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Measures of central tendency in the disparity of elements in full transformation semi groups

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Abstract

This research work explores the measures of central Tendency in full transformation semi group we investigate the distance, mean and mode in the disparity of elements in full transformation semi group $D_s T_n$ and its subsemigroup of order-preserving $D_s O_n$, order-decreasing $D_s D_n$ and the combined order - preserving and order-decreasing $D_s C_n$ full transformation semi groups. The method of polynomial is used to generate the general nth term for the sequences obtained. Tables were constructed and the results obtained shows the movements of elements in the disparity of full transformation semi group from the domain to its co-domain.

Keywords: Measures of central tendency, distance, mean, mode

1. Introduction

The concept of Measures of central tendency dates back to ancient civilizations where mathematician and philosophers like Aristotle, Blaise Pascal, Isaac Newton used various methods to calculate the middle of a set of numbers. Measures of Central Tendency are statistical measure that represents the average values of sets of elements in a semigroup. Recently, some authors like James and Peter (2011), Rotimi Kehinde et.al (2020), Francis Obinna (2022) have used different measures of central tendencies such as distance, work done and average work done to determine the values of semigroups.

In this research, we used the distance, mean and mode to describe the movement of element in the domain and co-domain of each semigroup. Finite element formulation ovides necessary flexibility in taking care of different behavior of distinctly different subregions.

The Distance

The distance covered between the movements of elements in this semigroup is described as

$$d(i, j) = |(i - j)| + |(j - \alpha j)| + |(\alpha(i) - \alpha(j))| + |(i - \alpha(i))|$$

The Mean

This is given by the sum of the distance covered in the sets as it varies with the order of semi groups and is given as: $\overline{M}_n = \frac{d(i,j)}{n}$.

The Mode

This is determined by the frequency of distance covered by the movements of elements in the semi group at any degree.

2. Methodology

2.1 Measures of central tendency in full transformation semi group $D_s T_n$

2.1.1 Distance

The distance covered between the movements of elements in the semi group is described as

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$$d(i, j) = |(i - j)| + |(j - \alpha(j))| + |(\alpha(i) - \alpha(j))| + |(i - \alpha(i))|$$

For all $i, j \in D_s T_n$

Below are transformations on $D_s T_2$:

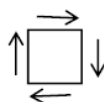
$$\left\{ \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \right\} = 2 \text{ Elements}$$

We will compute the distance between each transformation using the definition of distance function above,

$$d_1 \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} = |2 - 1| + |1 - 1| + |2 - 1| + |2 - 2| = 2$$



$$d_2 \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} = |2 - 1| + |2 - 1| + |2 - 1| + |2 - 1| = 4$$



Now, the total distance in $D_s T_2$ is obtained by summing up the distance covered by each transformation in $D_s T_2$,

Hence, $DD_s T_2 = |d_1 + d_2| = |2 + 4| = 6$ with 2 different Structures

2.1.2 Mean

The mean in $D_s T_n$ is obtained by the sum of the distance covered in the sets as it varies with the order of semigroups and is given as: $\overline{M}_n = \frac{DD_s T_n}{n}$.

since $DD_s T_2 = 6$ and $n = 2$, then, $\overline{M}_{D_s T_2} = \frac{6}{2} = 3$

2.1.3 Mode in $D_s T_n$

The frequency of distance covered by the movements of elements in $D_s T_n$ is obtained from the most occurrence in $|d_1 + d_2|$ of each element in $DD_s T_n$. This follows from the elements in $DD_s T_2$ that $MD_s T_2 = 2$

2.2 Measures of Central Tendency in Full Transformation Semi group $D_s O_n$

2.2.1 Distance

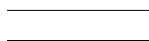
Transformations on $D_s O_3$:

$$\left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 3 \end{pmatrix} \right\} = 7 \text{ Elements}$$

$$|d_1 \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix}| = |3 - 1| + |3 - 2| + |2 - 1| + |1 - 1| = 4$$



$$|d_2 \begin{pmatrix} 1 & 3 \\ 1 & 3 \end{pmatrix}| = |3 - 1| + |3 - 3| + |3 - 1| + |1 - 1| = 4$$



