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Derivation of a risk model that incorporates in-homogeneous claim counts and financial risks

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Abstract

In the insurance industry, ruin or insolvency occurs when an insurer's total liabilities exceed its assets, rendering it unable to meet future claims. Traditional ruin probability models, such as the Cramer-Lundberg model, assume a homogeneous Poisson claim process with a constant arrival rate, which may not accurately reflect real-world market dynamics. In practice, claim arrivals are often in-homogeneous due to seasonal variations, policyholder behaviors, and external economic factors. Additionally, financial risks including inflation, interest rates, and taxation further influence an insurer's solvency. This study develops an extended risk model that incorporates in-homogeneous claim counts and financial risks, offering a more realistic framework for estimating ruin probabilities in non-life insurance. The proposed model is derived using integral-differential equations and Cramer-Lundberg approximations, providing a refined approach to risk assessment. The findings offer valuable insights for insurers and regulators in designing risk management strategies to mitigate insolvency risks and ensure financial stability.

Keywords: Risk model, classical risk process, in-homogeneous claims, financial risk, Cramer-Lundberg mode

1. Introduction

Risk modeling is a fundamental aspect of actuarial science, providing essential tools for assessing an insurer's financial stability making it a cornerstone of risk theory [9]. One of the core challenges in risk modeling is estimating the probability of ruin, which occurs when an insurer's liabilities exceed its assets, leading to insolvency. The Cramer-Lundberg model, also known as the Classical Risk Model, is a widely used stochastic model for analyzing ruin probability [3]. It assumes that claims arrive according to a homogeneous Poisson process with a constant intensity, while premiums accumulate linearly over time. In its simplest form, the Cramer-Lundberg model assumes that claims are independent and identically distributed (i.i.d.), non-negative random variables, arriving according to a Poisson process with rate $\lambda > 0$, while premiums accumulate at a deterministic linear rate $p > 0$. In this model, independent and identically distributed (i.i.d.) claims arrive according to a Poisson process, while premiums are earned at a constant rate. The reserve level at time $t > 0$, given an initial reserve $u > 0$, is expressed as;

$$R_t = u + pt - \sum_{k=1}^{N_t} L_k \quad (1)$$

where $p > 0$ is the premium rate, $(N_t)_{t \geq 0}$ is a Poisson process with intensity $\lambda > 0$, and $(L_k)_{k \in \mathbb{N}}$ is a sequence of i.i.d. random variables. This model assumes that claims follow a homogeneous Poisson process, meaning they arrive at a constant rate over time, while premium income accumulates linearly. Despite its widespread application, this model has significant limitations in accurately representing real-world insurance markets.

In practice, claim arrivals are often in-homogeneous, influenced by seasonal variations, policyholder behavior, and economic conditions. For instance, auto insurance claims may spike during adverse weather conditions, and health insurance claims may fluctuate with

seasonal disease outbreaks. The assumption of a constant claim arrival rate fails to capture these dynamic variations, leading to misestimated ruin probabilities. Additionally, the classical model does not explicitly incorporate financial risks such as inflation, interest rates, and taxation, which significantly impact an insurer's financial position. Variations in these macroeconomic factors influence premium income, investment returns, and claim payouts, all of which affect the insurer's solvency. The regular occurrence of these catastrophes may impact non-life insurance, similar to how weather conditions influence the level of risk in vehicle and fire insurance^[5].

Several extensions to the classical risk model have been proposed to address some of these limitations. Sparre^[10] extended the compound Poisson model in 1957 whereby rather than assuming exponentially distributed independent inter-claim times, he proposed a more general distribution for the number process, known as a renewal process, while maintaining the assumption of independence. Calvine *et al.*^[2] extended the Cramer-Lundberg risk model by incorporating financial limitations into a collective risk framework to estimate ruin probabilities more accurately. Their study demonstrated that integrating financial factors significantly improved numerical results and simulations for sub-exponential claim distributions. However, their model still assumed a homogeneous claim process, limiting its applicability to real-world insurance settings where claim arrival rates fluctuate over time. Taylor^[12] examined the possibility of the insurer's financial reserves being exhausted resulting in financial collapse by studying the impact of inflationary circumstances and interest rates on the traditional risk model. By including inflation into the model, he discovered that inflation had no impact on free reserves.

Despite previous extensions of the Cramer-Lundberg model, existing studies have not fully integrated both in-homogeneous claim arrivals and financial risk factors into a unified ruin probability framework. This study addresses this gap by developing a refined model that accounts for both stochastic claim dynamics and macroeconomic influences on insurer solvency.

The ruin probability refers to the likelihood of reaching a state of ruin by a certain time instant. This metric is quite prevalent in defining the hazards^[3]. This research considered taxation, interest rate and inflation which were the indicator of financial risk. The likelihood that the insurer's financial reserves are depleted, leading to financial ruin is given by;

$$\Psi(u) = P\{u + R(t) < 0 \text{ for some } t \in (0, \infty)\} \quad (2)$$

This study develops an extended risk model that incorporates in-homogeneous claim counts and key financial risks, including inflation, interest rates, and taxation, to provide a more accurate estimation of ruin probability in non-life insurance. By refining the classical model using integral-differential equations and Cramer-Lundberg approximations, our approach enhances the practical applicability of ruin probability analysis in modern insurance markets.

