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E-Bayesian estimation and e-posterior risk for Erlang distribution for known shape parameter

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Abstract

Bayesian estimation is one of the central areas of research in theoretical and mathematical statistics. It has found application in various areas of mathematical finance, medicine, physics, etc. This paper presents a study on expectation of Bayesian estimation (E-Bayesian) on erlang distribution by fixing the shape parameter. E-Bayesian is calculated by taking expectation of bayesian estimate which is calculated by bayesian estimation and MLE combined. Least squared error loss function has been used for the estimation. E-Posterior risk has also been presented in this paper. It is calculated by taking expectation of posterior risk. In end, monte carlo simulation has been performed and the result has been tabulated.

Keywords: E-Bayesian estimation, E-Posterior risk, Erlang distribution, loss function, Monte Carlo simulation, maximum likelihood estimation

I. Introduction

Erlang Distribution is a queuing distribution. It was first proposed by Agner Krarup Erlang. Erlang distribution has following probability density function: -

$$f(x; k, \lambda) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!} \quad \text{for } x, \lambda \geq 0 \quad (1)$$

Where λ a rate parameter setting and k is is shape parameter setting.

Erlang distribution main usage comes in queueing theory, reliability analysis, mathematical biology, etc. Erlang distribution has found its way into statistical research and has been extensively used as well. Kindly refer to Therrar Kadri *et al.* [1], Erik Levén *et al.* [2], Aslam *et al.* [3], M. M. E. Abd El-Monsef [4], etc.

The foundation for Bayesian Statistics was first laid by Thomas Bayes in 18th century. Bayesian Statistics involves prior and posterior distribution. Prior distribution has to do with investigator's belief about the concerned parameter whereas posterior distribution has to do with updated belief after incorporating new data. The are linked with the following relation -

$$\text{posterior} \propto \text{likelihood} \times \text{prior}$$

II. Erlang Distribution Graph

Graph has been shown for the variable values indicated below -

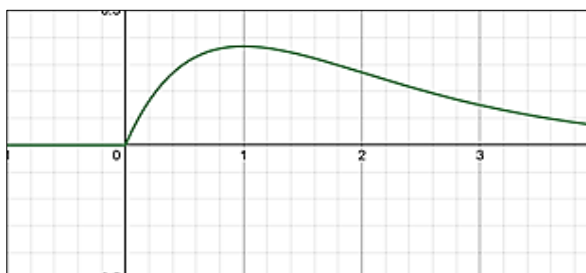


Fig 1: Shape Parameter $K = 2$, Rate Parameter $\lambda =$

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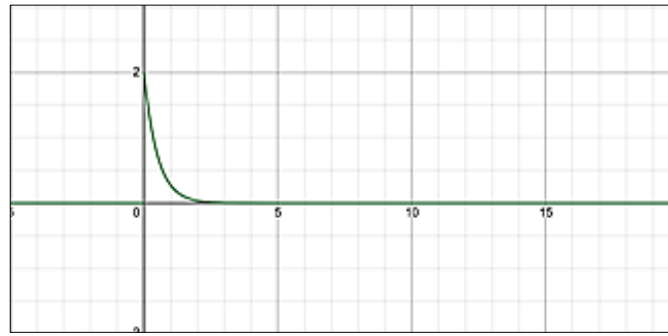


Fig 2: Shape Parameter $K = 1$, Rate Parameter $\lambda = 2$

III. Bayesian Estimation

Let $X_1, X_2, X_3, \dots, X_n$ be observed random variables for the distribution (1). The likelihood function (LF) can be written as

$$L(\lambda|\bar{x}) = \prod_{i=1}^n \frac{\lambda^k x_i^{k-1} e^{-\lambda x_i}}{(k-1)!} \quad (2)$$

Where $\bar{x} = \{x_1, x_2, \dots, x_n\}$. Let $p(\bar{x} | \lambda)$ be the prior distribution. On simplifying -

$$L(\lambda|\bar{x}) = p(\bar{x} | \lambda) = \frac{\lambda^{nk} e^{-\lambda \sum_{i=1}^n x_i}}{(k-1)!} \prod_{i=1}^n x_i^{k-1} \quad (3)$$

For prior distribution we will use gamma distribution.