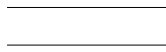
$$|d_3 \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix}| = |3 - 1| + |3 - 2| + |2 - 1| + |1 - 1| = 4$$



$$|d_4 \begin{pmatrix} 1 & 3 \\ 1 & 3 \end{pmatrix}| = |3 - 1| + |3 - 3| + |3 - 1| + |1 - 1| = 4$$



$$|d_5 \begin{pmatrix} 1 & 3 \\ 1 & 3 \end{pmatrix}| = |3-1| + |3-3| + |3-1| + |1-1| = 4$$



$$|d_6 \begin{pmatrix} 1 & 3 \\ 2 & 3 \end{pmatrix}| = |3-1| + |3-3| + |3-2| + |2-1| = 4$$



$$|d_7 \begin{pmatrix} 1 & 3 \\ 2 & 3 \end{pmatrix}| = |3-1| + |3-3| + |3-2| + |2-1| = 4$$



$$DD_s O_3 = |d_1 + d_2 + d_3 + d_4 + d_5 + d_6 + d_7| = |4 + 4 + 4 + 4 + 4 + 4 + 4| = 28 \text{ with 2 different Structures}$$

2.2.2 Mean

$$\bar{MD}_s O_3 = \frac{DD_s O_3}{n} = \frac{28}{3} = 9.3$$

2.2.3 Mode

Considering the elements in $DD_s O_3$ it is observed that $MD_s O_3 = 4$.

2.3 Measures of Central Tendency in Full Transformation Semigroup $D_s D_n$

2.3.1 Distance

Transformations on $D_s D_3$

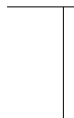
$$\left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \right\} = 5 \text{ Elements}$$



$$|d_1 \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix}| = |3-1| + |3-2| + |2-1| + |1-1| = 4$$



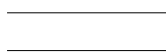
$$|d_2 \begin{pmatrix} 1 & 3 \\ 1 & 3 \end{pmatrix}| = |3-1| + |3-3| + |3-1| + |1-1| = 4$$



$$|d_3 \begin{pmatrix} 2 & 3 \\ 2 & 1 \end{pmatrix}| = |3-2| + |3-1| + |2-1| + |2-2| = 4$$



$$|d_4 \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix}| = |3-1| + |3-2| + |2-1| + |1-1| = 4$$



$$|d_5 \begin{pmatrix} 1 & 3 \\ 1 & 3 \end{pmatrix}| = |3-1| + |3-3| + |3-1| + |1-1| = 4$$

$$DD_s D_3 = |d_1 + d_2 + d_3 + d_4 + d_5| = |4 + 4 + 4 + 4 + 4| = 20, \text{ with 3 different structures}$$

2.3.2 Mean

$$\bar{M}D_s D_3 = \frac{DD_s D_3}{n} = \frac{20}{3} = 6.7$$

2.3.3 Mode

From the elements in $DD_s D_3$, it is observed that $MD_s D_3 = 4$.

2.4 Results of measures of central tendency in full transformation semigroup $D_s C_n$

2.4.1 Distance

Transformations on $D_s C_4$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 \end{pmatrix} \right\} = 8 \text{ Element}$$

$$|d_1 \begin{pmatrix} 1 & 4 \\ 1 & 2 \end{pmatrix}| = |4 - 1| + |4 - 2| + |2 - 1| + |1 - 1| = 6$$



$$|d_2 \begin{pmatrix} 1 & 4 \\ 1 & 3 \end{pmatrix}| = |4 - 1| + |4 - 3| + |3 - 1| + |1 - 1| = 6$$



$$|d_3 \begin{pmatrix} 1 & 4 \\ 1 & 2 \end{pmatrix}| = |4 - 1| + |4 - 2| + |2 - 1| + |1 - 1| = 6$$



$$|d_4 \begin{pmatrix} 1 & 4 \\ 1 & 3 \end{pmatrix}| = |4 - 1| + |4 - 3| + |3 - 1| + |1 - 1| = 6$$



$$|d_5 \begin{pmatrix} 1 & 4 \\ 1 & 3 \end{pmatrix}| = |4 - 1| + |4 - 3| + |3 - 1| + |1 - 1| = 6$$



$$|d_6 \begin{pmatrix} 1 & 4 \\ 1 & 2 \end{pmatrix}| = |4 - 1| + |4 - 2| + |2 - 1| + |1 - 1| = 6$$



$$|d_7 \begin{pmatrix} 1 & 4 \\ 1 & 3 \end{pmatrix}| = |4 - 1| + |4 - 3| + |1 - 1| + |1 - 1| = 6$$



$$|d_8 \begin{pmatrix} 1 & 4 \\ 1 & 3 \end{pmatrix}| = |4 - 1| + |4 - 3| + |3 - 1| + |1 - 1| = 6$$



$$DD_s C_4 = |d_1 + d_2 + d_3 + d_4 + d_5 + d_6 + d_7 + d_8| = |6 + 6 + 6 + 6 + 6 + 6 + 6 + 6| = 48, \text{ with 2 different structures}$$

2.4.2 Mean

$$\bar{M}D_s C_4 = \frac{DD_s C_4}{n} = \frac{48}{4} = 12$$

2.4.3 Mode

From the elements in $DD_s C_4$, it is observed that $MD_s C_4 = 6$.

