

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
NAAS Rating (2026): 4.49
Maths 2026; 11(1): 28-34
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<https://www.mathsjournal.com>
Received: 12-10-2025
Accepted: 23-11-2025

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Annihilator in ternary semiring

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DOI: <https://www.doi.org/10.22271/math.2026.v11.i1a.2237>

Abstract

We investigate the concept of Moore-Penrose inverses by relating annihilator and ideal in Ternary Semiring. We discussed some outcomes and also characterized the annihilator using Moore Penrose inverses.

Keywords: Regular, annihilator, g- inverses, ternary semiring

1. Introduction

In 1932, Lehmer ^[1] explored ternary algebraic system theory. He studied “triplexes”, which are ternary algebraic systems that finally turn out to be ternary groups. A generalization of ternary rings are called ternary semirings, as presented by Dutta *et al* ^[2]. Ternary semiring is a superset of semiring. Negative integers form a ternary semiring but it does not form a semiring.

In 1920, E.H. Moore established the concept of the generalized inverse of a matrix, defining the unique generalized inverse through the use of matrices projectors. Penrose demonstrated in 1955 that Moore’s inverse is the unique matrix that can satisfy four equations ^[3]. The notion of regular element was first introduced by Von Neumann in 1936 ^[4]. We present some properties of generalized inverses, elements in Ternary semirings that we use throughout the article and particular generalized inverses are defined or studied by means of their associated principal or annihilator ideals. The several characterizations of these inverses are given. Some new properties are obtained and some known results are generalized.

2. Preliminaries

In this work, T represents the Ternary Semiring; it is abbreviated as T.S.R. throughout this paper.

Definition 2.1. The term “ternary semiring” refers to $T \neq \emptyset$, a binary addition, and a ternary multiplication called juxtaposition if $(T, +)$ is a semigroup that commutes that fulfilling the requirements listed below:

- $(abc)de = a(bcd)e = ab(cde)$,
- $(a + b)cd = acd + bcd$,
- $a(b + c)d = abd + acd$,
- $ab(c + d) = abc + abd$, for all a, b, d, d, e in T .

Remark: $\mathbb{Z}^-$ is a natural example of a ternary semiring, which has ternary multiplication and regular addition.

Definition 2.2. If an element a exists in T such that $ababa = a$, then a is considered regular. The element b is represented as a^- and is also known as the generalized inverse or 1-inverse of the element ' a ' and the set of all 1-inverses is denoted by $a\{1\}$. If every element in a T.S.R is regular, then we say T.S.R is regular.

Definition 2.3. If there is an element a in $T \exists babab = b$, then a is considered anti-regular. The name for element b is 2-inverse of a and set of all 2-inverses of a is denoted by $a\{2\}$.

The both conditions in Definition (2.2) and Definition (2.3) satisfied by a single element b is called 1-2 inverse and is denoted by $\hat{a}$. In Definition (2.2), if the ' b ' is unit then ' a ' is called unit regular.

Definition 2.4. An element $a \in T$ has Moore penrose inverse if there exists an $b \in T$ such that the following equations hold:

$$(1) \ ababa = a \quad (2) \ babab = b \quad (3) \ (abe)^* = abe \quad (4) \ (eba)^* = eba.$$

Where e is identity element, here b is unique and denoted by $a^\dagger$.

In Definition (2.4), the element b is satisfy the equations (1) and (3), then we say that element is $\{1,3\}$ -inverse of ' a ' and b satisfy the equation (1) and (4) then we say that b is $\{1,4\}$ -inverse of ' a '.

Definition 2.5. An involutuin of $*$ of T is an involuturory anti-automormhism,

$$i.e., (a^*)^* = a, (a + b)^* = a^* + b^*, (abc)^* = c^* b^* a^*,$$

$$a^* = 0 \text{ iff } a = 0 \forall a, b, c \text{ in } T$$

Definition 2.6. An element $m \in T$ and the right annihilator of m in T is defined by $m^\circ = \{x \in T | xmt = 0 \text{ for some } t \in T\}$.