Gamma distribution has two parameter which we will assume to be known parameter. It has the following pdf

$$g(\lambda|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda}, \lambda > 0 \quad (4)$$

Joint distribution of λ and $\bar{x}$ is -

$$f(\lambda, \bar{x}) = \frac{\lambda^{nk} e^{-\lambda \sum_{i=1}^n x_i}}{(k-1)!} \prod_{i=1}^n x_i^{k-1} \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda} \quad (5)$$

The marginal distribution is -

$$f(\bar{x}) = \int_0^\infty \frac{\lambda^{nk} e^{-\lambda \sum_{i=1}^n x_i}}{(k-1)!} \prod_{i=1}^n x_i^{k-1} \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda} d\lambda \quad (6)$$

$$f(\bar{x}) = \prod_{i=1}^n x_i^{k-1} \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{1}{(k-1)!} \frac{\Gamma(nk + \alpha)}{(\sum_{i=1}^n x_i + \beta)^\alpha} \quad (7)$$

Posterior distribution has the following mathematical structure -

$$q(\lambda | \bar{x}) = \frac{(\sum_{i=1}^n x_i + \beta)^\alpha}{\Gamma(nk + \alpha)} \lambda^{nk + \alpha - 1} e^{-\lambda(\sum_{i=1}^n x_i + \beta)} \quad (8)$$

Using the squared loss error function, and differentiating for reducing the value of loss function we get -

$$\hat{\lambda}(\alpha, \beta) = \frac{nk + \alpha}{\sum_{i=1}^n x_i + \beta} \quad (9)$$

According to Ali *et al* [10] and Han [7], the posterior risk of bayesian estimate $\hat{\lambda}(\alpha, \beta)$ is $R[\hat{\lambda}(\alpha, \beta)] = \text{Var}(\lambda | x)$.

$$R[\hat{\lambda}(\alpha, \beta)] = \frac{nk + \alpha}{(\sum_{i=1}^n x_i + \beta)^2} \quad (10)$$

IV. E-Bayesian Estimation and E-Posterior Risk

The E-Bayesian Estimation of λ becomes

$$\widehat{\lambda}_{EB} = E(\lambda|\bar{x}) = \int \int_p \hat{\lambda}(\alpha, \beta) \pi(\alpha, \beta) d\alpha d\beta \quad (11)$$

Where p is domain or precinct of the prior parameter α and β . The E-Posterior Estimation of λ becomes -

$$E - R(\widehat{\lambda_{EB}}) = \int \int_p R[\hat{\lambda}(\alpha, \beta)] \pi(\alpha, \beta) d\alpha d\beta \quad (12)$$

For simplification, this paper assumes the prior parameters to be independent with bivariate density function -

$$\pi(\alpha, \beta) = \pi_1^*(\alpha) \times \pi_2^*(\beta) \quad (13)$$

The above assumption has also been used for the calculation of E-Bayesian for record statistics in the paper Okasha & Wang^[5] from the same paper, we take the following distribution for α and β (this will further strengthens their work)

$$\pi_1(\alpha, \beta) = \frac{1}{s^*B(u, v)} \alpha^{u-1} (1-\alpha)^{v-1}, 0 < \alpha < 1, 0 < \beta < s \quad (14)$$

$$\pi_2(\alpha, \beta) = \frac{2\beta}{s^2B(u, v)} \alpha^{u-1} (1-\alpha)^{v-1}, 0 < \alpha < 1, 0 < \beta < s \quad (15)$$

I. E-Bayesian Estimation of λ

The following distribution, as mentioned earlier, has been chosen -

$$\pi_1(\alpha, \beta) = \frac{1}{s^*B(u, v)} \alpha^{u-1} (1-\alpha)^{v-1}, 0 < \alpha < 1, 0 < \beta < s \quad \pi_2(\alpha, \beta) = \frac{2\beta}{s^2B(u, v)} \alpha^{u-1} (1-\alpha)^{v-1}, 0 < \alpha < 1, 0 < \beta < s$$

