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Ratio type exponential estimator for finite population mean using known parameters of auxiliary variates in double sampling procedure

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Abstract

This paper presents a ratio type exponential estimator for finite population mean using known parameters of auxiliary variates in double sampling procedure. The bias and mean squared error expressions have been derived up to the first order of approximation. The developed estimator has been compared with some relevant considered estimators theoretically as well as empirically. It has been established that the proposed estimator is more efficient than other considered estimators.

Keywords: Double sampling, ratio type exponential estimator, population mean, auxiliary variates, bias and mean squared error

1. Introduction

An auxiliary variate in survey sampling is a secondary variate that is correlated with the study variable. It plays a crucial role in survey sampling for improving the efficiency of estimators and reducing sampling error. For example, in a population survey, if researchers want to estimate household income, they might use education level as an auxiliary variate since it is correlated with income. It is assumed by the ratio method of estimation that knowledge about the population mean of auxiliary variate is known. In some cases, however, it is unknown. Double sampling is used in this instance. In double sampling, in first phase a large preliminary sample is selected from the population to estimate the population mean of the auxiliary variate x . Then in second phase the required size of n sample drawn from the first phase sample (case I) or directly from the population (case II) and the information collected on both the auxiliary variate as well as study variate. Bahl and Tuteja (1991) ^[1] worked out ratio and product type exponential estimator for population mean in simple random sampling. Singh and Vishwakarma (2007) ^[4] studied Bahl and Tuteja (1991) ^[1] estimators in double sampling. Tailor and Sharma (2009) ^[5] developed ratio-cum-product estimator of finite population mean in double sampling when coefficient of variation and coefficient of kurtosis of auxiliary variates are known. Tailor and Sharma (2013) ^[6] developed ratio-cum-product estimator of finite population mean in double sampling when coefficient of variation and coefficient of kurtosis of auxiliary variable are known. Lakhare and Tailor (2017) ^[3] developed an improved ratio-cum-product estimators of finite population mean using known parameters of two auxiliary variates in double sampling. Mehta and tailor (2020) ^[7] developed chain ratio-type estimator of population mean in double sampling. Gupta and Tailor (2021) ^[2] developed a ratio in ratio type exponential strategy for the estimation of population mean.

Consider the following notations

$\Omega = \{\Omega_1, \Omega_2, \dots, \Omega_N\}$, a finite population of size N .

y = Study variate.

x = Auxiliary variate.

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$\bar{Y} = \frac{1}{N} \sum_{i=1}^N y_i$ is a population mean of the study variate.

$\bar{X} = \frac{1}{N} \sum_{i=1}^N x_i$ is a population mean of auxiliary variate.

$\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$ is an unbiased estimator of population mean $\bar{Y}$.

$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ is an unbiased estimator of population mean $\bar{X}$.

$\bar{z} = \frac{1}{n} \sum_{i=1}^n z_i$ is an unbiased estimator of population mean $\bar{Z}$.

$\bar{x}' = \frac{1}{n'} \sum_{i=1}^{n'} x_i$ is an unbiased estimator of population mean $\bar{X}$ based on the sample of size n' .

$\bar{z}' = \frac{1}{n'} \sum_{i=1}^{n'} z_i$ is an unbiased estimator of population mean $\bar{Z}$ based on the sample of size n' .

The usual ratio estimator of population mean $\bar{Y}$ in double sampling is defined as

$$\hat{Y}_R^d = \bar{y} \left(\frac{\bar{x}'}{\bar{x}} \right) \quad (1.1)$$

The mean squared error of the usual ratio estimator to the first degree of approximation in both the cases are given as

Case I

$$MSE(\hat{Y}_R^d)_I = \bar{Y}^2 \left[\frac{C_y^2}{n} + f_3 C_x^2 (1 - 2K_{yx}) \right] \quad (1.2)$$

Case II

$$MSE(\hat{Y}_R^d)_{II} = \bar{Y}^2 \left[\frac{C_y^2}{n} + \frac{C_x^2}{n} (1 - 2K_{yx}) + \frac{C_{x'}^2}{n'} \right] \quad (1.3)$$

