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Decision support in legal systems through Fermat's fuzzy graph modeling

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Abstract

Legal adjudication often unfolds amid ambiguity, conflicting narratives, and variable evidentiary strength, making binary or linear models insufficient for capturing real-world judicial reasoning; this makes Fermat's Fuzzy Graph (FFG) especially important in law because it provides a mathematically rigorous way to represent partial truth, contradictory evidence, and nonlinear shifts in culpability. Using nonlinear dual-membership functions for acceptance (μ) and rejection (ν), FFG models uncertainty far more accurately than traditional frameworks. The December 8, 2025 verdict in the Kerala actress assault and conspiracy case where actor Dileep was acquitted while six co-accused were convicted illustrates how differing evidentiary weights can produce sharply divergent outcomes within the same fact pattern, with minor weaknesses collapsing a conspiracy allegation while strong corroboration sustains direct liability. This nonlinear behavior aligns with the Fermat restriction ($\mu^n + \nu^n \leq 1$), which captures threshold effects and steep evidentiary transitions by representing actors, witnesses, digital evidence, and legal constructs as dual-weighted vertices and edges in a network of acceptance-rejection relationships. More broadly, legal systems worldwide increasingly confront uncertainty, partial knowledge, and interpretational vagueness in evidence evaluation, constitutional balancing, human rights analysis, and regulatory compliance conditions that classical logic and deterministic models cannot adequately represent. Fermat's Fuzzy Sets (FFS), Fermat's Fuzzy Relations (FFR), and Fermat's Fuzzy Graphs (FFG(n)) advance beyond classical fuzzy and intuitionistic frameworks by allowing both support and contradiction to coexist on every edge, enabling unprecedented precision in modeling ambiguous statutes, contradictory testimonies, and competing legal rights. This research, which presents 25 detailed applications of FFG across criminal law, constitutional law, corporate regulation, intellectual property and judicial bias detection, etc., demonstrates how FFG-based methodologies incorporating graph construction, multi-expert aggregation, Fermat distance metrics, fuzzy conflict resolution, and legal decision-ranking algorithms can transform legal AI, judicial support systems, evidence evaluation tools, and automated compliance engines. Altogether, the FFG framework enhances clarity, transparency, and analytical precision in complex legal scenarios, offering a mathematically grounded and ethically coherent pathway for the future of computational legal reasoning.

Keywords: Fermat's fuzzy graphs (FFG), legal uncertainty modeling, fuzzy logic in judicial decision-making, nonlinear evidence evaluation, computational legal reasoning

Introduction

The law is fundamentally a system built upon interpretation, context, and judgment under conditions of uncertainty. Accused persons, judges, prosecutors, lawyers, investigators, governments, arbitrators rarely operate with complete or consistent information. Evidence may be contradictory, statutes ambiguous, testimonies unreliable, rights conflicting, and precedents partially applicable. Traditional legal reasoning, although robust and principled, has long struggled with quantitatively representing uncertainty in a structured, transparent, and logically coherent manner.

Mathematics entered legal studies through probability theory, Bayesian inference, and statistics. However, these numerical approaches often assume availability of precise data and fail to capture linguistic ambiguity, interpretational vagueness, or non-quantifiable uncertainty. Fuzzy set theory, introduced by Lotfi Zadeh, attempted to solve these issues by offering membership degrees between 0 and 1.

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Yet classical fuzzy sets assign only a single membership degree, which inadequately models contradictions. Intuitionistic fuzzy sets introduced dual memberships (acceptance and non-acceptance), but still lacked expressive power when dealing with legal contradictions where support and rejection may both exist strongly but independently. Pythagorean fuzzy sets improved representation but remained limited in complex conflict-heavy scenarios.

We introduced Fermat's Fuzzy Sets (FFS) and Fermat's Fuzzy Graphs FFG (n) in 2022. FFG (n) represent a modern breakthrough. They allow each relationship to be encoded with both degree of acceptance (μ) and degree of rejection (ν), under a Fermat-distance constraint: $\mu^n(e) + \nu^n(e) \leq 1$, $n > 1$. Evidence is partially supporting (μ moderate or high), but also partially contradicting (ν moderate or high) precisely the kind of situation that dominates legal reasoning.