Mean of the independent priors are

a) π_1^* distribution

$$\hat{\alpha} = E\alpha = \frac{u}{u+v}$$

$$\hat{\beta} = E\beta = \frac{s}{2}$$

b) π_1^* distribution

$$\hat{\alpha} = E\alpha = \frac{u}{u+v}$$

$$\hat{\beta} = E\beta = \frac{2s}{3}$$

Both of these distributions follow our assumption of independent bivariate density as presented in the equation (10). The calculation for E-bayesian for the respective distributions have been done below

a) $\pi_1(\alpha, \beta)$ Distribution -

$$\begin{aligned} \widehat{\lambda_{EB\pi_1}} &= \int \int_p \hat{\lambda}(\alpha, \beta) \pi_1(\alpha, \beta) d\alpha d\beta \\ \widehat{\lambda_{EB\pi_1}} &= \frac{1}{sB(u, v)} \int_0^1 \int_0^s \left(\frac{nk + \alpha}{\sum_{i=1}^n x_i + \beta} \right) \alpha^{u-1} (1-\alpha)^{v-1} d\beta d\alpha \\ \widehat{\lambda_{EB\pi_1}} &= \frac{1}{s} \left(nk + \frac{u}{u+v} \right) \ln \left(\frac{\sum_{i=1}^n x_i + s}{\sum_{i=1}^n x_i} \right) \end{aligned} \quad (16)$$

b) $\pi_2(\alpha, \beta)$ Distribution -

$$\begin{aligned} \widehat{\lambda_{EB\pi_2}} &= \int \int_p \hat{\lambda}(\alpha, \beta) \pi_2(\alpha, \beta) d\alpha d\beta \\ \widehat{\lambda_{EB\pi_2}} &= \frac{1}{s^2B(u, v)} \int_0^1 \int_0^s \left(\frac{nk + \alpha}{\sum_{i=1}^n x_i + \beta} \right) 2\beta \alpha^{u-1} (1-\alpha)^{v-1} d\beta d\alpha \\ \widehat{\lambda_{EB\pi_2}} &= \frac{2}{s} \left(nk + \frac{u}{u+v} \right) \left[1 - \frac{\sum_{i=1}^n x_i}{s} \ln \left(\frac{\sum_{i=1}^n x_i + s}{\sum_{i=1}^n x_i} \right) \right] \end{aligned} \quad (17)$$

II. E-Posterior Risk Estimation of λ

a) $\pi_1(\alpha, \beta)$ Distribution -

$$E - R(\widehat{\lambda_{EB\pi_1}}) = \frac{1}{sB(u, v)} \int_0^1 \int_0^s \left(\frac{nk + \alpha}{(\sum_{i=1}^n x_i + \beta)^2} \right) \alpha^{u-1} (1-\alpha)^{v-1} d\beta d\alpha$$

$$= \frac{1}{s} \left(nk + \frac{u}{u+v} \right) \left(\frac{1}{\sum_{i=1}^n x_i} - \frac{1}{\sum_{i=1}^n x_i + s} \right) \quad (18)$$

b) $\pi_2(\alpha, \beta)$ Distribution

$$E - R(\widehat{\lambda_{EB\pi_2}}) = \frac{1}{s^2 B(u, v)} \int_0^1 \int_0^s \left(\frac{nk + \alpha}{(\sum_{i=1}^n x_i + \beta)^2} \right) 2\beta \alpha^{u-1} (1 - \alpha)^{v-1} d\beta d\alpha$$

$$E - R(\widehat{\lambda_{EB\pi_2}}) = \frac{2}{s^2} \left(nk + \frac{1}{2} \right) \left(\ln \left(1 + \frac{s}{\sum_{i=1}^n x_i} \right) - \frac{s}{s + \sum_{i=1}^n x_i} \right) \quad (19)$$

V. Monte Carlo Simulation

Erlang distribution can be generated using the following transformation -

$$X_i \sim \text{Exponential}(\lambda), \sum_{i=1}^k X_i \sim \text{Erlang}(k, \lambda)$$

For our simulation the following steps are used

- We take fixed values for our prior (u, v) and (0, s). With these values we generate α and β from (14) and (15).
- We fix our shape parameter k = 3.
- For generating $\hat{\lambda}(\alpha, \beta)$ we take mean of $\pi_1^*(\alpha)$ and $\pi_2^*(\beta)$. This will give our (α, β) .
- With (α, β) , we generate the prior (4).
- With the formulas, generate bayesians $\hat{\lambda}(\alpha, \beta)$, $\widehat{\lambda_{EB\pi_1}}$ and $\widehat{\lambda_{EB\pi_2}}$ and posteriors $R[\hat{\lambda}(\alpha, \beta)]$, $R(\widehat{\lambda_{EB\pi_1}})$ and $R(\widehat{\lambda_{EB\pi_2}})$.