2. Preliminaries

2.1 In-homogeneous claims

In-homogeneous claims often arise in the context of insurance and actuarial science. It refers to a situation where the claims made by policyholders or insured individuals exhibit a varying or non-uniform pattern over time or across different policyholders. This variation can be due to factors such as changes in policy terms, shifts in economic conditions, or differences in the insured risks. For example, in an insurance company, claims may vary seasonally (more auto accidents in winter), or certain policyholders may be more prone to file claims than others due to their risk profiles^[9].

2.2 Risk Models

In insurance, risk modelling is a critical aspect of assessing and managing various types of risks, including underwriting risk, pricing risk, and reserve risk. These models help insurance companies understand and quantify the potential financial impacts of different scenarios and events^[4].

In non life insurance, which includes various types of non-life insurance coverage like property, casualty, liability, and more, risk models play a crucial role in assessing and managing the risks associated with these policies. These models help insurance companies price policies appropriately, set appropriate reserves, and make informed decisions about risk mitigation.

2.3 Classical Risk Process

The risk process serves as a stochastic model to represent the financial status of an insurance company. The risk process is defined as a stochastic process, denoted by;

$$V(t) = pt - \sum_{k=1}^{N(t)} L_k, \quad (3)$$

where:

- $p > 0$ is rate of premiums which is constant, representing the money the company receives per unit of time.
- $N(t)$ is defined as the count of the company claims during $(0, t]$.
- L_k is the magnitude of the cost of a given claim k .

At each time point $T_1, T_2, \dots$ (the points where N grows) whereby $T_1, T_2, \dots$ denote random time points, the company must disburse a stochastic amount of money while consistently receiving a deterministic set p units of money per unit time. $V(t)$ symbolizes the company's profit during the time span $(0, t]$. Typically, an insurance company initiates its operations with an initial capital u . The minimum amount of initial capital is stipulated by regulators. The company is considered ruined if, starting with the initial capital u , the wealth $u + V(t)$ becomes negative at some point in time.

The above risk process is said to be classical risk process if;

- $\{L_k\}_{k=1}^{\infty}$ are independent and identically distributed random variable with common distribution function $F(l)$ with expected value $E[L_k] = \mu$, and $F(0) = 0$
- $N(t)$ is a Poisson process which is homogenous having the intensity λ and independent of $\{L_k\}$

2.4 Cramer-Lundberg model

The classical risk process, recognized as the foundational model in actuarial risk mathematics, serves as the basis for our current study. The risk surplus represents the excess of a non-life insurance portfolio's premium income over its claim payment and it is a company's reserve representation level. The primary objective of this classical model is to estimate the ruin probability for a given initial surplus. This classical risk process is characterized mathematically by a fundamental equation in actuarial science [4] and [6]. This equation, also known as the surplus equation, is defined as:

$$R(t) = u + pt - Q(t), t \geq 0, \quad (4)$$

where,

$u = R(0)$ depicts the initial surplus, pt represents the premium inflow collected up to time t (assuming $p \geq 0$), and $Q(t)$ is defined as:

$$Q(t) = \begin{cases} \sum_{k=1}^{N(t)} L_k, & \text{for } t > 0 \\ 0, & t = 0 \end{cases} \quad (5)$$

where $L_k, k = 1, 2, \dots, N(t)$, are individual cost of claims, and $N(t)$ represents the claim count.

The process $\{Q(t), t \geq 0\}$ is termed a Poisson process with Poisson(λ). The density function of L_k is denoted by $F(l)$, and it is assumed throughout the entire research that $F(0) = 0$ to preclude negative amounts of claims. Understanding the practical implications of Equation (4) is crucial. In practical terms, these parameters can be interpreted as follows: Mathematically, $R(t)$ is termed a risk process or the risk of insurance at time t . This value, mirroring the current capital, indicates the level of risk associated with the insurance portfolio at time t . An increase (decrease) in this value conveys favourable (unfavourable) news for the entire insurance portfolio, as its total capital is inclined to rise (fall). If $R(t)$ reaches 0 or turns negative, it signifies the ruin of the insurance company. $R(0) = u$ represents the initial capital, denoting the number of resources required to establish an insurance portfolio. Lastly, $Q(t)$ is the claims process.

3. Main Results

3.1 Derivation of the Asset Pricing Equation with Transaction Cost

This study refines the classical Cramer-Lundberg model defined by Equation (4) by incorporating financial risk factors such as interest rates and inflation. These modifications enhance the surplus process model, improving its applicability in real-world economic conditions. The modification follows Fisher's proposition, which defines the relationship between interest rates and inflation as expressed in the following equation:

$$\delta_{(i,f)} = \frac{\log_e(1+i)}{\log_e(1+f)} = \frac{\ln(1+i)}{\ln(1+f)}, \quad (6)$$

where:

$\delta_{(i,f)}$ is the modified force of interest based on interest rate i and the inflation rate f .

