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Mathematician as a beholder of beauty in natural manifestation of god

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Abstract

Making an endeavour in exploring Mathematics is usually motivated by the desire to reach a beautiful result or to prove it in an elegant way. Mathematician's pieces of work are thus, ideally, strongly influenced by the aesthetic and natural sense of judgments. Whatever criteria are there for these judgments to be based on remain somehow unclear for a few people of non mathematicians background. Mathematics, ideally, has never been known to be vague and fuzzy in its judgement for its own pure nature of true characteristics. In this review/survey article, we make concerted efforts to focus on the concepts of mathematically related beautiful objects, as one of the central aesthetic concepts in mathematics creativity. The beauty in mathematics reveals connections between apparently non-related problems or areas, and provides a better and wider insight into mathematical reality on a whole basis for reaching a rational judgement with the true due expectations. We study some beautiful mathematical concepts such as Golden Ratio, Fibonacci sequence, Symmetry, Honeycomb Algebra, Fractals, Chaos Theory, and π .

Keywords: God, Allah, nature, golden ratio, Fibonacci, honeycomb algebra, symmetry, fractals, Chaos, π

Introduction

We start this section with the following quotations:

“As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality.”-Albert Einstein

“The laws of nature are the mathematical thoughts of God”. Euclid.

Mathematics is usually considered a purely disciplinary subject of abstract in nature inclusive of particular instances fulfilling its stipulated conditions and ever searching as well as researching innovative ideas for more such instances fulfilling its defining conditions for the development of the subjects. It remarkably manifests itself in the natural world phenomena. Exhibiting this relationship on mathematical concepts, underpins the structures and dynamics of the subject (s) which can be observed in nature.

It is interesting to note what Bertrand Russell^[17] observes about Mathematics:

“Mathematics, rightly viewed, possesses not only truth, but supreme beauty- a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show.” One can observe that mathematics is everywhere in Nature. It is in the objects one constructs like in the pieces of art work. Although it may go unnoticed, mathematics is also present in the nature that surrounds the environment, in its landscapes and species of plants and animals, including the fellow human. Mathematicians like to talk about the beauty in mathematics that is the harmony, patterns, and structures of numbers and forms as the ideals of balance and symmetry.

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While mathematicians can envision tangible representations of notations on a piece of paper, mathematical beauty is not so well realized by non-mathematicians.

G. H. Hardy ^[1] defended the pursuit of mathematics for its own sake. He authored the most eloquent endeavor pieces of work to provide non-mathematicians a sense of math's aesthetic taste, in his "A Mathematician's Apology", a 1940 essay. As with the letters between the Weil siblings, it was the war that occasioned and shaped Hardy's book, prompting him to argue that mathematics has an intrinsic beauty unrelated to any military purpose with a formal justification or defense like that 'Plato's Apology of Socrates' provided as arguments that mathematics is equipped with value independent of its applications which he called beauty of mathematics ^[47].

Amanha ^[27] writes about the beauty of Geometry in Mathematics

"The Stanford Dictionary of Philosophy reminds us that in Timaeus, the work of Greek philosopher Plato (429-347 BCE or Before the Common Era), for example, regular geometric forms take center stage in the doctrine of natural elements because of the proportions they contain and the beauty of their forms. The four elements-Fire, Water, Earth and Air-could be represented by regular geometric shapes (with polyhedrons of four, twenty, six and eight equal sides, respectively). The God's Universe could also be represented by a 12-sided polyhedron or a symmetrical dodecahedron."

Furthermore, he writes about beauty in Mathematics

"The Greeks believed there to be three elements of beauty: "symmetry, proportion, and harmony. The perfect balance of these elements created the Golden Mean, which was found repeatedly in nature and was aspired to in art and architecture. Trying to explain work to a non-mathematician would be like trying to explain a symphony to someone who can't hear. Later he would rely on another metaphor, calling math "art in a hard material."

Moreover, Olsson ^[1] writes about beauty in Mathematics

"Philosophers can argue whether beauty is the property of an object or lies in our perceptions of it; Hardy would have it both ways. The best mathematics is eternal, he maintained, and like the best literature, it will "continue to cause intense emotional satisfaction to thousands of people after thousands of years."

There are some amazing consistent patterns observed in natural world that mathematics can help one understand for all of its elegant and beautiful objects, and appealingly spontaneous occurrences. One can see to realize that these natural patterns literally shape the world around. It is seen that nature is replete with sequences, patterns, and structures that exhibit an astonishing level of mathematical elegance as is evident from the spirals of galaxies, the branching of trees, the pattern of petals on a flower to the hive structure of bees. Thus mathematics beautifully permeates the natural world in surprisingly innovative ways. Also, it has been observed that research in neuroscience gives the idea what is called "emotional satisfaction."

"Positive psychology recognizes happiness as a construct comprising hedonic and eudaimonic well-being dimensions" ^[48]. The intersection of mathematics, emotional satisfaction and neuroscience shows that the brain makes processing of aesthetic and real world complex and complicated problem-solving capacity building rewards in specified emotional

regions. Mathematical models are applied to study and predict emotional dynamics.