3. Results

3.1 Proposition: Let $\alpha \subseteq D_s T_n$, then, there is a measure of central tendency C_n in each of the subsemigroup, $\alpha \subseteq S$ which is a semigroup such that

$$d(i, j) = |(i - j)| + |(j - \alpha(j))| + |(\alpha(i) - \alpha(j))| + |(i - \alpha(i))|$$

$$\geq |W^+(\alpha) - W^-(\alpha)|, \text{ where } W^+(\alpha) = \max(\text{Im}(\alpha)) \text{ and } W^-(\alpha) = \min(\text{Im}(\alpha)) \text{ for all } i, j \in D_s T_n$$

Proof: Since $\alpha \subseteq D_s T_n$ such that, $S = \{D_s T_n, D_s O_n, D_s D_n, D_s C_n\}$ and $x, y \in S$, this means that

$|(x - y) + (x - x\alpha) + (x\alpha - y\alpha) + (y - y\alpha)|$ Has measures of central tendency C_n in $|W^+(\alpha) - W^-(\alpha)|$ and since the absolute difference are always non-negative, the results of transformation on S is also non-negative, therefore, C_n is in $|W^+(\alpha) - W^-(\alpha)|$ for all $x, y \in S$. Hence the proof.

3.1 Results in $D_s T_n$

Table 3.1: The table displays the sequence of distance obtained in $D_s T_n$

n	$ S = \frac{221n^4}{3} - \frac{1940n^3}{3} + \frac{6271n^2}{3} - \frac{8530n}{3} + 1326$
1	0
2	6
3	116
4	870
5	4576

Table 3.2: The Table displays the sequence of Structures in $D_s T_n$

n	$ S = \frac{n^4}{24} - \frac{3n^3}{4} + \frac{95n^2}{24} - \frac{21n}{4} + 2$
1	0
2	2
3	5
4	7
5	7

Table 3.3: The table displays the sequence of mean obtained in $D_s T_n$

n	$ S = \frac{331n^4}{30} - \frac{5513n^3}{60} + \frac{8753n^2}{30} - \frac{11839n}{30} + \frac{917}{5}$
1	0
2	3
3	$\frac{2295}{60}$
4	$\frac{13028}{60}$
5	$\frac{54889}{60}$

Table 3.4: The table displays the sequence of mode obtained in $D_s T_n$

n	$ S = 2n - 2$
1	0
2	2
3	4
4	6
5	8

3.1 Theorem: Let $C_n \geq D_s T_n$, then,

1. $d \subset C_n \geq D_s T_n$
2. $\bar{m} < d \subset C_n$
3. $m < d \subset C_n$

Proof: If C_n is a measure of central tendency in $D_s T_n$, then there exists $d, \bar{m}, m \in D_s T_n$ such that $d, \bar{m}, m \subset C_n$, by definition of

$D_s T_n = |W^+(\alpha) - W^-(\alpha)| = p$ where $p = \{1, 2\}$,

1. $d(x, y) = |x - y|$ is the domain set of α and

$|\alpha x - \alpha y|$ is the image set α , it implies

that $|x - y| + |\alpha x - \alpha y| \geq |W^+(\alpha) - W^-(\alpha)|$, thus, $d(x, y) \geq |W^+(\alpha) - W^-(\alpha)|$

2. Since $d(x, y) \geq D_s T_n$, $\bar{m}$ takes the average form of $d(x, y)$ in its n^{th} values and hence, $\frac{d(x, y)}{n} < d(x, y)$

3. Since m takes value from $d(x, y)$ for the frequency of movement in $D_s T_n$ hence, $m < d \subset C_n$

Remark 3.1: $D_s T_2$ clearly explains theorem 3.1

3.2 Results in $D_s O_n$

Table 3.5: The table displays the sequence of distance obtained in $D_s O_n$

n	$ S = n^4 - 2n^3 - n^2 + 4n - 2$
1	0
2	2
3	28
4	126
5	368

Theorem 3.2: Given that the sequence 0,2,28,126,368,... has the general n^{th} term

$n^4 - 2n^3 - n^2 + 4n - 2$, then the sequence is a subsemigroup in DD_sO_n which is closed and contains an identity element under the operation $+$.

