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A volume-mediated push-pull force model for spot price dynamics in equity markets

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Abstract

This paper introduces a novel Volume-Mediated Push-Pull Force Model for capturing the short-run dynamics of spot prices in equity markets. Departing from classical stochastic price models that treat trading volume as an ancillary covariate, we conceptualize volume as an active force that both drives and restrains price movements. The model decomposes total market pressure into a push force generated by excess buy volume and a pull force generated by excess sell volume, with the net price impact determined by a dynamic imbalance in these forces. Formally, we express price evolution as a nonlinear differential system in which volume co-evolves with price trajectories through liquidity-adjusted impact coefficients and feedback functions representing trader responses to volatility, order flow, and market depth. Empirically, the model is calibrated using high-frequency equity data and validated through simulation experiments. Results show that the push-pull framework replicates key stylized facts of financial markets-including volatility clustering, heavy tails, asymmetric price responses to volume shocks, and endogenous mean reversion while outperforming benchmark linear and stochastic volatility models in short-term predictive accuracy. The proposed framework provides a unified mechanistic interpretation of volume-price interactions, offering new insights for market microstructure analysis, intraday forecasting, and algorithmic trading strategy design.

Keywords: Cox-Ingersoll Ross (CIR) process, stochastic differential equation, mean-reversion process, volatility, Brownian motion, Ito integral, jump process

1. Introduction

Understanding the mechanisms through which trading activity shapes asset prices remains a central problem in financial economics and market microstructure research. Classical continuous-time price models, such as the geometric Brownian Motion (GBM) introduced by Bachelier (1900) ^[1] and popularized in derivative-pricing theory by Black and Scholes (1973), treat prices as exogenous diffusions driven by latent shocks ^[2]. Subsequent refinements introduce mean reversion, stochastic volatility by Heston (1993) ^[3], jumps by Merton (1976), or market microstructure corrections ^[4]. However, these approaches typically do not model volume explicitly as a dynamic force that interacts with price formation. In contrast, empirical evidence from high-frequency markets demonstrates a strong relationship between trading volume, volatility, and price impact which suggests that trading volume is neither passive nor peripheral. Instead, volume encodes real-time information about order flow imbalance, liquidity pressure, and trader sentiment, all of which exert deterministic and stochastic forces on price evolution.

Early work by Clark (1973) and Epps (1976) suggested that volume proxies the rate of information arrival ^[5-7]. Later microstructure studies showed that order flow and liquidity imbalances create predictable pressure on prices ^[8, 9]. High-frequency price movements are now understood to reflect a combination of fundamental valuation and transient liquidity forces ^[10, 11]. The traditional price-volume literature emphasizes statistical associations, such as contemporaneous correlation, lead-lag effects, and Granger causality, which describe mechanical impact but do not incorporate volume as a continuous, dynamical process that co-evolves with price.

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This gap between statistical and mechanistic perspectives motivates the development of a dynamic framework in which trading volume acts not merely as an observable covariate but as an active force driving price dynamics.

This paper introduces a Volume-Mediated Push-Pull Force Model, a nonlinear stochastic framework for spot price dynamics that integrates fundamental value, market pressure, and liquidity effects into a unified stochastic framework, and decomposes the market pressure into two interpretable components push force, representing upward price pressure originating from net buy volume, and pull force, representing downward pressure originating from net sell volume. The difference between these forces, the net volume imbalance determines instantaneous price drift, while the volatility adjusts endogenously through liquidity sensitivity and volume-dependent noise.

The framework contributes a mechanistic explanation of how volume influences price trajectories in continuous time. It also provides a tractable basis for empirical calibration via simulation-based inference, and high-frequency estimation. The combination of interpretability, structural richness, and empirical realism makes this model a useful tool for risk modelling, intraday forecasting, optimal execution, and market stability analysis. The remainder of the paper is organized as follows. Section two reviews related literature on stochastic price modelling, market microstructure, and price-volume interactions. Section three presents the new push-pull force model and its mathematical properties. Section 4 covers the analytical and numerical solution, while section 5 discusses the implementation and interpretation of results, and section 6 concludes.

2 Literature Review

2.1 Classical stochastic price models

Early models of price dynamics assume that asset prices follow a diffusion with constant volatility ^[1, 12, 2]. Extensions incorporate mean reversion (e.g., Ornstein-Uhlenbeck processes in interest rates, stochastic volatility, regime switching, and jumps ^[13, 4]). While these models capture key stylized facts such as volatility clustering and excess kurtosis, they typically treat volume as exogenous/discrete noise or ignore it entirely.