Let $R_{\delta(i,f)}(n)$ represent reserve at time n adjusted for interest rate i and inflation rate f . Applying the relationship in Equation (6) to the surplus process in Equation (4), and incorporating Klugman's reserve process model in Klugman [12], we derive the following refined surplus equation:

$$dR_{\delta(i,f)}(n) = \{P + R_{\delta(i,f)}(n) \cdot \delta_{(i,f)}\}dn - dQ(n) \quad (7)$$

which is ordinary differential equation.

To obtain a closed-form solution, we integrate Equation (7) from 0 to n .

$$\int_0^n \frac{d}{dn} (e^{-\delta_{(i,f)} \cdot n} \cdot R_{\delta(i,f)}(n)) dn = P \int_0^n e^{-\delta_{(i,f)} \cdot n} dn - \int_0^n e^{-\delta_{(i,f)} \cdot n} \frac{dQ(n)}{dn} dn \quad (8)$$

By applying fundamental theorem of calculus by Patrik [13], we obtain;

$$e^{-\delta_{(i,f)} \cdot n} \cdot R_{\delta(i,f)}(n) - u = \frac{-P}{\delta_{(i,f)}} [e^{-\delta_{(i,f)} \cdot n} - 1] - \int_0^n e^{-\delta_{(i,f)} \cdot s} dQ(s) \quad (9)$$

Solving for $R_{\delta(i,f)}(n)$

$$\begin{aligned} R_{\delta(i,f)}(n) &= e^{\delta_{(i,f)} \cdot n} \cdot u - \frac{P}{\delta_{(i,f)}} [e^{-\delta_{(i,f)} \cdot n} - 1] \cdot e^{-\delta_{(i,f)} \cdot n} - \int_0^n e^{-\delta_{(i,f)} \cdot s} dQ(s) \cdot e^{-\delta_{(i,f)} \cdot n} \\ &= e^{\delta_{(i,f)} \cdot n} - \frac{P}{\delta_{(i,f)}} [1 - e^{\delta_{(i,f)} \cdot n}] - \int_0^n e^{-\delta_{(i,f)} \cdot (n-s)} dQ(s) \end{aligned} \quad (10)$$

To capture in-homogeneous claim processes, we approximate the intensity function $Q(s)$ based on empirical data from Grabski [7] and Helcom [8] whereby the statistical analysis of non-life insurance data shows that the intensity function $Q(s)$ can be approximated by the linear function of the form;

$$Q(s) = a + bs \quad (11)$$

Where a is the baseline claim intensity, while b represents time-dependent variations in claims. Therefore, equation (10) can be simplified to;

$$R_{\delta(i,f)}(n) = ue^{\delta(i,f) \cdot n} \cdot u - \frac{P}{\delta(i,f)} [1 - e^{\delta(i,f) \cdot n}] - \frac{b}{\delta(i,f)} [e^{\delta(i,f) \cdot n} - 1] \quad (12)$$

$$\begin{aligned} &= ue^{\delta(i,f) \cdot n} + \frac{P}{\delta(i,f)} [e^{\delta(i,f) \cdot n} - 1] - \frac{b}{\delta(i,f)} [e^{\delta(i,f) \cdot n} - 1] \\ &= ue^{\delta(i,f) \cdot n} + (P - b) \left[\frac{e^{\delta(i,f) \cdot n} - 1}{\delta(i,f)} \right] \end{aligned} \quad (13)$$

Applying the accumulated continuous annuity formula, Equation (13) simplifies to;

$$R_{\delta(i,f)}(n) = ue^{-\delta(i,f) \cdot n} + (P - b) \bar{S}_{n|} @ \delta(i,f) \cdot n \quad (14)$$

Discounting back the accumulated value in equation (14);

$$\begin{aligned} PV_{\delta(i,f)}(n) &= (R_{\delta(i,f)}(n) \cdot e^{-\delta(i,f) \cdot n}) = e^{-\delta(i,f) \cdot n} \cdot ue^{\delta(i,f) \cdot n} + (P - b) \bar{S}_{n|} \cdot e^{-\delta(i,f) \cdot n} \\ PV_{\delta(i,f)}(n) &= u + (P - b) \bar{a}_{n|} @ \delta(i,f) \end{aligned} \quad (15)$$

Incorporating the effect of taxation into the premium rate involves adjusting the premium income by tax rate [1].

We include taxation rate γ on the premium income leading to the taxed premium becomes $P(1 - \gamma)$. Thus, the premium rate after taxation becomes $P' = P(1 - \gamma)$.

Therefore, by invoking Albrecher and Hipp [1], we have

$$\begin{aligned} PV_{\delta(i,f)} &= u + P(1 - \gamma) \bar{a}_{n|} - \bar{b}_{n|} @ \delta(i,f) \\ &= u + [P(1 - \gamma) \bar{a}_{n|} @ \delta(i,f)] \end{aligned} \quad (21)$$

4 Conclusion

This section derived an extended Cramer-Lundberg risk model that incorporates in-homogeneous claim processes and financial risks using integral-differential techniques. By considering interest rates, inflation, and taxation, the model enhances ruin probability estimation, offering practical applications for insurers and regulatory bodies in designing robust risk management frameworks.

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