Singh and Vishwakarma (2007) ^[4] developed exponential ratio type estimator using auxiliary variable in double sampling is defined as

$$\hat{Y}_{Re}^d = \bar{y} \exp \left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}} \right) \quad (1.4)$$

The mean squared error of the Singh and Vishwakarma (2007) ^[4] ratio type exponential estimator in Double sampling to the first degree of approximation in both the cases are given as

Case I

$$MSE(\hat{Y}_{Re}^d)_I = \bar{Y}^2 \left[\frac{C_y^2}{n} + \frac{f_3 C_x^2}{4} (1 - 4K_{yx}) \right] \quad (1.5)$$

Case II

$$MSE(\hat{Y}_{Re}^d)_{II} = \bar{Y}^2 \left[\frac{C_y^2}{n} + \frac{C_x^2}{4n} (1 - 4K_{yx}) + \frac{C_{x'}^2}{4n'} \right] \quad (1.6)$$

The variance of general estimator of population mean under simple random sampling without replacement is given as

$$V(\bar{y}) = \frac{\bar{Y}^2 C_y^2}{n} \quad (1.7)$$

2. Proposed Estimator

Motivated by Singh and Vishwakarma's (2007) ^[4] exponential ratio type estimator and Mehta and Tailor (2020) ^[7] chain ratio type estimator in double sampling, we developed an exponential type ratio estimator based on the assumption that coefficient of kurtosis of the second auxiliary variate is known as

$$\hat{Y}_{PT} = \bar{y} \exp \left[\frac{\bar{x}' \left(\frac{\bar{Z} + \beta_2(Z)}{\bar{z}' + \beta_2(Z)} \right) - \bar{x}}{\bar{x}' \left(\frac{\bar{Z} + \beta_2(Z)}{\bar{z}' + \beta_2(Z)} \right) + \bar{x}} \right] \quad (2.1)$$

To obtained bias and mean squared error of the proposed estimator $\hat{Y}_{PT}$, let us assume that

The relative error terms are

$e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}},$	$e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}},$	$e'_1 = \frac{\bar{x}'_1 - \bar{X}}{\bar{X}},$	$e_2 = \frac{\bar{z} - \bar{Z}}{\bar{Z}},$	$e'_2 = \frac{\bar{z}' - \bar{Z}}{\bar{Z}},$
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Such that,

$\bar{y} = \bar{Y}(1 + e_0),$	$\bar{x} = \bar{X}(1 + e_1),$	$\bar{x}' = \bar{X}(1 + e'_1),$	$\bar{z} = \bar{Z}(1 + e_2),$	$\bar{z}' = \bar{Z}(1 + e'_2)$
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Now for Case I, when in second phase the required sample of size n drawn from the first phase sample

$$E(e_0) = E(e_1) = E(e'_1) = E(e_2) = E(e'_2) = 0 \text{ and}$$

$E(e_0^2) = \frac{C_y^2}{n},$	$E(e_1^2) = \frac{C_x^2}{n},$	$E(e_1'^2) = \frac{C_x^2}{n'},$
$E(e_2^2) = \frac{C_z^2}{n},$	$E(e_2'^2) = \frac{C_z^2}{n'},$	
$E(e_0 e_1) = \frac{\rho_{yx} C_y C_x}{n},$	$E(e_0 e'_1) = \frac{\rho_{yx} C_y C_x}{n'},$	$E(e_0 e_2) = \frac{\rho_{yz} C_y C_z}{n},$
$E(e_0 e'_2) = \frac{\rho_{yz} C_y C_z}{n'},$	$E(e_1 e'_2) = \frac{\rho_{xz} C_x C_z}{n'},$	$E(e_1' e_2') = \frac{\rho_{xz} C_x C_z}{n'},$