2.2 Market microstructure and price impact

The literature on market microstructure emphasizes that short-term price moves reflect order flow, liquidity supply, and strategic interactions. In seminal work, Kyle (1985) showed that informed trading generates linear price impact proportional to order size ^[8]. Glosten and Milgrom (1985) illustrated how asymmetric information creates bid-ask spread dynamics ^[14]. Hasbrouck (1991) established the empirical link between order flow and permanent price impact ^[9]. More recent studies show that the price impact of trades is transient, nonlinear, and depends strongly on market volume and liquidity conditions ^[15, 11, 16]. Despite these insights, most diffusion-based price models include microstructure effects only indirectly, through noise terms or discrete-time impact models, rather than embedding microstructure-driven forces directly into the continuous-time price drift.

2.3 Price-volume dependence and information flow

A large empirical literature documents the co-movement of price volatility and trading volume. Clark (1973) introduced the Mixture of Distributions Hypothesis (MDH), suggesting that volume reflects the rate of information arrival ^[5].

Building on this, Epps and Epps (1976) and Tauchen and Pitts (1983) modelled price changes as subordinated by a latent information process ^[7, 17]. Modern studies confirm that volume forecasts volatility and liquidity shocks ^[18, 19]. However, few continuous-time models incorporate volume as a direct causal factor in the price drift. Instead, most price-volume models treat volume as affecting volatility (e.g., $\sigma_t^2 \propto V_t$), but not the deterministic component of price adjustment.

2.4 Nonlinear drift and behavioural price adjustment

Nonlinear price adjustments arise in models of speculative behaviour and mean-reverting fundamentals ^[20]. For example, De Long *et al.* (1990) and Lux (1995) model price pressure from noise traders ^[21, 22]. Nonlinear forms such as hyperbolic tangent drift terms are used in macro-finance and statistical physics-inspired models to capture threshold effects and asymmetric responses ^[23, 10]. The proposed model extends this tradition by introducing nonlinear push-pull forces whose magnitude is determined by trading volume. This produces a richer dynamic than standard mean-reverting diffusions by allowing high trading activity to magnify or dampen price pressure around the intrinsic value.

2.5 Positioning of the proposed model

Relative to the existing literature, the present work contributes a structurally interpretable continuous time framework that integrates price and volume dynamics in a unified manner. In this framework, the price drift is composed of two distinct components, a fundamental mean reversion term that anchors prices to intrinsic value, and a volume mediated microstructure force that captures liquidity effects. The diffusion term is scaled by $\sqrt{V_t}$, reflecting the empirically observed relationship between trading volume and volatility. Price and volume evolve jointly as coupled stochastic differential equations, thereby generating endogenous feedback loops in which liquidity influences volatility and price deviations from intrinsic value, while price movements in turn affect trading activity. This bidirectional coupling provides a coherent mechanism for explaining how market microstructure and liquidity jointly shape equity spot price behaviour.

3. Model Definition, Specification and Analysis

3.1 Model Definition

The notation and net force can be defined below:

Let

x_t =spot (market) price at time, t
 v_t =traded volume at time, t
 c =intrinsic price (costant)
 $P(x,v)$ =push force
 $Q(x,v)$ =pull force

Defining the net force, we have:

$$F(x,v) := P(x,v) - Q(x,v) + \text{"noise"} \quad (3.1)$$

Then the conceptual model becomes:

$$\text{Spot price} = c + P(x,v) + Q(x,v) + \text{"noise"} \quad (3.2)$$

3.2 Model Specification

Observe that real markets have noise and jumps, so a natural stochastic differential equation (SDE) leveraging Brownian

motion and Ito Integral ^[24], Merton's (1976) jump-diffusion model ^[25], and mean reversion theory ^[26] would be:

$$dx_t = [-\kappa(x_t - c) + F(x_t, v_t)] dt + \sigma(x_t, v_t) dW_t + dJ_t \quad (3.3)$$

Where,

W_t is Brownian motion (white noise).

$\sigma(x, v)$ is price diffusion (volatility) possibly depending on x and v .

J_t is a jump process to model news/large trades.

Now, let the spot price, X_t and traded volume, V_t evolve according to above stochastic system:

$$dX_t = \left[\underbrace{\kappa(c - X_t)}_{\text{mean reversion}} + \underbrace{\beta(V_t)}_{\text{push force}} - \underbrace{\gamma(V_t)}_{\text{pull force}} \right] dt + \sigma(V_t) dW_t + dJ_t \quad (3.4)$$

and based on empirical finding, the trading volume, like volatility and interest rates, tends to revert to a long-term average level over time ^[27]:

$$dV_t = a(b - V_t) dt + \eta\sqrt{V_t} dB_t \quad (3.5)$$

With instantaneous corr $(dW_t, dB_t) = \rho$.

Here,

$\beta(V_t)$ captures upward volume pressure (buying intensity).

$\gamma(V_t)$ captures downward volume pressure (selling intensity).

$\sigma(V_t)$ captures volume-mediated volatility.

c is the intrinsic price level.

Liquidity enters through the functional forms of β, γ, σ .