Similarly defined the left annihilator

$${}^\circ m = \{x \in T | xtm = 0 \text{ for some } t \in T\}.$$

Definition 2.7. For any $t_1, t_2 \in T$ and $i \in I$, an additive subsemigroup I of T is considered a left (right, lateral) ideal of T if $t_1 t_2 i$ (similarly $i t_1 t_2, t_1 i t_2$) $\in I$. I is said to as an ideal T if I is both a left, right, and lateral ideal of T .

Results in ternary semiring

Proposition 3.1. ^[6] Let $m, m^- \in T$, Then the following holds:

- $m^\circ = (1 - m^- m m^-)T = (m^- m 1 m^-)^\circ$
- ${}^\circ m = T(1 - m m^- m m^-) = {}^\circ (m m^- 1 m m^-)$ Where 1 is multiplicative identity.

Proof:

$$(i) \quad m^\circ = \{(1 - m^- m m^-)x : m(1 - m^- m m^-)x = 0 \text{ for some } x \in T\}$$

$$= m 1 x - m^- m m^- m x$$

$$\Rightarrow (1 - m^- m m^-)T;$$

$$m^\circ = (1 - m^- m m^-)T \text{ and}$$

$$(m^- m 1 m^-)^\circ = \{(1 - m^- m m^-)x : m^- m m^- m(1 - m^- m m^-)x = 0$$

$$\Rightarrow m^- m m^- m x - m^- m m^- m m^- m x = 0$$

$$(m^- m 1 m^-)^\circ = (1 - m^- m m^-)T$$

$$\therefore m^\circ = (1 - m^- m m^-)T = (m^- m 1 m^-)^\circ$$

$$(ii) \quad {}^\circ m = \{x(1 - m m^- m m^-) : (x - x m m^- m m^-)m = 0 \text{ for some } x \in T\}$$

$$x 1 m - x m m^- m m^- m = 0$$

$${}^\circ m = T(1 - m m^- m m^-) \text{ and}$$

$${}^\circ (m m^- 1 m m^-) = \{x - x m m^- m m^- : (x - x m m^- m m^-)m m^- m m^- = 0\}$$

$$\Rightarrow T(1 - m m^- m m^-)$$

$$\therefore {}^\circ m = T(1 - m m^- m m^-) = {}^\circ (m m^- 1 m m^-)$$

Proposition.3.2 ^[6] Let $m \in T$. The the following are holds

- y is a 1-3 inverse of $m \Leftrightarrow m^* y^* m^* m y = m^*$
- y is a 1-4 inverse of $m \Leftrightarrow m y m m^* y^* = m$
- y is a 2-3 inverse of $m \Leftrightarrow y m y y^* m^* = y$
- y is a 2-4 inverse of $m \Leftrightarrow y^* m^* y^* y m = y^*$.

Proof:

The Moore-Penrose inverse(y)