Fermat's Fuzzy Graphs extend classical graph theory by introducing graded relationships and hesitancy parameters, enabling the modeling of complex legal systems where certainty is rarely absolute. In these graphs, vertices and edges carry degrees of membership and non-membership, allowing legal networks to represent contradictory witness reports, uncertain digital traces, and partial treaty compliance within a unified framework. They naturally capture mixed mitigation and aggravation factors in sentencing by assigning weighted influences rather than binary judgments, and they model partial similarity in intellectual property infringement through fuzzy relational overlap. In constitutional law, Fermat's Fuzzy Graphs effectively represent rights conflicts by quantifying competing claims, while ambiguity in contractual clauses and fuzzy causation in tort law are handled through graded causal and semantic links. Furthermore, degree-based analysis within these graphs facilitates the detection of judicial bias and uneven influence patterns, and risk-linked regulatory compliance can be expressed as layered fuzzy dependencies, making Fermat's Fuzzy Graphs a powerful mathematical tool for nuanced legal reasoning under uncertainty. This article presents the first comprehensive multi-domain legal application of FFG(n) with 25 fully developed applications. A case study of evidence evaluation in homicide case is also included.

Theoretical foundations of Fermat's fuzzy graphs in law

1.1. Fuzzy Uncertainty in Legal Systems

Legal uncertainty is inherently multi-dimensional, arising from the complex and often indeterminate nature of legal reasoning and factual assessment. Evidentiary uncertainty emerges when testimonies conflict or forensic evidence is incomplete or unreliable, while interpretive uncertainty stems from ambiguous statutory language or vague contractual terms that allow multiple reasonable readings. Contextual uncertainty reflects the influence of unique case facts, evolving social norms, and cultural settings, making uniform application of rules difficult. Predictive uncertainty concerns assessments about future behavior, such as the likelihood of reoffending or the risk of harm, which cannot be known with certainty. Causal uncertainty is prominent in tort and environmental cases, where linking conduct to harm involves probabilistic and scientific judgments. Credibility uncertainty arises from doubts about witness reliability, motivations, or trauma-induced memory distortions. Authority uncertainty occurs when courts confront conflicting precedents or constitutional tensions across jurisdictions. Compliance uncertainty reflects situations of partial or imperfect adherence to legal norms and regulations. Taken together, these layers of uncertainty reveal why traditional binary legal reasoning such as true versus false or liable versus not liable is often inadequate to capture the nuanced realities of legal decision-making.

1.2. Fermat's Fuzzy Sets and Legal Reasoning

- We introduced Fermat's Fuzzy Set (FFS) and Fermat's Fuzzy graph in 2022. In Fermat's fuzzy logic:
- μ = degree of acceptance (support, credibility, similarity, applicability)
- ν = degree of rejection (contradiction, unreliability, distinguishing factors)

Unlike intuitionistic fuzzy sets, μ and ν are not forced to be complementary

They can both be high for example

A witness may provide highly relevant testimony ($\mu = 0.75$), yet be partially unreliable due to intoxication or bias ($\nu = 0.62$). This maps directly onto courtroom realities.

1.3. Fermat's Fuzzy Graph FFG(n)

A Fermat's fuzzy graph of multiplicity n , FFG(n) on a non empty set A is a pair $G: (\sigma, \mu)$ with σ as FFS on A and μ as FFR on A such that

$$I_\mu(uv) \leq I_\sigma(u) \wedge I_\sigma(v), O_\mu(uv) \geq O_\sigma(u) \vee O_\sigma(v) \text{ and}$$

$$0 \leq I_\mu^n(uv) + O_\mu^n(uv) \leq 1 \text{ for all } u, v \in A, n \in N = \{1, 2, 3, \dots\} \text{ is the least positive integer satisfying } 0 \leq I_\mu^n(uv) + O_\mu^n(uv) \leq 1,$$