Now we substitute all the values in (2.1), we have

$$\hat{Y}_{PT} = \bar{Y}(1 + e_0) \exp \left[\frac{\bar{X}(1 + e'_1) \left\{ \frac{\bar{Z} + \beta_2(Z)}{\bar{Z}(1 + e'_2) + \beta_2(Z)} \right\} - \bar{X}(1 + e_1)}{\bar{X}(1 + e'_1) \left\{ \frac{\bar{Z} + \beta_2(Z)}{\bar{Z}(1 + e'_2) + \beta_2(Z)} \right\} + \bar{X}(1 + e_1)} \right]$$

$$\hat{Y}_{PT} = \bar{Y}(1 + e_0) \exp \left[\frac{(1 + e'_1) \left\{ \frac{1}{1 + \lambda e'_2} \right\} - (1 + e_1)}{(1 + e'_1) \left\{ \frac{1}{1 + \lambda e'_2} \right\} + (1 + e_1)} \right]$$

$$\hat{Y}_{PT} - \bar{Y} = \bar{Y} \left[\frac{e'_1}{2} - \frac{e_1}{2} - \frac{e_1'^2}{8} + \frac{3e_1^2}{8} - \frac{e_1 e'_1}{4} - \frac{\lambda e'_2}{2} - \frac{\lambda e'_1 e'_2}{4} + \frac{\lambda e_1 e'_2}{4} + \frac{3\lambda^2 e_2'^2}{8} + e_0 + \frac{e_0 e'_1}{2} - \frac{e_0 e_1}{2} - \frac{\lambda e_0 e'_2}{2} \right] \quad (2.2)$$

To obtain the bias of the proposed estimator $\hat{Y}_{PT}$ for Case I, we will take expectation on both sides in eq. (2.2), we have

$$\text{Bias}(\hat{Y}_{PT}) = \bar{Y} \left[\left(\frac{C_x^2}{8n} - \frac{C_x^2}{8n'} \right) (3 - 4K_{yx}) + \frac{C_z^2}{8n'} (3\lambda^2 - 4K_{yz}) \right] \quad (2.3)$$

Where,

C_x is the coefficient of variation of X, C_y is the coefficient of variation of Y and C_z is the coefficient of variation of Z.

ρ_{yx} is the correlation coefficient between y and x, ρ_{yz} is the correlation coefficient between y and z and, ρ_{xz} is the correlation coefficient between x and z.

$C_x^2 = \frac{S_x^2}{\bar{X}^2},$	$C_y^2 = \frac{S_y^2}{\bar{Y}^2},$	$C_z^2 = \frac{S_z^2}{\bar{Z}^2},$
$\rho_{yx} = \frac{S_{yx}}{S_y S_x},$	$\rho_{yz} = \frac{S_{yz}}{S_y S_z},$	$\rho_{xz} = \frac{S_{xz}}{S_x S_z},$
$f_1 = \left(\frac{1}{n} - \frac{1}{N} \right),$	$f_2 = \left(\frac{1}{n'} - \frac{1}{N} \right),$	$f_3 = \left(\frac{1}{n} - \frac{1}{n'} \right),$
$K_{yx} = \rho_{yx} \frac{C_y}{C_x},$	$K_{yz} = \rho_{yz} \frac{C_y}{C_z},$	$K_{xz} = \rho_{xz} \frac{C_x}{C_z},$
$\lambda = \frac{\bar{Z}}{\bar{Z} + \beta_2(Z)},$		

To obtain the mean square error of the proposed estimator $\hat{Y}_{PT}$ for Case I, we will take square and expectation on both sides in eq. (2.2), we have

$$E(\hat{Y}_{PT} - \bar{Y})^2 = \bar{Y}^2 E \left[e_0 + \frac{e_1^2}{2} - \frac{e_1}{2} - \frac{\lambda e'_2}{2} \right]^2$$

$$MSE(\hat{Y}_{PT}) = \bar{Y}^2 E \left[e_0^2 + \frac{e_1'^2}{4} + \frac{e_1^2}{4} + \frac{\lambda^2 e_2'^2}{4} + \frac{2e_0 e'_1}{2} - \frac{2e_0 e_1}{2} - \frac{2\lambda e_0 e'_2}{2} - \frac{2e_1 e'_1}{4} - \frac{2\lambda e'_1 e'_2}{4} + \frac{2\lambda e_1 e'_2}{4} \right]$$

$$MSE(\hat{Y}_{PT}) = \bar{Y}^2 \left[\frac{C_y^2}{n} + \frac{C_x^2}{4} (1 - 4K_{yx}) f_3 + \frac{\lambda C_z^2}{4n'} (\lambda - 4\lambda K_{yz}) \right] \quad (2.4)$$