Table 1: Simulation for s = 100, u = 2, v = 3 and rate = 0.1

$R(\widehat{\lambda_{EB\pi_2}})$	0.067062 83	0.019857 7	0.019457 15	0.010254 9	0.009190 167	0.006285 464	0.007284 657	0.003059 813	0.001391 289	0.000708 606	0.000669 768	0.000333 383	0.000173 824	8.21223 E-05	6.63526 E-05
$R \pi_2$ [$\hat{\lambda}(\alpha, \beta)$]	0.000833 332	0.000398 182	0.000401 123	0.000211 755	0.000188 986	0.000128 13	0.000148 453	0.000061 77	0.000027 954	1.42056E -05	1.34231E -05	6.67459E -06	3.47832E -06	1.64287 E-06	1.32733 E-06
$R(\widehat{\lambda_{EB\pi_1}})$	0.005268 939	0.000551 991	0.000484 027	0.000233 802	0.000200 248	0.000131 995	0.000152 735	6.24981E -05	0.000028 103	1.42429E -05	1.34536E -05	6.68208E -06	3.48031E -06	1.64332 E-06	1.32762 E-06
$R \pi_1$ [$\hat{\lambda}(\alpha, \beta)$]	0.001080 981	0.000454 081	0.000440 992	0.000225 253	0.000197 004	0.000131 278	0.000151 966	6.24332E -05	2.80965E -05	0.000014 242	0.000013 453	6.68201E -06	3.4803E- 06	1.64332 E-06	1.32762 E-06
$\widehat{\lambda_{EB\pi_2}}$	0.056175 2	0.048992 93	0.068495 7	0.055792 59	0.074494 48	0.087012 66	0.104682 9	0.095894 15	0.091409 25	0.092235 61	0.100251 9	0.100017 4	0.102129 6	0.09927 317	0.09976 613
$\hat{\lambda}_{\pi_2}(\alpha, \beta)$	0.046735 68	0.047272 07	0.067262 56	0.055368 06	0.074238 38	0.086910 48	0.104568 8	0.095872 31	0.091404 53	0.092234 4	0.100250 9	0.100017 1	0.102129 5	0.09927 315	0.09976 612
$\widehat{\lambda_{EB\pi_1}}$	0.097197 16	0.057484 9	0.076264 4	0.059631 38	0.077810 17	0.089207 78	0.107223 4	0.096935 35	0.091877 59	0.092472 99	0.100476 1	0.100128 7	0.102187 6	0.09930 056	0.09978 826
$\hat{\lambda}_{\pi_1}(\alpha, \beta)$	0.060624 54	0.053908 43	0.073947 95	0.058897 31	0.077388 14	0.089045 97	0.107043 81	0.096901 81	0.091870 43	0.092471 16	0.100474 6	0.100128 4	0.102187 5	0.09930 054	0.09978 825
n	1	2	4	5	10	20	25	50	100	200	250	500	1000	2000	2500