$$mymym = m \text{ --- (1)}$$

$$ymymy = y \text{ --- (2)}$$

$$(mye)^* = mye \text{ --- (3)}$$

$$(yme)^* = yme \text{ --- (4)}$$

$$\text{Using (1) and (3), } mymym = m \Rightarrow (mye)(mye)m = m$$

$$\Rightarrow (mye)^*myem = m \Rightarrow (ey^*m^*myem)^* = m^*$$

$$\Rightarrow m^*y^*m^*my = m^* \text{ --- (5)}$$

$$\text{Using (1) and (4), } myem(yme)^* = m \Rightarrow myeme^*m^*y^* = m$$

$$\Rightarrow mymm^*y^* = m \text{ --- (6)}$$

$$\text{Using (2) and (3), } ymyemye = y \Rightarrow ymye(mye)^* = y$$

$$\Rightarrow ymyee^*y^*m^* = y$$

$$\Rightarrow ymyy^*m^* = y \text{ --- (7)}$$

$$\text{Using (2) and (4), } ymeymey = y \Rightarrow (yme)^*ymey = y$$

$$e^*m^*y^*ymey = y \Rightarrow (m^*y^*ymy)^* = y^*$$

$$\Rightarrow y^*m^*y^*ym = y^* \text{ --- (8)}$$

From Equations (5), (6), (7) and (8) we get

$$y \text{ is a 1-3 inverse of } m \Leftrightarrow m^*y^*m^*my = m^*$$

$$y \text{ is a 1-4 inverse of } m \Leftrightarrow mymm^*y^* = m$$

$$y \text{ is a 2-3 inverse of } m \Leftrightarrow ymyy^*m^* = y$$

$$y \text{ is a 2-4 inverse of } m \Leftrightarrow y^*m^*y^*ym = y^*.$$

Proposition 3.3.^[7] Let $m \in T$. Then we have the following results:

1. m is $\{1,3\}$ invertible with $\{1,3\}$ inverse y iff $y^*m^*mym = m$
2. m is $\{1,4\}$ invertible with $\{1,4\}$ inverse y iff $mxmm^*y^* = m$.

Theorem 3.1.^[8] Let $m, n \in T$ and $n^- \in n\{1\}$ Then

$$Tem \subseteq Tn^-n \text{ iff } n^\circ \subseteq m^\circ$$

Proof: Suppose $Tem \subseteq Tn^-n$

There exist $t \in T$ such that $m = tn^-n$ and $n^\circ = \{x | nsx = 0 \text{ for some } s \in T\}$

$$\text{So we have } msx = 0 \Rightarrow (tn^-n)sx = tn^-(nsx) = 0$$

$$\therefore n^\circ \subseteq m^\circ$$

Converse,

Since $n \in T$ is regular and $n^\circ \subseteq m^\circ$

$$ns(m - nn^-n) = 0$$

$$\text{We have } ms(m - nn^-n) = 0 \Rightarrow msm - msnn^-n = 0$$

$$msm = msnn^-n$$

$$\therefore Tem \subseteq Tn^{-}n$$

Theorem 3.2. ^[8] Let $m, y \in T$. Then $meT \subseteq nn^{-}T$ iff ${}^{\circ}n \subseteq {}^{\circ}m$

Proof: Take $meT \subseteq nn^{-}T \exists t \in T$ such that $m = nn^{-}t$ and

$${}^{\circ}n = \{x \in T | xsn = 0 \text{ for some } s \in T\}$$

$$\text{So we have } xsm = xs(nn^{-}t) = (xsn)n^{-}t = 0$$

$$\therefore {}^{\circ}n \subseteq {}^{\circ}m$$

Conversely,

Since ${}^{\circ}n \subseteq {}^{\circ}m$, n is regular

$$\text{suppose } {}^{\circ}n = (m - nn^{-}n)sn = 0, \text{ we have } (m - nn^{-}n)sm = 0$$

$$msm - nn^{-}nsm = 0 \Rightarrow msm = nm n^{-}nsm$$

$$\Rightarrow meT \subseteq nn^{-}T$$

Theorem 3.3. ^[8] Let $m, y \in T$. The following statements are equivalent

1. $yeT = mm^{-}T$ and $Tey = Tm^{-}m$ for $m^{-} \in m\{1\}$
2. ${}^{\circ}y = {}^{\circ}m$ and $y^{\circ} = m^{\circ}$
3. $yeT \subseteq mm^{-}T$ and $Tey \subseteq Tm^{-}m$
4. ${}^{\circ}m \subseteq {}^{\circ}y$ and $m^{\circ} \subseteq y^{\circ}$