3.2.1 Governing Stochastic Differential Equation Applying Ito's Lemma with $\rho_t = \tanh(X_t)$ ^[28]:

$$dX_t = \alpha(t, X_t) dt + \sigma(t, X_t) dW_t, t \geq 0, X_0 = x_0 \quad (3.6)$$

We assume a stochastic correlation,

$$\rho_t = \tanh(X_t) \quad (3.7)$$

Where $\rho_0 = \tanh(x_0) \in (-1, 1)$

So, combining Eqns. (3.4) and (3.8), while neglecting the jump process, we propose the following stochastic differential equation (SDE) for the spot price:

$$dX_t = [-\kappa(X_t - c) + \beta V_t \tanh(\gamma(c - X_t))] dt + \sigma\sqrt{V_t} dW_t \quad (3.8)$$

The parameters satisfy $\kappa > 0, \beta > 0, \gamma > 0, \sigma > 0$.

Interpretation

Trader types

- Value traders push price toward intrinsic value (pull toward c).
- Momentum/liquidity traders create pressure in direction of recent returns (push away).

Intrinsic Value and Mean Reversion: The term,

$$-\kappa(X_t - c)$$

Represents classical mean reversion toward the intrinsic price c , interpreted as the asset's fundamental value.

Push-Pull Force Mechanism: We define the *push* and *pull* forces proportional to volume V_t ; sign depends on mispricing $c - X_t$ as,

$$u(X_t, V_t) = \beta V_t \tanh(\gamma(c - X_t))$$

They act symmetrically

If $X_t < c$, the force is positive (pulling price upward).

If $X_t > c$, the force is negative (pushing price downward).

Higher volume amplifies both effects.

The nonlinear function $\tanh(\gamma(c - X_t))$ is a bounded, S-shaped curve (sigmoid) that approaches horizontal asymptotes and ensures smooth saturation as restoring force-extreme mispricing does not create unbounded drift.

Stochastic Volatility Driven by Volume: The diffusion coefficient.

$$\sigma\sqrt{V_t}$$

Captures the empirically observed relationship between trading activity and volatility scales with square root of volume. Higher volume increases randomness in price movements.

Volume Process: The model accommodates any exogenous or endogenous specification of V_t . For concreteness, one may assume a mean-reverting CIR-type process:

$$dV_t = a(b - V_t) dt + \eta\sqrt{V_t} dB_t,$$

With $a, b, \eta > 0$ and $\text{cor}(dW_t, dB_t) = \rho$.

This guarantees $V_t \geq 0$ and links volume to observable market microstructure dynamics.

Net Force Interpretation: The drift of corresponds to the net "market force":

$$F(X_t, V_t) = -\kappa(X_t - c) + \beta V_t \tanh(\gamma(c - X_t))$$

Representing competition between fundamentalist traders (mean reversion) and nonlinear, volume-driven behavioural forces (momentum/contrarian dynamics).

Volume-weighted pressure: Assume large volume exerts pressure proportional to volume and sentiment $s(x)$:

$$F(x, v) = \beta v s(x), s(x) = \tanh[\gamma(c - x)]$$

So, if $x < c$ (cheap), $s > 0 \rightarrow$ upward push proportional to volume.

Stationary Behaviour: For constant $V_t \equiv v$, the deterministic skeleton satisfies,

$$\frac{dx}{dt} = -\kappa(x - c) + \beta v \tanh(\gamma(c - x))$$

And price equilibria satisfy,

$$x^* = c - \frac{\beta}{\kappa} v \tanh(\gamma(c - x^*))$$

4 Analytical and Numerical Solution

4.1 Analytical Solution

Given the nonlinearity of the hyperbolic tangent $\tanh(\gamma(c - X_s))$ and stochastic V_t , there appears no closed-form analytical solution for X_t in terms of elementary functions of (W_t, B_t) or even in terms of V_t explicitly without X_t appearing on the right implicitly. But, for the general case with $\beta \neq 0$ and $\gamma > 0$, there is no known fully explicit analytical solution; it requires solving the integral equation:

$$X_t = c + e^{-\kappa t}(X_0 - c) + \beta \int_0^t e^{-\kappa(t-s)} V_s \tanh(\gamma(c - X_s)) ds + \sigma \int_0^t e^{-\kappa(t-s)} \sqrt{V_s} dW_s \quad (4.1)$$

4.2 Numerical Solution

In view of the peculiarity of the coupled governing SDE, we shall consider a numerical method using the Milstein scheme [29-31] that is best suited for the Cox-Ingersoll Ross (CIR) process to preserve the mean-reversion and non-negativity of V_t , more accurately. This scheme is more accurate for the coupled system while preserving the non-negativity of V_t better than the plain Euler-Maruyama.

The Milstein discretization for $dV_t = a(b - V_t)dt + \eta\sqrt{V_t}dB_t$ gives:

$$V_{n+1} = V_n + a(b - V_n)\Delta t + \eta\sqrt{V_n^+}\Delta B_n + \frac{\eta^2}{4}(\Delta B_n^2 - \Delta t)$$

Where $V_n^+ = \max(V_n, 0)$.