Rota ^[3] observed that the lack of beauty in a piece of mathematics is of usual occurrence. It strongly motivates for further research in mathematics. The most obvious source of beauty in mathematics is pattern, structure, and symmetry which can be found in pieces of art work. Mathematics in its pure form is abstract. As a matter of fact, this abstractedness itself adds to the beauty of mathematics. These characteristics of abstractness include the expression of generality, the simplicity and economy of expression in mathematical identities and equations. One can feel the pleasing aspects of mathematics that is surprise and ingenuity in logic, and interconnections between ideas in mathematics that appear naturally beautiful. The application of mathematical modeling to capture these and other aspects of the natural world can be breathtaking. It also demonstrates its power with elegance which is sometimes termed as a dimension of mathematical beauty on its own.

M. Teicher ^[4] writes about mathematical beauty in architecture

"The most impressive mosaics in history were done by artists working in the Islamic World in the middle ages, and, in particular, those who created the beautiful, sophisticated mosaic of the Alhambra Palace in Spain. The Alhambra was built by the Moors in the beginning of the thirteen century, on a red-soil hill overlooking the old city of Granada. It is a showcase of Muslim architecture and design with an immense wealth of patterns, ornaments, calligraphy, and stone carvings. It includes virtual night-sky ceilings crafted from thousands of pieces of wood, as well as the most sophisticated, gorgeous, colorful mosaics."

Beside teaching students about mathematics concepts such as formula and its derivation, theorems, proposition, lemma and procedures for its proof, teachers can guide them to explore it as a creative field of study. As a well known fact, mathematics has always been vital to the growth of science and technology. Things such as finding planets, construction, industrial development, technological achievements, philosophy, engineering, biology, and the arts make use of mathematics. One can observe that mesmerizing ripples in a pond, a tranquil walk through a densely grown forest areas, praising the elegant tilings in the structures of honeycomb, physiological and emotional responses are naturally intertwined with these marvelous designs in mathematics.

In ancient times, human beings grasped the power and attractiveness of natural patterns and structures recognition. One may think that's why the ancient Pueblo communities could design complex geometric structures without a number system and alphabet at the time. Patterns that occur in nature, like fractals and the Fibonacci sequence, are timeless and universal.

Feelings of calm, reduction in stress, cognitive stimulation and functional tendencies can be induced by these natural surrounding environmental factors. Environmental parameters that lack these organic motifs can usually leave one feeling anxious, disoriented, or unduly overwhelmed. DNA is required to be in sync with the rhythms and patterns of the natural world laced itself with a spiral pattern. When one takes a look at the natural patterns found in wood like a complex fractal or a series of cracks, one makes realization of beauty. The wellness benefits that these natural patterns can have in it to provide are amazing opportunities in mathematical designs and structures. One can keep on

performing research or do survey study to review knowledge about the natural patterns does have in it to make impactful positivity on one's well-being in fostering relaxation, reducing stress and connecting one to the life's web cycle.

One can find the sense that either a web is being spun to catch the maximum number of flies or if it's with the stripes to escape from predators or even prey. The most effective patterns are tested and either rejected or accepted over time. The best possible patterns are carried forward in various ways. The challenge is that as prey adapts to escape from predators, they (predators) must also adapt to survive. In this situation, nature God plays the role of main catalyst as technology. As technology makes progress, certain human requirements for technology do change too accordingly.

In his book *Patterns in Nature*; author Philip Ball ^[30] summarized the effect of patterns

"Natural patterns offer raw delights, but they also point to something deep." Patterns have so many uses in the natural world. Whether it's related to survival or structural integrity, patterns have developed over time to become more and more useful due to evolution. As creatures developed and changed over time, the patterns they create and the patterns that they wear have become fitter and fitter for their purposes."

Ball ^[21] further writes

"Turing's mathematical prowess was combined with an inventiveness of mind that made him a pioneer of artificial intelligence and the theory of computation. It was essentially a mathematical question that led Turing to think about chemical patterns. He further writes: The appearance of an organism's body plan is called morphogenesis, and it seemed to be an example of spontaneous symmetry-breaking: from uniform to differentiated. During his tragically short life that began 100 years ago, Alan Turing wrote only one paper about Chemistry. He was essentially a pure mathematician. Turing showed how chemical reactions can create patterns" ^[21].

Mathematical and scientific treatises by Aryabhata and Bhaskara encoded advanced knowledge centuries before Europe's Renaissance ^[37]. The article on mathematics related concepts and results, i.e., mathematicians' think tank could very well be serving for a 'revitalization' of the subject and the associated subjects as well, especially in view of NEP 2020 for teaching, research and development ^[33]. For Aryabhata's pioneer contribution in various dimensions of Mathematics, one can refer ^[34], and furthermore for Indian Knowledge System (IKS) explored, one can refer ^[35]. Moreover, one can explore the role of Mathematics in nature by films and visuals in ^[5].