Proof: (i) $\Rightarrow$ (ii) $yeT = mm^{-}T$ and $Tey = Tm^{-}m$

$$\text{Since } {}^{\circ}y = \{x | xsy = 0\}, {}^{\circ}m = \{x | xsm = 0\}$$

$$yeT = mm^{-}T \Rightarrow y = mm^{-}u \text{ or } yeu = m \text{ (} m^{-} \text{ is unity)}$$

$$Tey = Tm^{-}m \Rightarrow y = um^{-}m \text{ or } uey = m$$

$${}^{\circ}y = xsy = xs(um^{-}m) = x(sum^{-})m = xtm = {}^{\circ}m \text{ where } t = sum^{-}$$

$$\therefore {}^{\circ}y = {}^{\circ}m$$

$$y^{\circ} = ysx = (mm^{-}u)sx = m(m^{-}us)x = mvx = m^{\circ} \text{ where } v = m^{-}us$$

$$\therefore y^{\circ} = m^{\circ}$$

$$(ii) \Rightarrow (iii) {}^{\circ}y = {}^{\circ}m \Rightarrow xsy = x(sum^{-})m$$

$$\Rightarrow Tey \subseteq Tm^{-}m \text{ and } y^{\circ} = m^{\circ} \Rightarrow ysx = m(m^{-}us)x$$

$$\Rightarrow yeT \subseteq mm^{-}T$$

$$(iii) \Rightarrow (iv) Tey \subseteq Tm^{-}m, \exists u \in T \text{ such that } y = um^{-}m \text{ and by using definition of annihilator of } y, ysx = 0 \text{ for some } s, x \in T$$

$$\text{So } um^{-}msx = um^{-}(msx) = 0$$

$$\therefore m^{\circ} \subseteq y^{\circ}$$

Similarly, we get ${}^{\circ}m \subseteq {}^{\circ}y$

$$(iv) \Rightarrow (i) {}^{\circ}y = (y - mm^{-}m)sy = 0 \Rightarrow ysy = mm^{-}msy \Rightarrow yeT \subseteq mm^{-}T$$

$$\text{Similarly, } Tey = Tm^{-}m$$

Theorem 3.4. ^[8] Let $m, y \in T$, The following statements are equivalent

1. y is Moore-Penrose inverse of m
2. $yeT = m^{*}y^{*}T$ and $Tem^{*} = Tmy$

3. ${}^\circ y = {}^\circ(m^*)$ and $y^\circ \subseteq (m^*)^\circ$
4. $yeT \subseteq m^*y^*T$ and $Tem^* \subseteq Tmy$
5. ${}^\circ(m^*) \subseteq {}^\circ y \subseteq$ and $(m^*)^\circ \subseteq y^\circ$

Proof: (i) $\Rightarrow$ (ii)

From the equations (5),(6), (7) and (8) $m^*y^*m^*my = m^*$, $ymyy^*m^* = y$

$$m^* = m^*y^*m^*my \Rightarrow Tem^* = Tmy$$

$$y = m^*y^*ymy \Rightarrow yeT = m^*y^*T$$

(ii) $\Rightarrow$ (iii) $yeT = m^*y^*T \Rightarrow y = m^*y^*u$ (for some $u \in T$)

$$Tem^* = Tmy \Rightarrow m^* = umy$$

Since ${}^\circ y$, ${}^\circ m$ are annihilator of y and m ,

$$\Rightarrow xsm^* = xs(umy) = (xsu)m^* = tmy = 0, \text{ for some } xsu = t \in T$$