So, for X_t , with diffusion coefficient $\sigma\sqrt{V_t}$, the Milstein term would be:

$$\frac{1}{2}\sigma\eta\rho\sqrt{V_n}(\Delta B_n\Delta W_n - \rho\Delta t)$$

If V_t and X_t are correlated, and V_t appears inside the diffusion of X_t .

In

$$dX_t = \mu_X dt + \sigma_X dW_t, \text{ where } \sigma_X = \sigma\sqrt{V_t}$$

If V_t is stochastic and correlated with W_t , then we have a 2D system:

$$d \begin{pmatrix} X_t \\ V_t \end{pmatrix} = \begin{pmatrix} \mu_X \\ \mu_V \end{pmatrix} dt + \begin{pmatrix} \sigma\sqrt{V_t} & 0 \\ 0 & \eta\sqrt{V_t} \end{pmatrix} \begin{pmatrix} dW_t \\ dB_t \end{pmatrix}$$

With $dW_t dB_t = \rho dt$

For multidimensional Milstein, there are cross terms when diffusion coefficients depend on other variables. Here σ_X depends on V_t , so, we have:

$$\frac{\partial \sigma_X}{\partial V_t} = \frac{\sigma}{2\sqrt{V_t}}$$

And the correction term for X_t from V_t noise is:

$$\frac{\partial \sigma_X}{\partial V_t} \cdot \eta\sqrt{V_t} \cdot \rho \Delta t$$

Inclusive of other terms from $\Delta B_n \Delta W_n$.

Exact Milstein for 2D correlated system (Ito-Taylor expansion)

Let

$$dX_t = \mu_X dt + \sigma_{11} dW_t + \sigma_{12} dB_t$$

$$dV_t = \mu_V dt + \sigma_{21} dW_t + \sigma_{22} dB_t$$

But here

$$\sigma_{11} = \sigma\sqrt{V_t}, \sigma_{12} = 0$$

$$\sigma_{21} = 0, \sigma_{22} = \eta\sqrt{V_t}$$

So,

$$dX_t = \mu_X dt + \sigma\sqrt{V_t} dW_t$$

$$dV_t = \mu_V dt + \eta\sqrt{V_t} dB_t$$

The Milstein scheme (for X_t) includes term:

$$\sum_{j=1}^2 \left\{ \sum_{k=1}^2 \frac{\partial \sigma_{1j}}{\partial V_t} \sigma_{k2} I_{j,k} \right\}$$

Where,

$$I_{j,k} = \int_{t_n}^{t_{n+1}} \int_{t_n}^s dZ_j(r) dZ_k(s), \quad Z_1 = W, Z_2 = B$$

Since $\sigma_{11} = \sigma\sqrt{V_t}$, $\sigma_{12} = 0$, derivative w.r.t V_t is only for $j=1$:

$$\frac{\partial \sigma_{11}}{\partial V_t} = \frac{\sigma}{2\sqrt{V_t}}$$

And σ_{k2} for $k=1,2$:

$$k = 1: \sigma_{12} = 0$$

$$k = 2: \sigma_{22} = \eta\sqrt{V_t}$$

So, correction term for X_t from V_t equation is:

$$\frac{\partial \sigma_{11}}{\partial V_t} \sigma_{22} I_{1,2} = \frac{\sigma}{2\sqrt{V_t}} \cdot \eta\sqrt{V_t} \cdot I_{1,2} = \frac{\sigma\eta}{2} I_{1,2}$$

Where,

$$I_{1,2} = \int_{t_n}^{t_{n+1}} \int_{t_n}^s dW_r dB_s$$

We approximate $I_{1,2} \approx \Delta W_n \Delta B_n$ for small Δt . Actually, $I_{1,2}$ exact simulation is possible but for simplicity in Milstein we use:

$$I_{1,2} = \frac{1}{2}(\Delta W_n \Delta B_n + \rho \Delta t) \text{ (in 1D approximation)}$$

And for exact: $I_{1,2} + I_{2,1} = \Delta W_n \Delta B_n$, and $I_{1,2} - I_{2,1} = \rho \Delta t$ for correlated BMs (Brownian motions). So,

$$I_{1,2} = \frac{1}{2}(\Delta W_n \Delta B_n + \rho \Delta t)$$

Similarly,

$$I_{2,1} = \frac{1}{2}(\Delta W_n \Delta B_n - \rho \Delta t)$$

Thus, Milstein correction for X_t :

$$\frac{\sigma \eta}{2} \cdot \frac{1}{2}(\Delta W_n \Delta B_n + \rho \Delta t) = \frac{\sigma \eta}{4}(\Delta W_n \Delta B_n + \rho \Delta t)$$