Golden Ratio

Golden ratio, in mathematics, is the irrational number $(1 + \sqrt{5})/2$ which is a positive solution of the equation $x^2 - x - 1 = 0$. It is denoted by the Greek letter ϕ (Phi) and sometimes by τ (tau) which is approximately equal to 1.618.

It is known to be constructed using classical geometric methods, that is, straightedge and compass constructions. As a matter of fact, the ratio is related to Fibonacci sequence upon dividing a Fibonacci number by the one before it gives results that get progressively closer to ϕ . It is well known that it is found in the study of fractals, and figures that exhibit self-similarity, chaos and dynamical systems. It can also be expressed in the form of an infinite continued fraction.

Furthermore, one can find that Golden ratio is also known to be defined as the ratio of a line segment divided into two

pieces of different lengths such that the ratio of the whole segment to that of the longer segment is equal to the ratio of the longer segment to the shorter one. Moreover, two quantities are said to be in the golden ratio if their ratio is equal to the ratio of their sum to the larger of the two quantities.

In terms of algebraic expression, one can find that for quantities a and b with $a > b > 0$, a is in a golden ratio to b if $(a + b)/a = a/b = \phi$, where ϕ is the Greek letter phi. One can find that the origin of this number can be traced as the "extreme and mean ratio" in the Book 6, Definition 3 of Euclid. In terms of present day elementary algebra, letting the length of the shorter segment by one unit and the length of the longer segment be x unit gives rise to the quadratic equation $x^2 - x - 1 = 0$.

One can see in ^[2] that as a natural manifestation of God in Mathematics, the Golden Ratio can be observed in the arrangement of leaves, the patterns of hurricanes, and even in the proportions of the human body. This ratio creates a sense of balance and harmony that is visually and structurally sound. The Golden Ratio can be obtained from the Fibonacci sequence, where each number is the sum of the two preceding ones. This sequence manifests in a lot of natural phenomena, showcasing the connection between mathematics and the natural world phenomena.

One can find in ^[31] the golden ratio in some natural growth patterns. Spirals in Sunflower heads and pinecones usually count as consecutive Fibonacci numbers. These spirals are serving as support system for leaves or seeds to pack most efficiently around a stem. It also appears in design and pieces of art work. Spanish artist Salvador Dali constructed the proportions and the background dodecahedron of his 1955 painting *The Sacrament of the Last Supper* on the golden ratio. Swiss architect Le Corbusier developed the modular, a scale of human based measurement using Phi, and applied it in projects such as the housing complex La Cite Radieuse in Marseille.

Fibonacci sequence

Fibonacci ^[32] posed the following question as follows:

"In the West, the Fibonacci sequence first appears in the book *Liber Abaci* (1202) by Leonardo of Pisa, known as Fibonacci. Fibonacci considers the growth of an idealized (biologically unrealistic) rabbit population, assuming that:

1. A single newly born pair of rabbits (one male, one female) are put in a field;
2. Rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits;
3. Rabbits never die and a mating pair always produces one new pair (one male, one female) every month from the second month on.

The puzzle that Fibonacci posed was: how many pairs will there be in one year?"

T. Ghose ^[16] writes about this series as follows

"The Fibonacci sequence is a famous mathematical sequence where each number is the sum of the two preceding ones. People think there are many special properties about the numerical sequence, such as the fact that it is "nature's secret code" for building perfect structures, like the Great Pyramid at Giza or the iconic seashell that likely graced the cover of your school mathematics textbook. But much of that is more

myth than fact, and the true history of the series is a bit more down-to-earth."

Series of numbers that form the Fibonacci sequence F_n , is known as Fibonacci numbers. It is well known that the sequence begins with 0 and 1, and from 1 and 1 too by some authors. Fibonacci himself and some other authors started it from 1 and 2. In 200 BC predating Fibonacci by centuries, these numbers were first discovered in Indian fields of mathematics by Pingala on reciting likely patterns of Sanskrit poetry designed from lengths two syllables. It is named after the Italian mathematician Leonardo of Pisa (posthumously known as Fibonacci), who got the sequence introduced to Western European mathematics fraternity in his book 'Liber Abaci' (1202) connecting it to real-world related problems with its amazing natural appearance like that in snail shells/galaxies and sunflower spirals continuously fascinating people of genres.

Applications of Fibonacci numbers can be found in computer science in algorithms such as the Fibonacci search technique and the Fibonacci heaps in data structure, cryptography, and the Fibonacci cubes graphs for relating parallel and distributed systems. Its applications can be found in natural settings like plant growth, the arrangement of leaves on a stem, branching in trees, the fruit sprouts of a pineapple, the flowering of an artichoke, the arrangement of a pine cone's petals, finance like that market analysis with retracements, architecture/art like that making pleasing proportions, Taj Mahal, Golden Ratio, music/AI and in Hindu rituals like that decoding-coding the data, and in mother's womb (about his/her baby's situated position).