Table 2: Simulation for s = 100, u = 3, v = 2 and rate = 0.1

$R(\widehat{\lambda_{EB\pi_2}})$	0.067062 83	0.019857 7	0.019457 15	0.010254 9	0.009190 167	0.006285 464	0.007284 657	0.003059 813	0.001391 289	0.000708 606	0.000669 768	0.000333 383	0.000173 824	8.21223 E-05	6.63526 E-05
$R \pi_2$ [$\hat{\lambda}(\alpha, \beta)$]	0.000833 332	0.000398 182	0.000401 123	0.000211 755	0.000188 986	0.000128 13	0.000148 453	0.000061 77	0.000027 954	1.42056E -05	1.34231E -05	6.67459E -06	3.47832E -06	1.64287 E-06	1.32733 E-06
$R(\widehat{\lambda_{EB\pi_1}})$	0.005268 939	0.000551 991	0.000484 027	0.000233 802	0.000200 248	0.000131 995	0.000152 735	6.24981E -05	0.000028 103	1.42429E -05	1.34536E -05	6.68208E -06	3.48031E -06	1.64332 E-06	1.32762 E-06
$R \pi_1$ [$\hat{\lambda}(\alpha, \beta)$]	0.001080 981	0.000454 081	0.000440 992	0.000225 253	0.000197 004	0.000131 278	0.000151 966	6.24332E -05	2.80965E -05	0.000014 242	0.000013 453	6.68201E -06	3.4803E- 06	1.64332 E-06	1.32762 E-06
$\widehat{\lambda_{EB\pi_2}}$	0.056175 2	0.048992 93	0.068495 7	0.055792 59	0.074494 48	0.087012 66	0.104682 9	0.095894 15	0.091409 25	0.092235 61	0.100251 9	0.100017 4	0.102129 6	0.09927 317	0.09976 613
$\hat{\lambda}_{\pi_2}(\alpha, \beta)$	0.046735 68	0.047272 07	0.067262 56	0.055368 06	0.074238 38	0.086910 48	0.104568 8	0.095872 31	0.091404 53	0.092234 4	0.100250 9	0.100017 1	0.102129 5	0.09927 315	0.09976 612
$\widehat{\lambda_{EB\pi_1}}$	0.097197 16	0.057484 9	0.076264 4	0.059631 38	0.077810 17	0.089207 78	0.107223 4	0.096935 35	0.091877 59	0.092472 99	0.100476 1	0.100128 7	0.102187 6	0.09930 056	0.09978 826
$\hat{\lambda}_{\pi_1}(\alpha, \beta)$	0.060624 54	0.053908 43	0.073947 95	0.058897 31	0.077388 14	0.089045 97	0.107043 81	0.096901 81	0.091870 43	0.092471 16	0.100474 6	0.100128 4	0.102187 5	0.09930 054	0.09978 825
n	1	2	4	5	10	20	25	50	100	200	250	500	1000	2000	2500

Table 3: Simulation for $s = 100$, $u = 2$, $v = 3$ and rate = 10

$R(\lambda_{EB\pi_2})$	0.067062 83	0.019857 7	0.019457 15	0.010254 9	0.009190 167	0.006285 464	0.007284 657	0.003059 813	0.001391 289	0.000708 606	0.000669 768	0.000333 383	0.000173 824	8.21223 E-05	6.63526 E-05
R_{π_2} [$\hat{\lambda}(\alpha, \beta)$]	0.000833 332	0.000398 182	0.000401 123	0.000211 755	0.000188 986	0.000128 13	0.000148 453	0.000061 77	0.000027 954	1.42056E -05	1.34231E -05	6.67459E -06	3.47832E -06	1.64287 E-06	1.32733 E-06
$R(\lambda_{EB\pi_1})$	0.005268 939	0.000551 991	0.000484 027	0.000233 802	0.000200 248	0.000131 995	0.000152 735	6.24981E -05	0.000028 103	1.42429E -05	1.34536E -05	6.68208E -06	3.48031E -06	1.64332 E-06	1.32762 E-06
R_{π_1} [$\hat{\lambda}(\alpha, \beta)$]	0.001080 981	0.000454 081	0.000440 992	0.000225 253	0.000197 004	0.000131 278	0.000151 966	6.24332E -05	2.80965E -05	0.000014 242	0.000013 453	6.68201E -06	3.4803E- 06	1.64332 E-06	1.32762 E-06
$\lambda_{EB\pi_2}$	0.056175 2	0.048992 93	0.068495 7	0.055792 59	0.074494 48	0.087012 66	0.104682 9	0.095894 15	0.091409 25	0.092235 61	0.100251 9	0.100017 4	0.102129 6	0.09927 317	0.09976 613
$\hat{\lambda}_{\pi_2}(\alpha, \beta)$	0.046735 68	0.047272 07	0.067262 56	0.055368 06	0.074238 38	0.086910 48	0.104568 8	0.095872 31	0.091404 53	0.092234 4	0.100250 9	0.100017 1	0.102129 5	0.09927 315	0.09976 612
$\lambda_{EB\pi_1}$	0.097197 16	0.057484 9	0.076264 4	0.059631 38	0.077810 17	0.089207 78	0.107223 4	0.096935 35	0.091877 59	0.092472 99	0.100476 1	0.100128 7	0.102187 6	0.09930 056	0.09978 826
$\hat{\lambda}_{\pi_1}(\alpha, \beta)$	0.060624 54	0.053908 43	0.073947 95	0.058897 31	0.077388 14	0.089045 97	0.107043 14	0.096901 81	0.091870 43	0.092471 16	0.100474 6	0.100128 4	0.102187 5	0.09930 054	0.09978 825
n	1	2	4	5	10	20	25	50	100	200	250	500	1000	2000	2500