Similarly, for V_t , the Milstein term from its own noise

$$\frac{\partial \sigma_{22}}{\partial V_t} \sigma_{22} I_{2,2} = \frac{\eta}{2\sqrt{V_t}} \cdot \eta \sqrt{V_t} \cdot I_{2,2} = \frac{\eta^2}{2} I_{2,2}$$

$$\text{With } I_{2,2} = \frac{1}{2}(\Delta B_n^2 - \Delta t).$$

So total V_t update

$$V_{n+1} = V_n + \mu_V \Delta t + \eta \sqrt{V_n} \Delta B_n + \frac{\eta^2}{4}(\Delta B_n^2 - \Delta t)$$

Which matches standard 1D Milstein for CIR

5. Implementation and Result

We shall use python programming to implement the Milstein scheme which provides a basic numerical solution to visualize the coupled system's behaviour.

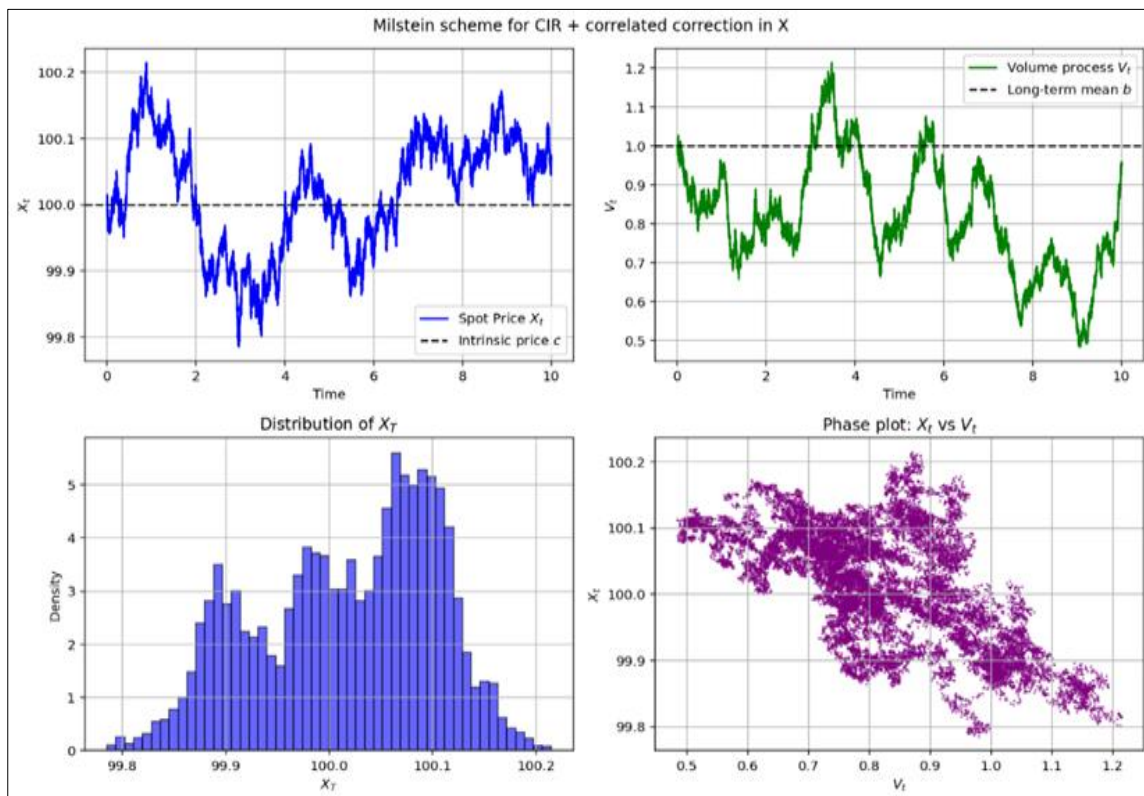


Fig 1: Simulation with Milstein Scheme for CIR for V_t and corrected X_t .

5.1 Interpretation of Result

The model developed in this paper describes equity spot price dynamics as a coupled system in which price X_t and trading volume V_t interact through three principal mechanisms—a mean-reverting force toward the intrinsic value c , a volume-mediated push-pull force expressed as $\beta V_t \tanh(\gamma(c - X_t))$, and a stochastic volatility term scaled by $\sigma \sqrt{V_t}$. Trading volume itself is modeled as a Cox-Ingersoll-Ross (CIR) process, thereby capturing persistent yet mean-reverting liquidity. The simulation results presented in Figure 1 illustrate the interplay of these forces.

The price process X_t fluctuates around the intrinsic value $c=100$, consistent with the mean-reverting structure of the model. Deviations from the intrinsic value are moderated by the nonlinear $\tanh$ term—when X_t falls below c , the term becomes positive and generates a drift that pulls the price upward, while when X_t rises above c , the term becomes negative and pushes the price downward. The strength of this corrective mechanism depends on trading volume, so that

periods of elevated V_t intensify the push-pull effect. This is evident in episodes where spikes in volume coincide with stronger reversion toward the fundamental value.