To obtain the bias of the proposed estimator $\hat{Y}_{PT}$ for Case II, we will take expectation on both sides in eq. (2.2), we have

$$Bias(\hat{Y}_{PT}) = \bar{Y} \left[-\frac{C_x^2}{8n'} + \frac{C_x^2}{8n} (3 - 4K_{yx}) + \frac{\lambda C_z^2}{8n'} (3\lambda - 2K_{xz}) \right] \quad (2.5)$$

To obtain the Mean square error of the proposed estimator $\hat{Y}_{PT}$ for Case II, we will take square and expectation on both sides in eq. (2.2), we have

$$MSE(\hat{Y}_{PT}) = \bar{Y}^2 \left[\frac{C_y^2}{n} + \frac{C_x^2}{4n} (1 - 4K_{yx}) + \frac{\lambda C_z^2}{4n'} (\lambda - 2K_{xz}) + \frac{C_x^2}{4n'} \right] \quad (2.6)$$

3. Efficiency Comparison

In this section the efficiency of proposed estimator has been compared with all other efficiency of existing estimators such as the variance of general estimator of population mean under simple random sampling without replacement, the MSE of usual unbiased ratio estimator in double sampling, MSE of Singh and Vishwakarma (2007) ^[4] ratio type exponential estimator in double sampling.

In Case I,

$$V(\bar{y}) = \frac{\bar{Y}^2 C_y^2}{n} \quad (3.1)$$

$$MSE(\hat{Y}_R^d)_I = \bar{Y}^2 \left[\frac{C_y^2}{n} + f_3 C_x^2 (1 - 2K_{yx}) \right] \quad (3.2)$$

$$MSE(\hat{Y}_{Re}^d)_I = \bar{Y}^2 \left[\frac{C_y^2}{n} + \frac{f_3 C_x^2}{4} (1 - 4K_{yx}) \right] \quad (3.3)$$

Comparison of (3.1) and (2.4) shows that the proposed estimator $\hat{Y}_{PT}$ would be more efficient than the variance of general estimator of population mean under simple random sampling without replacement if

$$MSE(\hat{Y}_{PT})_I - V(\bar{y}) < 0$$

$$\frac{C_x^2}{C_z^2} < \frac{\lambda(4\lambda K_{yz} - \lambda)}{f_3 n' (1 - 4K_{yx})} \quad (3.4)$$

Comparison of (3.2) and (2.4) shows that the proposed estimator $\hat{Y}_{PT}$ would be more efficient than the usual unbiased ratio estimator if

$$MSE(\hat{Y}_{PT})_I - MSE(\hat{Y}_R^d)_I < 0$$

$$\frac{C_x^2}{C_z^2} < \frac{\lambda(4K_{yz} - \lambda)}{f_3 n' (4K_{yx} - 3)} \quad (3.5)$$

Comparison of (3.3) and (2.4) shows that the proposed estimator $\hat{Y}_{PT}$ would be more efficient than the Singh and Vishwakarma (2007) ^[4] estimator if

$$MSE(\hat{Y}_{PT})_I - MSE(\hat{Y}_{Re}^d)_I < 0$$

$$\frac{\lambda}{4} < K_{yz} \quad (3.6)$$

In Case II,

$$V(\bar{y}) = \frac{\bar{Y}^2 C_y^2}{n} \quad (3.7)$$

$$MSE(\hat{Y}_R^d)_{II} = \bar{Y}^2 \left[\frac{C_y^2}{n} + \frac{C_x^2}{n} (1 - 2K_{yx}) + \frac{C_x^2}{n'} \right] \quad (3.8)$$

$$MSE(\hat{Y}_{Re}^d)_{II} = \bar{Y}^2 \left[\frac{C_y^2}{n} + \frac{C_x^2}{4n} (1 - 4K_{yx}) + \frac{C_x^2}{4n'} \right] \quad (3.9)$$