Table 4: Simulation for $s = 100$, $u = 3$, $v = 2$ and rate = 10

$R(\lambda_{EB\pi_2})$	0.136886	0.2780693	0.4180728	0.5953399	0.7511271	1.274768	1.201227	1.658954	2.057022	2.092104	2.051581	1.675682	1.160169	0.663388 4	0.569296 6
R_{π_2} [$\hat{\lambda}(\alpha, \beta)$]	0.0010522 92	0.0019447 54	0.0036097 42	0.0045512 55	0.0081930 18	0.015414 54	0.017012 03	0.027331 35	0.038302 04	0.042267 31	0.042018 99	0.034894 33	0.023997 94	0.013553 18	0.011594 48
$R(\lambda_{EB\pi_1})$	0.0478018 4	0.1273654	0.0933236 6	0.1927965	0.0884933 4	0.117431 8	0.078495 65	0.079092 54	0.075074 53	0.060932 11	0.056562 18	0.040356 08	0.025694 56	0.013977 87	0.011884 13
R_{π_1} [$\hat{\lambda}(\alpha, \beta)$]	0.0013978 89	0.0025863 9	0.0047817 58	0.0060443 7	0.0107527 3	0.020092 54	0.021830 08	0.034195 42	0.046217 56	0.048604 53	0.047595 64	0.037813 74	0.025156 03	0.013896 94	0.011837 24
$\lambda_{EB\pi_2}$	0.0693608	0.1284118	0.2374567	0.2998963	0.5417566	1.029575	1.176235	2.04516	3.340423	4.893845	5.44836	7.042082	8.324959	8.917073	9.237319
$\hat{\lambda}_{\pi_2}(\alpha, \beta)$	0.0534012 2	0.0982403	0.1852967	0.231216	0.4370642	0.846543 8	1.001127	1.813801	3.088964	4.698496	5.276747	6.951243	8.28814	8.905925	9.229412
$\lambda_{EB\pi_1}$	0.17653	0.3480087	0.5457606	0.7539385	1.049683	1.854466	1.897364	2.954278	4.358554	5.81915	6.326827	7.69726	8.749579	9.149497	9.435075
$\hat{\lambda}_{\pi_1}(\alpha, \beta)$	0.0709394 3	0.1306529	0.2454591	0.3070703	0.5736145	1.103453	1.284661	2.269324	3.727331	5.402951	5.977064	7.532815	8.688105	9.131812	9.422653
n	1	2	4	5	10	20	25	50	100	200	250	500	1000	2000	2500

VI. Conclusion

As you can see from table caption the parameter study has been done on rate parameter, and two parameters of prior distribution. Following can be observed

- On inverting (u, v) from (2, 3) to (3, 2), the bayesian and e-bayesian converges to rate parameter slowly.
- For increasing rate parameter, a larger sample is required to reduce the error for converging to the rate parameter
- In all study e-bayesian study has better perform than traditional likelihood estimation method. (Kindly refer to last row of each study)
- As the sample size increases, the posterior risk decreases in all. Thus, this is in good agreement with the theory. Also, the e-posterior risk has been far lower than the posterior risk.

VII. Further Reading

E-Bayesian Estimation has seen a keen interest by other research and author in the field of mathematics and statistics. The analysis presented in this paper has been done for exponential distribution Han ^[7]. For lomax distribution kindly refer Liu & Zhang ^[8]. Njomen *et al.* ^[9] has done an excellent work by relating competing risks and e-bayesian. The author has further referenced the work of Panda and Bhoi ^[6].

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