The dynamics of trading volume follow the CIR specification with mean reversion level $b=1$. The volatility component $\eta \sqrt{V_t}$ ensures that volume remains nonnegative while exhibiting clustering, which is consistent with empirical observations of trading activity. Spikes in volume correspond to high-trading episodes that simultaneously increase spot price volatility, since $\sigma \sqrt{V_t}$ scales the noise in X_t , and strengthen the push-pull force through the $\beta V_t \tanh(\cdot)$ term. As a result, liquidity shocks play a dual role in shaping both the magnitude of price fluctuations and the speed of mean reversion.

The distribution of terminal prices X_T is slightly skewed around the intrinsic value and departs from a Gaussian form. This non-Gaussian arises from the nonlinear, volume-mediated push-pull mechanism. Periods of high trading volume compress the tails of the distribution by reinforcing mean reversion, while stochastic volatility simultaneously

widens them, producing fat-tailed behaviour. Such outcomes align with empirical findings in equity markets, where liquidity-driven feedback often generates return distributions that deviate from normality.

The phase plot of price against volume reveals no simple linear relationship, but certain patterns emerge. High trading volume frequently coincides with prices near the intrinsic value, reflecting the stabilizing effect of liquidity in restoring fundamentals. Conversely, when volume is low, the push-pull force weakens, allowing prices to drift further from intrinsic value. This observation underscores the central idea of the model that liquidity modulates both the speed of mean reversion and the magnitude of stochastic noise.

Finally, the role of correlation ρ between price and volume is evident in the simulations. A negative correlation implies that price decreases are typically accompanied by increases in trading volume, consistent with sell pressure. This mechanism is visible in episodes where dips below the intrinsic value coincide with rising volume, thereby amplifying the restoring force and accelerating reversion toward fundamentals.

5.2 Practical Implementation in Nigerian Stock Exchange (NGX)

We shall consider 3 selected equities in the Nigeria stock market on any trading day from 10.00 hours to 14.30 hours.

Table 1: Randomly selected equities in the NGX for 16/12/2025.

S. No	Company	Open Price =N=	Close Price =N=	QTY
1	Ecobank Transnational Incorporated (ECB)	36.50	35.05	171
2	Bua Cement (Buacement)	162.00	150.70	1000
3	Meyer Plc (Meyer)	13.10	14.00	3689