Proof: Consider two arbitrary terms $an + am$ which are both in DD_sO_n , then,

$$an + am = (n^4 - 2n^3 - n^2 + 4n - 2) + (m^4 - 2m^3 - m^2 + 4m - 2)$$

since the operation on the two terms are present in the sequence of the full transformation semigroup then it is closed under the operation $+$ and contains an identity element 0 such that $an + 0 = an$ and $0 + an = an$ this means that 0 leaves every element in the sequence unchanged under the operation $+$ which is a binary operator that combines two elements to produce another element in the semigroup.

Table 3.6: The table displays the sequence of structures obtained in D_sO_n

n	$ S = \frac{n^4}{12} - n^3 + \frac{47n^2}{12} - 5n + 2$
1	0
2	1
3	2
4	2
5	2

Table 3.7: The table displays the sequence of mean obtained in D_sO_n

n	$ S = -\frac{1}{40}n^4 + \frac{27}{20}n^3 - \frac{153}{40}n^2 + \frac{17}{5}n - \frac{9}{10}$
1	0
2	1
3	$\frac{372}{40}$
4	$\frac{1260}{40}$
5	$\frac{2944}{40}$

Table 3.8: Sequence of mode obtained in D_sO_n

n	$ S = 2n - 2$
1	0
2	2
3	4
4	6
5	8

3.3 Results in $D_s D_n$ **Table 3.9:** The table displays the sequence of distance obtained in $D_s D_n$

n	$ S = \frac{n^4}{3} + \frac{10n^3}{3} - \frac{61n^2}{3} + \frac{104n}{3} - 18$
1	0
2	2
3	20
4	94
5	272

Table 3.10: The table displays the sequence of structures obtained in $D_s D_n$

n	$ S = \frac{n^4}{12} - \frac{7n^3}{6} + \frac{65n^2}{12} - \frac{25n}{3} + 4$
1	0
2	1
3	3
4	4
5	4

Table 3.11: The table displays the sequence of mean obtained in $D_s D_n$

n	$ S = -\frac{17n^4}{120} + \frac{1493n^3}{600} - \frac{5405n^2}{600} + \frac{383n}{30} - \frac{61}{10}$
1	0
2	1
3	$\frac{4101}{600}$
4	$\frac{14292}{600}$
5	$\frac{33015}{600}$

Table 3.12: Sequence of mode obtained in $D_s D_n$

n	$ S = 2n - 2$
1	0
2	2
3	4
4	6
5	8

3.4 Results in $D_s C_n$ **Table 3.13:** The table displays the sequence of distance obtained in $D_s C_n$

n	$ S = -\frac{1}{3}n^4 + \frac{13}{3}n^3 - \frac{35}{3}n^2 + \frac{35}{3}n - 4$
1	0
2	2
3	16
4	48
5	96

Table 3.14: The table displays the sequence of Structures obtained in $D_s C_n$

n	$ S = \frac{n^4}{12} - n^3 + \frac{47n^2}{12} - 5n + 2$
1	0
2	1
3	2
4	2
5	2

Table 3.15: The table displays the sequence of mean obtained in $D_s C_n$

n	$ S = -\frac{67}{1000}n^4 + \frac{567}{1000}n^3 - \frac{133}{1000}n^2 - \frac{1567}{1000}n + \frac{6}{5}$
1	0
2	1
3	$\frac{5184}{1000}$
4	12
5	$\frac{19040}{1000}$

Table 3.16: Sequence of mode obtained in $D_s C_n$

n	$ S = 2n - 2$
1	0
2	2
3	4
4	6
5	8

Theorem 3.3: Let $D_s C_n$ be a disparity of combined order -preserving and order-decreasing full transformation semigroup on a finite set X with n elements, suppose we have a subset $M \subseteq D_s C_n$, where each transformation $m_n \in M$ is defined as $m_n = 2n - 2$ for $n=1,2,3,\dots,n$, if the frequency of transformation in M is such that $f(m_n) > f(m_l)$ for all $l \neq n$, then, m_n is the most frequency of transformation on M .

Proof: Since the mode is the value that appears most frequently in the transformation, by assumption, m_n has the highest frequency among all transformations on M and since $f(m_n) > f(m_l)$ for all $l \neq n$, m_n is the unique transformation with the highest frequency. Therefore, $m_n = 2n - 2$ is the mode of the frequency in $M \subseteq D_s C_n$.

4. Conclusion

In this research, we have been able to obtain the measures of central tendency (Distance, Mean and Mode) of the set of elements in the disparity of full transformation semigroup, its subsemigroups and the structures of movements in each set of elements in the semigroups. These results describes the movement of elements in the semigroups from the domain and co-domain. we also derived the values for the n^{th} term of the sequences obtained, these results have a wide range of applications in Statistics, Social science, Engineering, Medicine computational algebra, coding theory, automata theory, game theory, it will also be of importance in some branches of physics and theoretical computer science.

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