$$\therefore {}^\circ y = {}^\circ(m^*)$$

$$yeT = m^*y^*T \Rightarrow m^* = yeu \quad (\because m^*y^*T = m^*eT = m^*)$$

$$m^*sx = (yeu)sx = ye(usx) = yev = 0 \text{ for some } v \in T$$

$$\therefore (m^*)^\circ = y^\circ$$

(iii) $\Rightarrow$ (iv) Let ${}^\circ y = \{m^*y^*u \mid (m^*y^*u)xy = 0\}$ and

$${}^\circ(m^*) = \{yeu \mid (yeu)xm^* = 0\}$$

We know that ${}^\circ(m^*) = {}^\circ y \Rightarrow (yeu)xm^* = (m^*y^*u)xy$

$$\Rightarrow yeT \subseteq m^*y^*T$$

Similarly $y^\circ = \{uy^*m^* \mid yx(uy^*m^*) = 0\}$ and

$$(m^*)^\circ = \{umy \mid m^*x(umy) = 0\}$$

$$\therefore (m^*)^\circ = y^\circ \Rightarrow m^*x(umy) = yx(uy^*m^*)$$

$$\Rightarrow Tem^* \subseteq Tmy$$

(iv) $\Rightarrow$ (v) Take $Tem^* \subseteq Tmy$ and $xsm^* = 0$ for some $s, x \in T$

$$\exists u \in T \text{ such that } y = m^*eu \Rightarrow xsy = (xsm^*)eu = 0$$

$$\therefore {}^\circ(m^*) \subseteq {}^\circ y$$

Similarly, we get $(m^*)^\circ \subseteq y^\circ$.

(v) $\Rightarrow$ (i) Suppose that $mymym = m \Rightarrow (mymym)^* = m^*$

$$\Rightarrow m^*y^*m^*y^*m^* = m^* \text{ --- (9)}$$

Follows (9), we see that $m^*(1 - y^*m^*y^*m^*) = 0$ and $(1 - m^*y^*m^*y^*)m^* = 0$

$$\Rightarrow m^*(1 - y^*m^*y^*m^*) \in (m^*)^\circ \subseteq y^\circ \text{ and}$$

$$(1 - m^*y^*m^*y^*)m^* \in {}^\circ(m^*) \subseteq {}^\circ y$$

Therefore, $y = ymyy^*m^*$ and $y = m^*y^*ymy$

$$\Rightarrow mye = m(ymyy^*m^*)e = emym(ymyy^*m^*)y^*m^* = emyy^*m^*y^*m^* = ey^*m^* = (mye)^*$$

$$\Rightarrow yme = y(my m m^* y^*)e = uymy(my m m^* y^*)m^* y^* = eym m^* y^* m^* y^* = em^* y^* = (yme)^*$$

$$\text{Finally, } ymymy = ymymymy(my)^* = ymyy^* m^* = y$$

$$\therefore y = m^\dagger$$

Lemma 3.1. ^[9] Let $m, n \in T$ be such that $n\{1\} \neq \emptyset$. Then

1. If $n^\circ \subseteq m^\circ$ and $Ten \subseteq Tem$, then $m\{1\} \neq \emptyset$
2. If ${}^\circ n \subseteq {}^\circ m$ and $neT \subseteq meT$, then $m\{1\} \neq \emptyset$

Proof: (i) Let $x \in n\{1\}$. Since $n^\circ \subseteq m^\circ$ and we have $S(1 - xnxn) \in n^\circ$

$$\text{We have } m(1 - xnxn) = 0 \Rightarrow m - mxnxn = 0$$

$$m = mxnxn \in mTnTn \subseteq mTmTm$$

$$\therefore m\{1\} \neq \emptyset.$$

(ii) since ${}^\circ n \subseteq {}^\circ m$ and $x \in n\{1\}$ and $(1 - nxnx) \in {}^\circ n$

$$\text{We have } (1 - nxnx)m = 0 \Rightarrow m - nxnxm = 0$$

$$m = nxnxm \in nTnTm \subseteq mTmTm$$

$$\therefore m\{1\} \neq \emptyset$$

Lemma 3.2. ^[9] Let $m, n \in T$ be such that $n\{1\} \neq \emptyset$.