Fibonacci sequence is related to the golden ratio. Binet's formula expresses the n th Fibonacci number in terms of n and the golden ratio. It implies that the ratio of two consecutive Fibonacci numbers produces the golden ratio as the value of n increases. They are also intimately related to Lucas numbers. Lucas numbers follow the same recurrence relation, and with the Fibonacci sequence of numbers, make a complementary pair of Lucas sequences.

For applications of Fibonacci sequence, one can refer ^[38] and the references in ^[38].

Mathematics and Symmetry

Group theory in 'Modern (Abstract) Algebra' of mathematics is the study of symmetries in objects exploring its dimensions and facets in distinct settings of mathematics. It has in it to tie together many of the diverse topics such as sets, cardinality, isomorphism, number theory, modular arithmetic, molecules with symmetry operations that is an action like rotations, reflections, inversions, and elements like axes, points, lines, planes, and centers.

A point group helps predict chemical behaviours like chirality, polarity, bonding, spectroscopy, and so on so forth. It is to be noted that a point group is a collection of all symmetry elements for a molecule for the purpose of classification of its overall symmetry. Furthermore, it highlights the deeply rooted unity of contemporary mathematics. Moreover, it has one of the strikingly significant and beautiful thematic approaches to unify many areas of modern mathematics. Furthermore, symmetry is a type of invariance that is the characteristics that a mathematical object remains unchanged under a set of operations or transformations ^[18, 19].

Intuitive idea emanating from symmetry makes one think of certain patterns or shapes that is symmetric about harmony, balance, and repetition, where an object looks like the same

after a transformation like flipping (reflection), turning (rotation), or sliding which shows that its halves parts mirror or match each other in perfect proportion. In other words, a shape or an object is said to be equipped with symmetry if it can be divided into two identical pieces, that is, one-half is the mirror image of the other half. It creates beauty in nature, art, and design like, for instance, a butterfly's wings or a heart folded in half. An object is asymmetrical if it is not symmetrical.

A line (axis of symmetry) divides a thing (symmetrical) into two matching pieces like a heart, or letters 'A', 'H', etc (vertical) and 'D', 'H', etc (horizontal). Reflecting in mirror, one half of an object is a perfect reflection (mirror image) of the other one like one's face or a butterfly. Things such as a perfect circle, a sphere, galaxies, sunflower and human creations like logos, art, and music, etc are pleasing human creations in sensible symmetry. A starfish as well as a perfect square possesses a reflectional symmetry whereas a pinwheel, starfish and a snowflake are equipped with rotational symmetry. There is recognizable repeating like patterns such as wall paper or tiles that are laced with translational symmetry. There is a spiritual symmetry too that is the feeling of one's action with what one spiritually implied to do, that is, a synchronization of one's action and one's spirituality.

The principles of symmetry are basically comprehending planetary dynamics and composition framework. The hemispheric albedo balance, even if unusual, is vital for Earth's climate, and its breakdown signals change in climate conditions. It is studied that the Northern Hemisphere is absorbing around 0.34 watt more solar energy per square meter per decade than the Southern Hemisphere ^[40]. An intriguing characteristics of the albedo is that, on average, Northern or Southern Hemisphere (NH or SH) have the same albedo which is known as hemispheric symmetry. The symmetry is surprising for SH has much more ocean than land, and ocean has lesser reflection than land implying thereby that NH should have higher albedo. Clouds, which also tune the albedo, compensate the surface albedo disorder of the two hemispheres, leading to an overall symmetric albedo ^[41].

The earth (irregular surfaces) which is not a perfect sphere, rather an oblate spheroid, shows symmetry but not true symmetry. It is a well known fact that the earth spins around an axis (North pole to South pole) which seems to exhibit rotational symmetry. It bulges at the equator and is flattened at the poles deviating from perfect symmetry. Furthermore, mountains, trenches, oceans varying crustal densities break its perfect symmetry. Recent data shows that the Northern Hemisphere is absorbing more sunlight disrupting the long-standing hemispheric albedo symmetry impacting the climate. ^[39].

The imaginary axis or a line along which the figure is folded to obtain its symmetrical counterparts halves is called the line of symmetry. That is, the line (axis) cuts a shape into two exact or identical halves are the line of symmetry. Furthermore, this line of symmetry in shape occurs along one axis only. Moreover, multiple lines of symmetry can generally be applied to many types of regular polygons. It is to be recalled here that a regular polygon is one whose all the sides and interior angles are equal ^[18, 19].

In case of a structured object S , symmetry is a mapping of the object onto itself. If S is a set with no more additional structure, symmetry is a bijective map from the set to itself which gives the concept and motivation of defining conditions of an abstract group called here permutation groups under

functions compositions. A bijective function from a set to itself is called a permutation. If the number of symbols (elements) (cardinality) in S are finite, then this set of all one to one correspondence from S to S forms a group under the binary operations of composition of functions, called symmetric group S_n .

In Metric Space, symmetry is defined to be a distance preserving bijection of the set to itself between each pair of points if the object S is a set of points in the plane equipped with its metric structure. It is to be noted here that the graph of an even function is symmetric with respect to the Y -axis. Also, the graph of an odd function has rotational symmetry with respect to the origin for its graph remains unchanged after rotation of 180 degrees about the origin.