Comparison of (3.7) and (2.6) shows that the proposed estimator $\hat{Y}_{PT}$ would be more efficient than the variance of general estimator of population mean under simple random sampling without replacement if

$$MSE\left(\hat{Y}_{PT}\right)_{II} - V(\bar{y}) < 0$$

$$\frac{C_x^2}{C_z^2} < \frac{\lambda(2K_{xz} - \lambda)}{n' \left[\frac{(1 - 4K_{yx})}{n} + \frac{1}{n'} \right]} \quad (3.10)$$

Comparison of (3.8) and (2.6) shows that the proposed estimator $\hat{Y}_{PT}$ would be more efficient than the usual unbiased ratio estimator if

$$MSE\left(\hat{Y}_{PT}\right)_{II} - MSE\left(\hat{Y}_R^d\right)_{II} < 0$$

$$\frac{C_x^2}{C_z^2} < \frac{\lambda(2K_{xz} - \lambda)}{n' \left[\frac{(4K_{yx} - 3)}{n} + \frac{3}{n'} \right]} \quad (3.11)$$

Comparison of (3.9) and (2.6) shows that the proposed estimator $\hat{Y}_{PT}$ would be more efficient than the Singh and Vishwakarma (2007) ^[4] estimator if

$$MSE\left(\hat{Y}_{PT}\right)_{II} - MSE\left(\hat{Y}_{Re}^d\right)_{II} < 0$$

$$\frac{\lambda}{2} < K_{xz} \quad (3.12)$$

4. Empirical Study

This section compared various estimators empirically. For this purpose, a natural population data set given by Murthy (1967) has been considered.

Table 1: Show Case I and Case II

Variables	Population [Source: Murthy (1967), P. 228]
y	Output
x	Fixed capital
z	Number of Workers

Parameters	Data
N	10
n	2
n'	4
$\bar{Y}$	1925.8
$\bar{X}$	214.4
$\bar{Z}$	51.8
C_y	0.319
C_x	0.349
C_z	0.014
ρ_{yx}	0.853
ρ_{yz}	0.89
ρ_{xz}	0.678
$\beta_2(x)$	1.88
$\beta_2(z)$	1.84

Estimators	Mean Squared Error		Percent Relative Efficiency	
	Case I	Case II	Case I	Case II
$\bar{y}$	188700.79731602		100	
$\bar{y}_R^d$	125533	175295	150.32	107.647
$\bar{y}_{Re}^d$	128884	97299.8	146.411	193.937
$\bar{y}_{PT}^d$	118784	93304.8	158.86	202.241