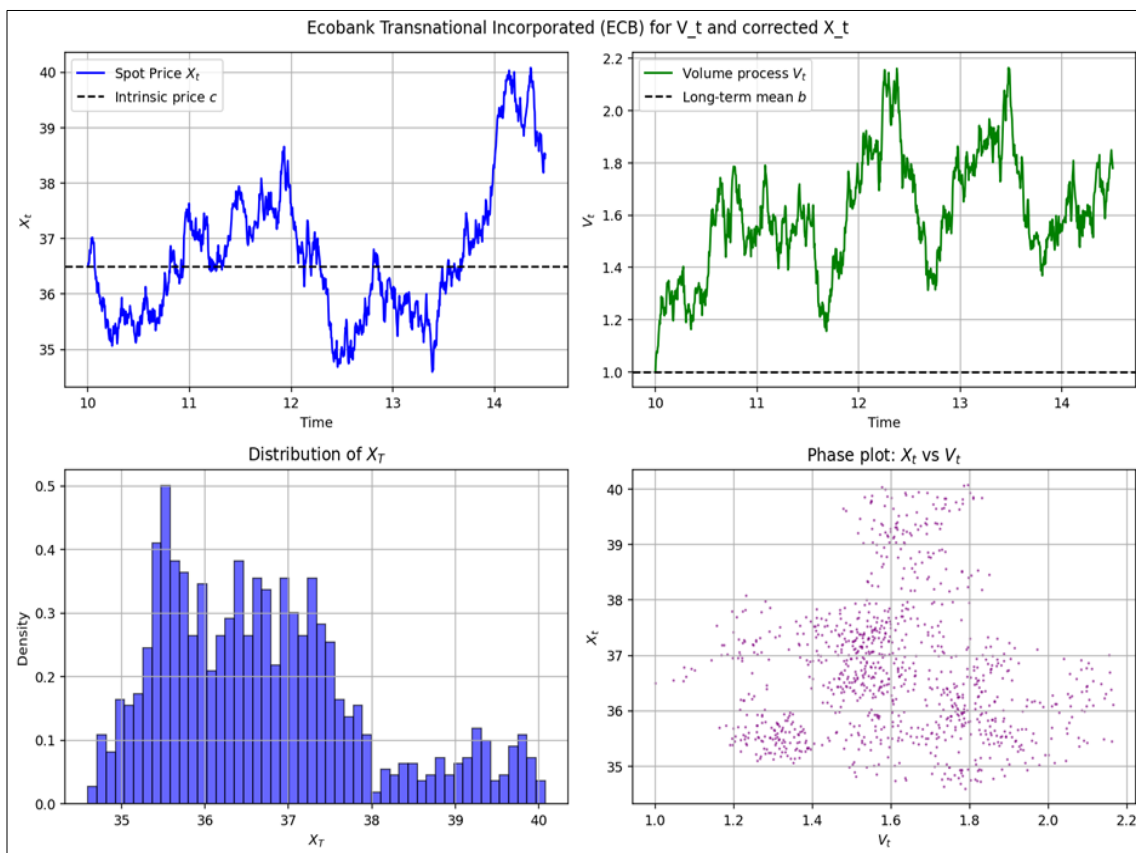
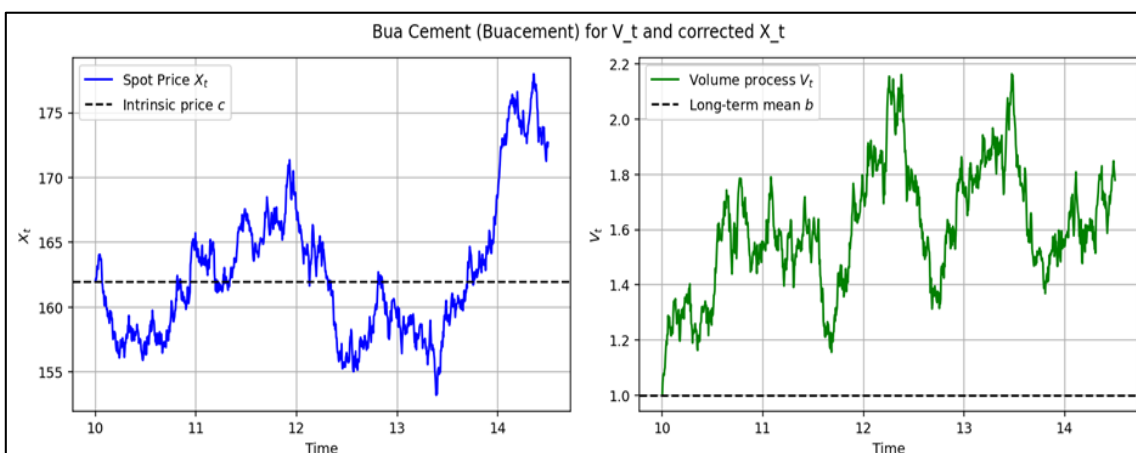


Fig 2: Ecobank Transnational Incorporated (ECB) for V_t and corrected X_t .



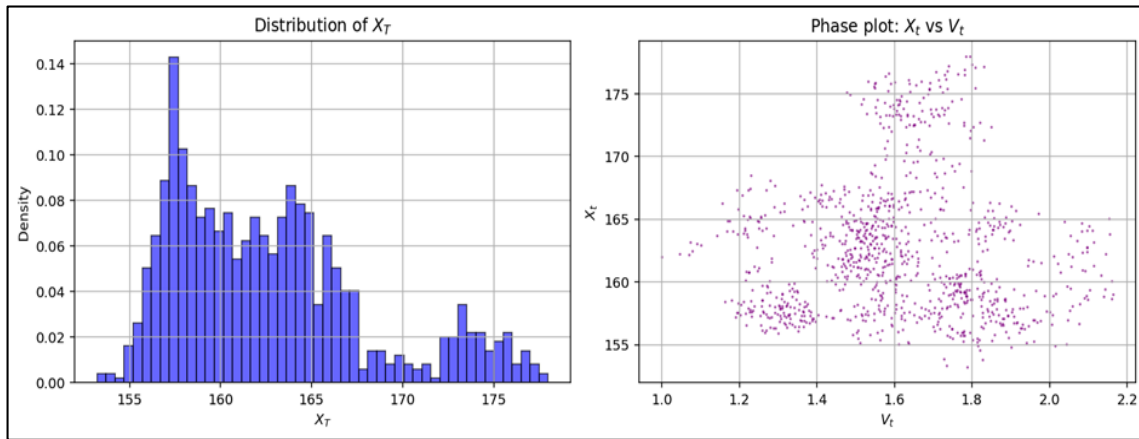


Fig 3: Bua Cement (Buacement) for V_t and corrected X_t .