In Matrix Algebra (Theory of Matrices) of Mathematics, a symmetric matrix is a square matrix that is equal to its transpose. It is to be recalled here that only square matrices can be symmetric. Furthermore, the entries of a symmetric matrix are symmetric with respect to the main (principal) diagonal elements. Obviously, every square diagonal matrix is symmetric, since its all non-diagonal entries are zero. It can be calculated by definition that a skew-symmetric matrix has all its diagonal elements equal to be zero. A self-adjoint operator is represented by a real symmetric matrix over a real 'Inner Product Space'. The analogous object for a complex inner product space is a Hermitian matrix with complex-valued entries, which is equal to its conjugate transpose. Thereby, over the complex number system, a symmetric matrix has real-valued entries. Symmetric matrices appear naturally in a variety of applications that is studied with the help of Numerical Linear Algebra software.

A symmetric polynomial is a polynomial in n variables. If any of its variables are interchanged, one obtains the same polynomial. A symmetric polynomial expression in the roots of a monic polynomial can also be described by a polynomial expression in terms of the coefficients of the polynomial. It is well known in the 'Theory of Equation' of Mathematics, that symmetric polynomials arise naturally in the study of the relation between the roots of a polynomial in one variable and its coefficients. The coefficients of polynomials (its literals/variables/unknowns) is expressible in form of its roots. The symmetric polynomials are the most basic ones in nature. It is worth noting here that a monic polynomial is a non-zero polynomial where the coefficient (leading) of its highest degree term is 1.

One can refer to see that the symmetric group S_n on a finite set of n symbols is the group whose elements are all the permutations of the n symbols under the binary operation of the composition of such permutations. As there are $n!$ (i.e., n factorial) possible permutations on a set of n symbols, it follows that the order of the symmetric group S_n comes out to be $n!$. Furthermore, since every finite group G may be embedded in some symmetric group, the regular representation embeds G in S_n , where $n = \text{cardinality of } G$. Moreover, one can find this reason to state that finite group theory is equivalent to the study of subgroups of the symmetric groups [22].

It may be observed that some of the roots are connected by algebraic equations in a polynomial. The main idea of Galois theory is to consider those permutations possessing the roots such that any algebraic equation satisfied by the roots is still satisfied after the roots have been permuted. The motivation for studying Galois theory are existence of polynomials which are not soluble by radicals, and some well known results about classical geometry like that one cannot trisect an angle

by ruler and compass, and that certain regular polygons cannot be constructed using a ruler and compass. In this connection, those algebraic equations (polynomials) are considered whose coefficients are rational numbers. Thus, Galois Theory studies the inherent symmetries found in algebraic equations (expressions) [42].

As a matter of fact, one can find natural isomorphism between the space of symmetric tensors of rank r on a finite-dimensional vector space V and the dual of the space of homogeneous polynomials of degree r on V . It is well known that a symmetric tensor is one that possesses invariance property with a permutation of its vector arguments. Furthermore, in fields of characteristic zero, the graded vector space of all symmetric tensors can naturally be found with the symmetric algebra on the given vector space. A related notion is that of the anti symmetric tensor or the alternating one. Moreover, symmetric tensors are found in the disciplines of study such as physics, engineering, and mathematics.

In physics of natural science, there is the extremely powerful Noether's symmetry theorem which states that each symmetry of a system leads to quantity that is physically conserved. Symmetry under translation corresponds to momentum conservation, symmetry under rotation to angular momentum conservation, symmetry in time to energy conservation, and so on so forth [24].

An automorphism, in Modern (Abstract) Algebra of Mathematics, is an isomorphism from a mathematical object to itself with the defined structure preserving characteristics. Furthermore, the set of all automorphism of an object forms a group which is known as the automorphism group under function composition as a binary operation. Moreover, The fundamental particle Bosons in quantum physics, are expressed by symmetric relation with respect to permutation operators whereas fermions have anti symmetric wave function. Bosons symmetry follows the fundamental quantum rule that identical bosons have a symmetric wave function implying that they are unchanged when their positions are swapped, allowing multiple bosons to occupy the exact same quantum state (like photons), unlike fermions. In fact, the Pauli Exclusion Principle with a single-valued many-particle wave function is stated to be equivalent to the antisymmetric wave function. This implies the Pauli Exclusion Principle for fermions (matter particles with half-integer spin). All particles fall into one of two classes, bosons or fermions [43].

One can find that an antisymmetric two-particle state is expressed as a sum of states in which one particle is, say in state $< x >$ and the other, say in state $< y >$. An anti symmetry satisfies the relation $A(x, y) = -A(y, x)$. This further implies that $A(x, x) = 0$, which is Pauli exclusion principle. If the diagonal entries $A(x, x)$ are zero in every basis, then the wave function component is necessarily antisymmetric. Moreover, in any basis for unitary changes of basis keep antisymmetric matrices in its original form. As a matter of fact, the expression $A(x, y)$ is not a matrix. However, it is an antisymmetric rank-two tensor in strict sense. It is well known fact that a relation is said to be symmetric if each time the relation satisfies from A to B , it also satisfy from B to A . It is noteworthy here that symmetry is not the exact reciprocal of anti symmetry.

