = \sum_{i=0}^{\infty} w_i(\lambda) \frac{1}{2^{\frac{v+q}{2}} \Gamma(\frac{v+q}{2} + i)} \int_0^{\infty} (\chi^2)^{\frac{v+q}{2} + i - 1} e^{-\frac{1}{2}\chi^2} d\chi^2$$

By using the probability density function, the following expression could be obtained:

$$E[S_R^2] = \frac{\sigma^2(v+q+2\lambda)}{m'} \quad (\text{A.3})$$

In order to solve, $E\left[\frac{1}{S_R^2}\right] = \int_0^{\infty} \frac{1}{s_R^2} f(s_R^2) ds_R^2$, let us take $\frac{(v+q)}{\sigma^2} S_R^2 = t_1$ and $\frac{\sigma^2}{(v+q)} \frac{1}{s_R^2} = \frac{1}{t_1}$

$$\begin{aligned} E\left[\frac{1}{S_R^2}\right] &= \frac{(v+q)}{\sigma^2} \int_0^{\infty} \frac{1}{t_1} f(t_1) dt_1 \\ &= \frac{m'}{\sigma^2} \sum_{i=0}^{\infty} w_i(\lambda) \frac{1}{2^{\frac{v+q}{2}} \Gamma(\frac{v+q}{2} + i)} \int_0^{\infty} (t_1)^{\frac{v+q}{2} + i - 1} e^{-\frac{1}{2}t_1} dt_1 \end{aligned}$$

$$E\left[\frac{1}{S_R^2}\right] = \frac{m'}{\sigma^2} \sum_{i=0}^{\infty} \frac{e^{-\lambda} \lambda^i}{i!} \frac{1}{(v+q+2(i-1))} \quad (\text{A.4})$$

Using equation (A.3) and (A.4) in equation (3.2), we get expression for risk of s_R^2 in (3.4).