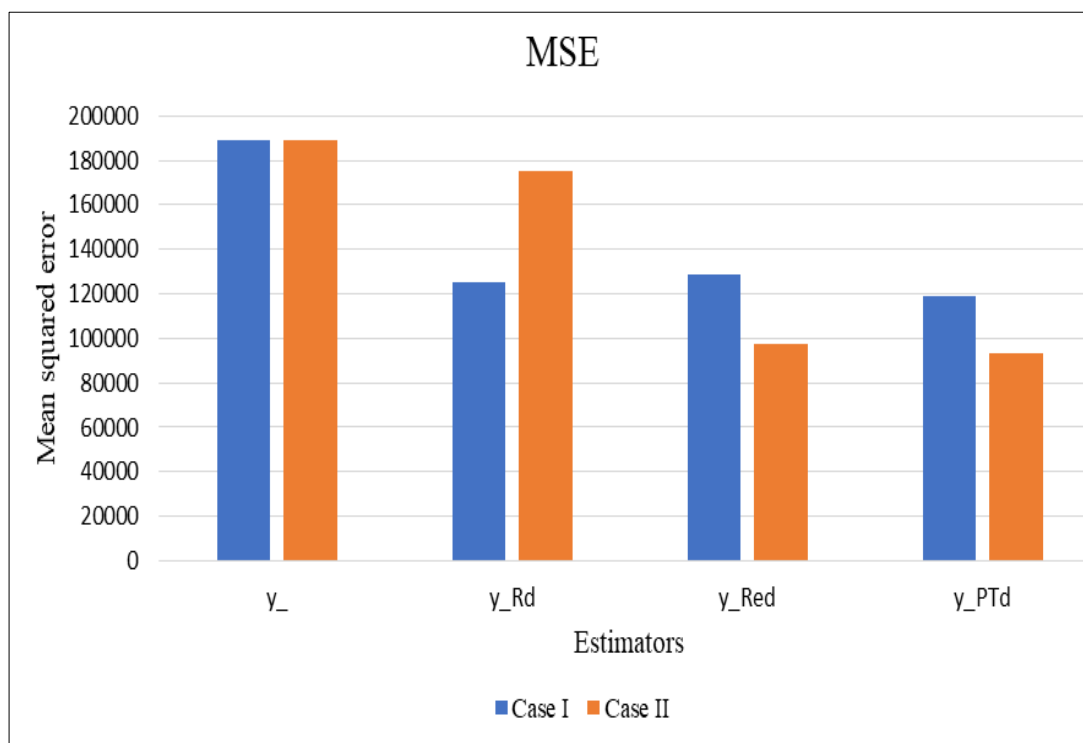


Fig 1: MSE

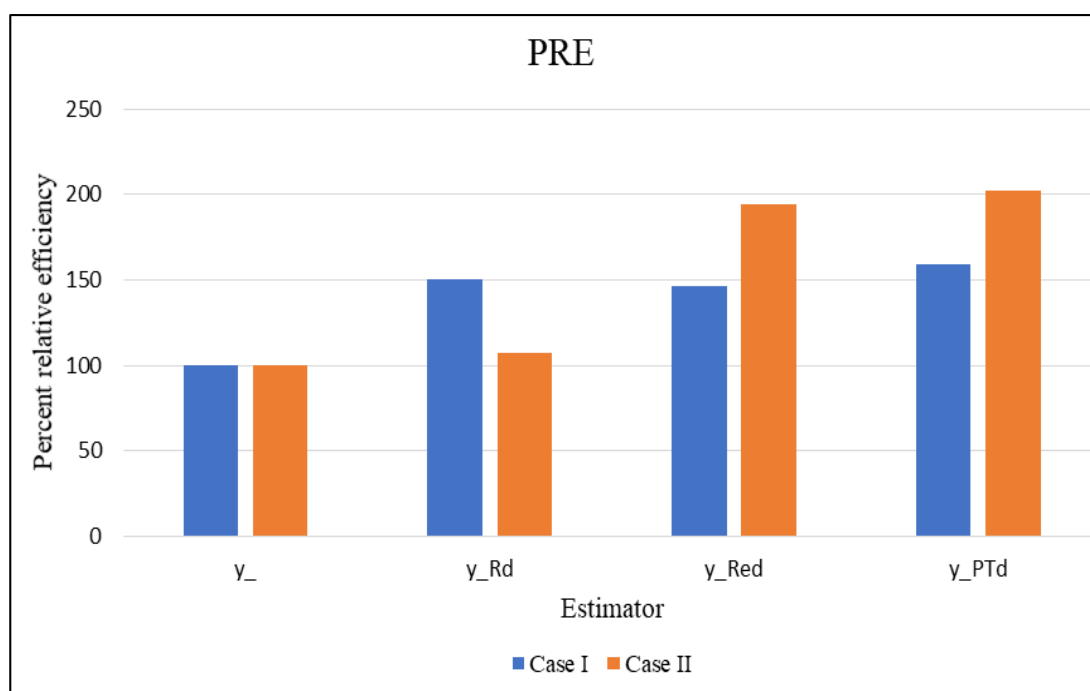


Fig 2: PRE

5. Conclusion

In this study, we have proposed a ratio type exponential estimator and we examined the properties of the proposed estimator such as bias, mean squared error and percent relative efficiency. We have compared the proposed estimator with the other considered estimator that shows that the proposed estimator is more efficient than the other existing estimator under Case I and Case II. These results are shown in Table 1 that the mean squared error of the proposed estimator is minimum and the percent relative efficiency of the proposed estimator is maximum among all other estimators. The result can be seen in the graphs shown in Figure 1 and Figure 2 that clearly indicates performance of estimators.

All things considered, our study makes a substantial theoretical contribution to the estimation of the population mean when double sampling is used. Therefore, it is advised that the proposed estimator can be utilized to estimate the population mean in practical contexts.

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