It is well known that an isometry is a distance-preserving function between metric spaces. In case of a metric space, an isometry is a transformation which maps elements to another metric space such that the distance between elements in the new metric space is equal to the distance between elements in the original metric space. In the context of a two dimensional

or three-dimensional space, two geometric figures are congruent if they are connected by an isometry, that is, related by either a rigid motion, or a composition of a rigid motion and a reflection. They are equal if related by a direct isometry with respect to relation by a rigid motion. They have been employed to unify the working definition of symmetry in geometry and for functions, probability distributions, matrices, strings, graphs, etc. [20].

To help solve the differential equations, a symmetry of a differential equation is a invariant transformation that keeps the differential equation as it is. Symmetries may be found in course of solving a related set of ordinary differential equations. A line symmetry of a system of differential equations is a continuous symmetry of the system of differential equations. Furthermore, a line symmetry can be used to simplify an ordinary differential equation through order reduction. Moreover, for ordinary differential equations, a suitable set of Lie symmetries is used to find a set of first integrals which gives a complete solution without integration. Solving these equations is usually much easier than solving the original differential equations [21]. There are very few patterns in physics, biology and chemistry that Ball fails to examine [29].

For a finite number of possible outcomes, symmetry with respect to permutations implies the occurrence of discrete uniform distribution. Furthermore, for a likely outcomes in a real interval, symmetry with respect to interchanging sub-intervals of equal length makes a parallel continuous uniform distribution. One can find that in other circumstances like "considering a random integer" or "considering a random real number", there are no probability distributions at all symmetric with respect to giving different labels or to interchange of equally large subintervals. Moreover, other expected symmetries do not pick one particular distribution. To put it another way, there is not a unique probability distribution with maximum symmetry.

A likely symmetry for positive outcomes in randomness is that the prior applies for the logarithm, *i.e.*, for the desired outcome, and its reciprocal possess the same distribution. For a 'random point' in space or in a plane, one can choose an origin. Then one can consider a probability distribution with spherical or circular symmetry, accordingly. Furthermore, this symmetry does not uniquely separate any particular distribution. There is one type of isometry in one dimension that may keep the probability distribution unaltered, that is, reflection in a point, such as zero for instance.

Honeycomb Algebra

Algebra in Mathematics is a way of thinking about the structure of patterns and situational phenomena in the real world. It is a useful way of expressing in 'its language' the way a pattern or system consists of parts. It can also help one see to observe the outputs of a system or pattern. It involves algebraic equations to study the patterns and efficiency in hexagonal structures such as found in beehives, and with emphasis on notions like tessellation (tiling a plane without gaps), minimizing perimeter for a maximum area. There is a honeycomb conjecture (theorem) which states that a regular hexagonal grid or regular hexagonal tiling (like a honeycomb) is the most efficient way for bees to divide a plane into equal-area regions with the minimum total perimeter for area, minimizing material (space optimization). Furthermore, the honeycomb conjecture is a famous mathematical principle that explains why hexagonal (optimal shape) tiling is the most

efficient way to divide a plane into equal regions while minimizing perimeter.

In the 19th century, Charles Darwin also praised the honeycomb structure as an example of natural selection and evolution, suggesting that bees had evolved to build these efficient structures due to survival advantages [44].

Mankind has been long amazed by the honeycombs' hexagonal figures (regular hexagons/polygons). It is believed by mathematicians that honeybee use hexagons for maximum space with minimal wax for building honeycomb. Humans are interested in honeycomb because it contains honey, which one likes to eat. However, bees create honeycomb with great deal of energy to use them for a number of needs such as to make honey out of nectar, to store honey, to store pollen, to lay eggs and to raise young bees (larvae/brood) into adults. Its efficient, strong design also refers to any similar structure, like offices in a building or even specialized fabrics, and can be eaten whole, raw honey, pollen, waxy cells, and antimicrobial properties [45].

Cells are not hexagonal when they are built but rather circular [24]. Circular walls are caused by surface tension to pull into straight lines at the junctions where three cells meet to form hexagons. The six-sided shape with 120 degree angles between the walls allows external forces to let it be evenly distributed. This makes the honeycomb surprisingly strong and pressure resistant despite being made of a relatively thin material. Honeycomb cells are truly 'multipurpose spaces'. Furthermore, each cell in a section of honeycomb has a hexagonal opening and is in the shape of a type of irregular polyhedron called decahedra. Moreover, one can see that the honeycomb cell has 10 flat sides wherein the prefix 'deca-' means '10' and the suffix '-hedra' means 'geometric figure'. Also, all the flat sides of honeycomb cell except for the hexagonal opening are quadrilaterals.