1. For $n^\circ = m^\circ$ and $Ten \subseteq Tem$, iff $m\{1\} \neq \emptyset$
2. For ${}^\circ n = {}^\circ m$ and $neT \subseteq meT$ iff $m\{1\} \neq \emptyset$

Remark 3.1. ^[9] Let T be regular Ternary semiring and $m \in T$. Then the following are holds:

1. If $x \in m\{1\}$ and $xeT = xmT$ then $x \in m\{1, 2\}$
2. If $x \in m\{2\}$ and $meT = mxT$, then $x \in m\{1, 2\}$

Proof: (i) Given $xeT = xmT \Rightarrow x = xmxmy$ for some $y \in T$ — — — (10)

Pre multiply 'm' on both sides in (10), we get

$$mxe = mxmxmye = mye$$

$$\therefore mxe = mye \text{ — — — (11)}$$

$$x = xmx(my) \Rightarrow x = xmxmx$$

$$\therefore x \in m\{2\}.$$

Hence $x \in m\{1, 2\}$.

(ii) $meT = mxT \Rightarrow m = mxmxy$ — — — (12)

Pre multiply 'x' on both sides in (11),

$$xme = xmxmxy = xye$$

$$\therefore xme = xye \text{ — — — (13)}$$

Substitute (13) in (12) $m = mxmxm$

$$\therefore x \in m\{1\}.$$

Hence $x \in m\{1, 2\}$.

Theorem 3.5 below can be seen as a generalization of [6, Theorem 2.13, pg-72].

Theorem 3.5. ^[10] Let $m, n, p \in T$ such that $(mnp)^- \in mnp\{1\}$. Then

1. $mn(mnp)^-mn(mnp)^-m = m \Leftrightarrow mnT = meT \Leftrightarrow {}^\circ(mnp) = {}^\circ m$
2. $p(mnp)^-np(mnp)^-np = p \Leftrightarrow Tnp = Tep \Leftrightarrow (mnp)^- = p^\circ$

Proof: Take $u = n(mnp)^-e$

$$\text{Since } mn(mnp)^-mn(mnp)^-m = m \text{ --- (14)}$$

$$mue = mn(mnp)^-mn(mnp)^-mue$$

$$\Rightarrow mn(mnp)^-e = mn(mnp)^-mn(mnp)^-mn(mnp)^-e$$

$$\Rightarrow mnT \subseteq meT \text{ --- (i)}$$

$$\text{From (14) we get } meT \subseteq mnT \text{ --- (ii)}$$

$$\text{From (i) and (ii) we get } mnT = meT$$

$$\text{To prove } mnT = meT \Leftrightarrow {}^\circ(mnp) = {}^\circ m$$

$$\because {}^\circ m = \{u: utm = 0\} \text{ and } {}^\circ(mnp) = \{u: utmnp = 0\}$$

$$\text{From (i) } mnp = mxe \text{ for some } x \in T$$

$$\Rightarrow utmnp = utmxe = (utm)xe = 0$$

$$\therefore {}^\circ(mnp) \subseteq {}^\circ m \text{ --- (iv)}$$

$$\text{From (iii) and (iv) we get } {}^\circ(mnp) = {}^\circ m$$

$$(2) \text{ we always have } p(mnp)^-np(mnp)^-np = p \Rightarrow Tnp = Tep \text{ --- (v)}$$

$$\text{From (v) } mnp = p(mnp)^-np(mnp)^-np$$

$$\Rightarrow mnptu = p(mnp)^-np(mnp)^-nptu$$

$$\Rightarrow mnptu = p(mnp)^-np(mnp)^-n(ptu) = 0$$

$$\therefore p^\circ \subseteq (mnp)^\circ$$

$$\text{Similarly, we get } (mnp)^\circ \subseteq p^\circ$$

$$\text{Hence } (mnp)^\circ = p^\circ.$$

4. Conclusion

In this paper, we characterize annihilators in ternary semirings by means of appropriate ideals and Moore-Penrose inverses.

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