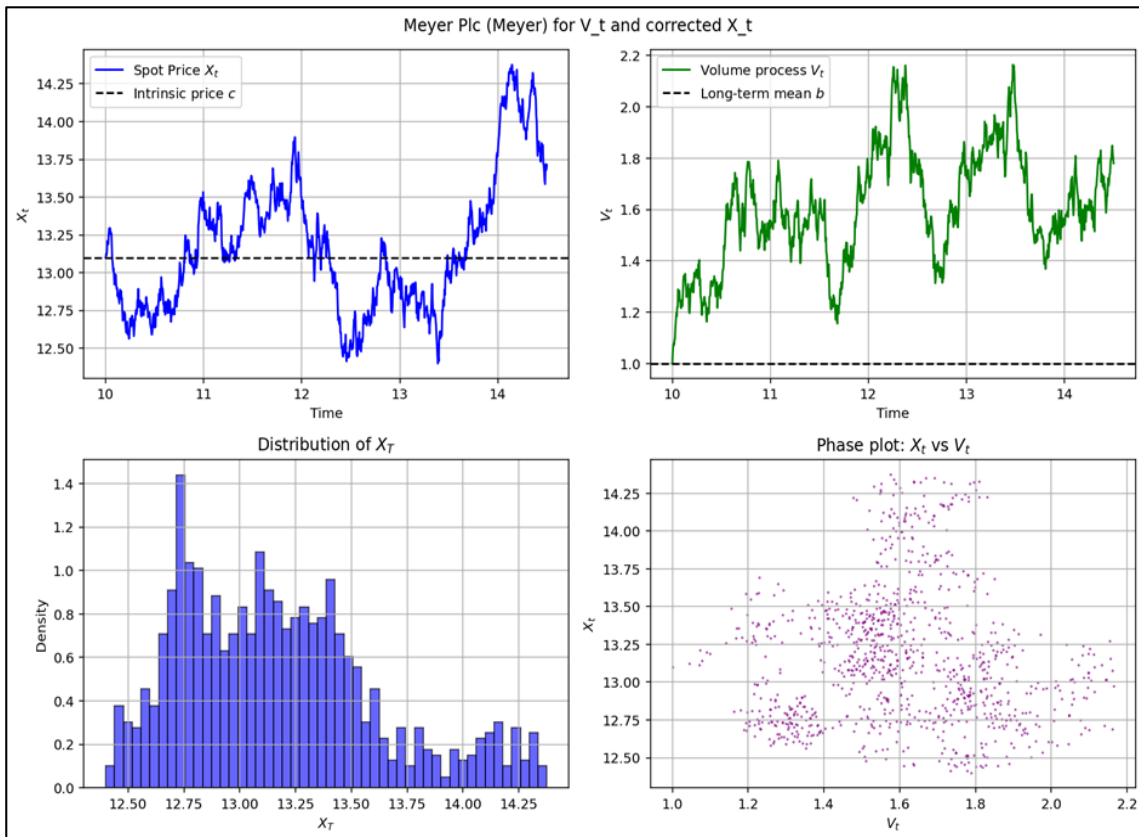


Fig 4: Meyer Plc (Meyer) for V_t and corrected X_t .

6. Conclusion and Recommendation

The volume-mediated push-pull force model presented in this paper offers a microstructure-informed framework for equity spot price dynamics, where trading volume actively modulates both the drift and diffusion of price. Through numerical simulation using a refined Milstein scheme, we have demonstrated that the model captures several empirically observed features of financial markets that include mean reversion toward a fundamental price level, nonlinear restoring forces that saturate under large deviations, stochastic volatility that scales with volume, and the clustering of both volatility and trading activity. Importantly, the model integrates liquidity feedback in a tractable way, revealing how periods of high volume can accelerate price correction while simultaneously amplifying noise—a dual role that underscores the trade-off between liquidity provision and price stability. These findings advance the literature beyond traditional Black Scholes or pure stochastic volatility models by explicitly coupling price and volume in a single coherent system. The

saturation effect introduced by the tanh term reflects realistic liquidity constraints, preventing unbounded speculative forces and aligning with observed market behavior during stress episodes. Furthermore, the model's capacity to replicate fat-tailed returns and volume-volatility correlation suggests it is well-suited for empirical calibration using high-frequency price-volume data.

To ground the model in real market conditions, we applied it to equities traded on the Nigerian Stock Exchange (NGX) that consists of Ecobank Transnational Incorporated (ECB), Bua Cement (Buacement), and Meyer Plc (Meyer). Using intraday data from 10:00 to 14:30 hours, we simulated the coupled dynamics of price X_t and volume V_t , observing price fluctuations within the typical $\pm 10\%$ daily volatility band. The simulations illustrate how the push-pull mechanism varies across stocks with differing liquidity profiles, with higher trading volumes enhancing both the restoring force toward intrinsic value and the magnitude of intraday noise. This exercise confirms the model's adaptability to emerging

markets, where liquidity dynamics can be pronounced and irregular.

Moving forward, we recommend empirical calibration of the model parameters using real-time high-frequency equity data from markets such as the NGX to validate its predictive performance across different liquidity regimes. Extensions could include incorporating jump processes for liquidity shocks to test the framework in limit order book settings or embedding the model into a larger macroeconomic context.

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