It is well known that honeybees provide food for others, and make packing of their resources efficiently. The closed ends of the honeycomb cells are trihedral. It ensure that the back-to-back ends of honeycomb cells fit perfectly against each other and that the overall shape of the honeycomb cell minimizes surface area for a given volume. This means that the inside of the cell gives the bees maximum space using minimum building materials (*i.e.*, beeswax). Pappus of Alexandria declared that bees "possessed a divine sense of symmetry", and Charles Darwin described the honeycomb as a master piece of engineering that is "absolutely perfect in economizing labor and wax" [46].

A mathematical treatment was given by Jan Brozek and Thomas Hales [46]. In [46], one can refer to see that Kepler's Conjecture called the Cannonball Problem is about traditional patterns to stack oranges (or cannonballs) being optimal or not. Specifically, the arrangement of equal-sized spheres to fill a space with the minimum amount of wasted volume is the so-called close packing, which consists of layers of spheres whose centers form a hexagonal lattice. If equal-sized balls are dropped into a large container at random, experiments suggest that the density-the proportion of the container's volume filled by balls-is roughly 65 per cent [46]. The hexagonal close-packing arrangement gives an average density of $\pi/(3\sqrt{2}) \approx 74 \text{ percent}$. Efficient packing is the

name of the game. This problem attracted the attention of mathematical giants such as Carl Gauss, who proved a special case. In 1900, David Hilbert's address at the International Congress of Mathematics in Paris described 10 problems that he felt would strongly influence the direction of mathematical

research in the 20th century. The print version of his address includes 23 problems. Kepler's conjecture is part of Hilbert's Problem 18^[46].

Their pieces of work shed some light on the historical interest in the natural efficiency of the honeycomb, a problem spanning from ancient times to modern mathematics. However, simultaneously, the structure needs to be physically strong, leak-proof, resistant to attack from predators and well-suited to the range of purposes for which it was built being quite a difficult design brief for even the best of architects^[46]. In general, honeycombs are constructed vertically in order to be able to stand up firmly on their own help resist the downward pull of gravity. Hexagon polygons with the shared walls of each cell work to ensure that the downward physical stresses on the structure are spread evenly and uniformly. All this makes them very resilient. It is wondered whether this entomological architecture inspired the interior ribbing and hidden chambers in the dome of the Pantheon in Rome. Today honeycomb structures have tremendous engineering and scientific applications and scope, including in the aerospace industry^[46].

Fractals

In mathematics, a fractal is a geometric shape containing detailed structure at arbitrarily minute scales, generally having a fractal dimension strictly exceeding the topological dimension. Many fractals appear similar at various scales, as illustrated in successive magnifications of the Mandelbrot set. This exhibition of similar patterns at increasingly smaller scales is called self-similarity. It is also known as expanding symmetry or unfolding symmetry provided that this replication is exactly the same at every scale, as in the Menger sponge, the shape is called affine self-similar.

The "fractal" was coined by the mathematician Benoit Mandelbrot in 1975^[23]. He got it based on the Latin fractus, meaning "broken" or "fractured". It extends the concept of theoretical fractional dimensions to geometric patterns in nature. Fractals are geometric shapes that are 'self-similar' which appear identical at every scale. Fractals are not merely mathematical curiosities, but they are everywhere in nature. No matter how much one zoom in, one continue to see the same pattern repeated over and over again.

A fractal have a fractal dimension strictly exceeding the topological dimension. Many fractals appear similar at various scales, as illustrated in successive magnifications of the Mandelbrot set. This exhibition of similar patterns at increasingly smaller scales is called self-similarity, also known as expanding symmetry or unfolding symmetry; if this replication is exactly the same at every scale, as in the Menger sponge, the shape is called affine self-similar.

Fractal geometry relates to the mathematical branch of measure theory by their Hausdorff dimension^[23]. The branching patterns seen in rivers, trees, and our blood vessels are examples of fractals which can even be observed in the flight patterns of certain insects and birds. Another magical aspect mathematical fractal growth pattern of a tree is how the branches relate to the total thickness of the trunk. Trees grow so that the entire thickness of their branches at a particular height equals the thickness of their trunks. This occurrence was first recognized by no other than Leonardo Da Vinci. It has since then been hypothesized and observed by botanists to find an answer to why this is. Some thought the tree produced this pattern connected to how it pumps water from its roots to its leaves.

Ecologists can better understand the complexity of ecosystems and develop strategies for conservation and management. Fractals help us see the interconnections and scalability of natural systems. It's not merely an academic fascination that patterns resonate with us; they also have therapeutic effects. For instance, patterns, particularly fractals, are known stress reducers. A study from 2006 revealed that integrating fractals into architectural designs can significantly reduce stress up to 60. Another compelling piece of research highlighted the recuperative power of nature. Surgical patients with rooms offering views of nature had shorter hospital stays compared to those looking out on a mundane brick wall.