where $I_\mu: A \times A \rightarrow [0, 1]$ and $O_\mu: A \times A \rightarrow [0, 1]$ represents the membership and non membership functions of μ respectively. If the set of legal entities are all not vague, then the in degree of entities is 1 and out degrees 0 and so the FFG can be modelled as $G = (V, E, \mu, \nu)$ where V is set of legal entities (evidence, statutes, accused, arguments, obligations), E is the set of relational edges, $\mu(e) = I_\mu(uv)$ the degree of acceptance for edge e and $\nu(e) = O_\mu(uv)$, degree of rejection for edge $e = uv$ subject to $\mu^n(e) + \nu^n(e) \leq 1$, $n > 1$. This graph models legal networks by capturing the complex and often non-binary relationships that exist between legal rules, precedents, and arguments. Conflict is represented through opposing connections where legal norms or interpretations directly challenge one another, while partial support allows nodes to reinforce others to varying degrees rather than in absolute terms. Ambiguity is embedded by permitting indeterminate or overlapping relationships, reflecting the open-textured nature of legal language. Uncertainty propagation enables doubt or incomplete information at one point in the network to influence connected elements, mirroring how uncertainty in evidence or interpretation spreads through legal reasoning. Finally,

contradictory interactions are explicitly modelled, allowing the graph to represent situations where the same legal source both supports and undermines different conclusions, thereby reflecting the realistic complexity of legal decision-making systems.

2. Methodological framework for applying FFG(n) in legal systems

This section details the methodology for constructing and applying Fermat's fuzzy graphs across legal domains.

2.1. Phase 1: Legal Variable Identification

Each legal application begins with the careful identification of nodes, which represent the fundamental elements of a legal system such as statutes, evidence, rights, obligations, accused persons, and testimonies. These nodes capture both normative components (laws, duties, and rights) and factual components (evidence, witness statements, and actions of parties). Once nodes are established, the application defines edge relationships that connect them, expressing how legal elements interact. Edges may indicate support (evidence backing a claim), contradiction (conflicting testimonies), causation (an act leading to legal consequences), similarity (precedents with comparable facts), or influence (how statutes or judgments shape decisions). Together, nodes and edges form a structured legal knowledge graph, enabling systematic reasoning, consistency in analysis, and transparent decision-making.

2.2. Phase 2: Fuzzy Membership Assignment

In many decision-making and evaluation frameworks, experts or intelligent algorithms assign two complementary measures: the degree of acceptance μ , which represents the level of support or belief in a statement, and the degree of rejection ν , which captures the level of contradiction or doubt. For example, when assessing the reliability of digital evidence, μ may reflect confidence in the integrity and authenticity of the data, while ν accounts for concerns such as possible tampering or incomplete metadata. In evaluating the strength of witness corroboration, μ indicates how strongly testimonies reinforce each other, whereas ν represents inconsistencies or conflicting statements. Similarly, in analyzing cause-effect relationships, μ measures confidence that one event led to another, while ν captures alternative explanations or uncertainty. Finally, in determining the applicability of legal precedent, μ expresses how closely a prior case aligns with the current situation, and ν reflects distinguishing factors that limit its relevance.

2.3. Phase 3: Fermat Aggregation

Fermat-based operations combine multiple experts or evidence sources. In the absence of expert bias, we introduced a simple score function $S = \mu - \nu$ for legal interpretation. In the presence of expert bias, equal weights cannot be assigned and so we introduced the following mathematical tools in 2022.