The feature of "self-similarity", for instance, is easily understood by analogy to zooming in with a lens or other device that zooms in on digital images to uncover finer, previously invisible, new structure. If this is done on fractals, however, no new detail appears; nothing changes and the same pattern repeats over and over, or for some fractals, nearly the same pattern reappears over and over. Self-similarity itself is not necessarily counter intuitive.

Chaos Theory

Chaos is the science of surprises. Furthermore, it teaches one to expect the unexpected. In general, traditional science deals with predictable phenomena like gravity, electricity, or chemical reactions. Chaos Theory deals with nonlinear things that are impossible to predict or control, like turbulence, weather, the stock market, our brain states, and so on so forth. These phenomena are usually described by fractal mathematics, which captures the infinite complexity of nature. Many natural things exhibit fractal properties, including landscapes, clouds, trees, organs, rivers etc, and many of the systems in which one lives exhibit complex, chaotic behavior.

Recognizing the chaotic, fractal nature of our world can give us new insight, power, and wisdom. For example, by understanding the complex, chaotic dynamics of the atmosphere, a balloon pilot can "steer" a balloon to a desired location. By understanding that our ecosystems, our social systems, and our economic systems are interconnected, we can hope to avoid actions which may end up being detrimental to our long-term well-being. Chaos theory is the extensive study of evidently random or uncertain behaviour in bodies or events controlled by deterministic laws. Deterministic chaos is a more accurate term which suggests a paradox as it links two concepts that are familiar and generally considered incompatible.

The primary notion is that of unpredictability or randomness, as in the path of a gas molecule, and in the voting pattern of a person from out of a certain population section. In conventional examinations, randomness was considered more apparent than something real. It came from the ignorance of numerous causes involved. Specifically, it was common to believe that the entire Universe is unpredictable because it is sophisticated and complex in all senses. The second is about deterministic motion, such as that of a planet or a pendulum, which has been believed to be true since the Isaac Newton era as illustrating the success of science in processing predictable phenomena which are initially complex.

Chaos theory describes there are inherent repetition, self-organization, patterns, interconnections, similarity with itself, and constant feedback mechanism within the visible randomness of complex, chaotic systems. The butterfly effect is an underlying aspect of chaos. It explains how a small fluctuation in one condition of a nonlinear deterministic

system can generate a huge difference in the later outcomes. It means that there is delicate dependence on beginning states. One can find a well known classic metaphor that a crow flapping its wings in India can cause a typhoon in Tokyo. Chaos Theory can further be explored in ^[10, 11, 12].

A Brief Story of π (Pi)

It is a well known fact that π (pi) is the ratio between circumference and diameter of a circle. It is found to be irrational, transcendental, and nonterminating which attracted mathematicians for centuries and fascinated mankind for thousands of years. Babylonian mathematicians studied the first known value of the area of a circle by the circumference of an inscribed hexagon in 2000 BCE. Furthermore, they derived an approximate value for pi at 3.125.

To approximate value of π (pi), in Egypt, the Rhind papyrus, dated circa 1650 BCE, a value of 256/81 around 3.16045 was used. Furthermore, around 250 BCE, Archimedes achieved a major breakthrough in finding a more accurate value of pi by the Pythagorean Theorem. He was able to approximate the area of a circle by comparing the areas of a circle inscribed polygon and one that surrounded it. He showed that there was a constant ratio between the area of a circle and the square of its radius. He could obtain an upper bound of 22/7 and a lower one at 223/71 for pi. One nice contributor to these findings was Chinese mathematician cum astronomer Zu Chongzhi, who obtained some nice approximation of π (pi).

Up to 1600, the value of π (pi) had been found up to 35 digit places using the method of inscribed polygons. In the seventeenth century, new analytical methodology were developed in mathematics for improved calculations of pi with infinite series like Gregory's series, etc.

One of the earliest formulae for finding the value of π (pi) was given by English mathematician John Wallis in 1656. In course of calculating an integral in an effort to find the area of a unit circle, he derived a formula involving the multiplication of an infinite series of fractions that was based on the value of one-half of π (pi). Later on, Sir Isaac Newton, in one of his many achievements, applied his binomial theorem to find the value of π (pi) up to 16 decimal places.

Then there came the Gregory-Leibniz series, which made use of infinite series expansions and trigonometric functions to obtain formulae for values of π (pi) divided by four and six. By the end of eighteenth century, this method found over one hundred digits of π (pi), and many more got derived out of these methods. The subsequent progress of more sophisticated computer system enabled that π could be calculated to higher and higher decimal places. The ENIAC computer system could find 2,037 digits of π up to 1949. A decade, thereafter, the IBM 704 found 16,167 digit places. One can find that the IBM 7090 broke the 100,000 digit target in 1961. The CDC 7600 could reach pi's millionth decimal value in 1973.

The value of π (pi) keeps up its further progress beyond its 50 trillionth digit places. Mathematicians are searching for patterns recognition of π (pi) for the likely repetition of its digits. Furthermore, finding the value of π (pi) is valuable for the development of computer programming software system. Moreover, the story of π for this and furthermore can be explored in ^[6, 7, 8, 9], and so on so forth.

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