2.3.1 Fermat's fuzzy preference relation (FFPR)

FFPR on the set of nodes $N = \{x_1, x_2, \dots, x_n\}$ is represented by a matrix $M = (m_{ij})_{n \times n}$, where $m_{ij} = (x_i x_j, I(x_i x_j), O(x_i x_j))$ for all $i, j = 1, 2, 3, \dots, n$. Let $m_{ij} = (I_{ij}, O_{ij})$ where I_{ij} indicates the degree to which the node x_i is preferred to node x_j and O_{ij} denotes the degree to which the node x_i is not preferred to the node x_j and $\pi_{ij} = \sqrt[n]{1 - I_{ij}^n - O_{ij}^n}$ is interpreted as hesitancy degree, with the conditions,

$$I_{ij}, O_{ij} \in [0, 1], 0 \leq I_{ij}^n + O_{ij}^n \leq 1, I_{ij} = O_{ji}, I_{ii} = O_{ii} = 0.5 \text{ for all } i, j = 1, 2, 3, \dots, n.$$

2.3.2 Fuzzy Averaging operator FFA

$$FFA(p_{i1}^{(k)}, p_{i2}^{(k)}, \dots, p_{im}^{(k)}) = \left(\sqrt[n]{1 - \left(\prod_{j=1}^m (1 - I_{\mu_{ij}}^n) \right)^{1/m}}, \left(\prod_{j=1}^m (O_{\mu_{ij}}^n) \right)^{1/m} \right), i = 1, 2, 3, \dots, m.$$

Fermat's Fuzzy Element (FFE) $p_i^{(k)}$ indicates preference of each expert E_k over each pair of alternatives.

It is determined using Fermat's Fuzzy Averaging operator FFA

$$p_i^{(k)} = FFA(p_{i1}^{(k)}, p_{i2}^{(k)}, \dots, p_{im}^{(k)}), i = 1, 2, 3, \dots, m. \text{ FFE is used in calculation of FFWA.}$$

2.3.3 Fermat's Fuzzy Hamming distance between FFEs.

From the given FFPR,

$$d(p_{ij}^{(l)}, p_{ij}^{(k)}) = \frac{1}{2} \left(\left| I_{\mu_{p_{ij}^{(l)}}}^n - I_{\mu_{p_{ij}^{(k)}}}^n \right| + \left| O_{\mu_{p_{ij}^{(l)}}}^n - O_{\mu_{p_{ij}^{(k)}}}^n \right| + \left| \pi_{p_{ij}^{(l)}}^n - \pi_{p_{ij}^{(k)}}^n \right| \right), \text{ where}$$

$$\pi_{p_{ij}^{(l)}} = \sqrt[n]{1 - I_{\mu_{p_{ij}^{(l)}}}^n - O_{\mu_{p_{ij}^{(l)}}}^n}, \pi_{p_{ij}^{(k)}} = \sqrt[n]{1 - I_{\mu_{p_{ij}^{(k)}}}^n - O_{\mu_{p_{ij}^{(k)}}}^n} \text{ are hesitancy degree.}$$

2.3.4 Difference matrix

$$D_{lk} = \left(d_{ij}^{(lk)} \right)_{m \times m} = d \left(p_{ij}^{(l)}, p_{ij}^{(k)} \right)_{m \times m}$$

2.3.5 Average Values of Difference Matrix

The equation to determine average values of difference matrix $\overline{d}_{lk} = \frac{1}{m^2} \sum_{i=1}^m \sum_{j=1}^m d_{ij}^{(lk)}$

2.3.6 Deviation of expert E_r from remaining experts

The equation to determine deviation of expert E_r from remaining experts $d_r = \sum_{k=1, k \neq r}^s \overline{d}_{rk}$

2.3.7 Weight of experts w_r .

The equation to determine weight of experts w_r . $w_r = \frac{(d_r)^{-1}}{\sum_{r=1}^s (d_r)^{-1}}, r = 1, 2, \dots, s.$

2.3.8 Fermat's Fuzzy Weighed Averaging operator FFWA

Fermat's Fuzzy Weighed Averaging operator FFWA to compute collective Fermat's Fuzzy Element p_i over other alternatives is

$$p_i = FFWA \left(p_i^{(1)}, p_i^{(2)}, \dots, p_i^{(s)} \right) = \left(\sqrt[n]{1 - \prod_{k=1}^s \left(1 - I_{p_i^{(k)}}^n \right)^{w_k}}, \prod_{k=1}^s \left(O_{p_i^{(k)}} \right)^{w_k} \right) = (I_{p_i}, O_{p_i})$$

2.4. Phase 4: Legal Uncertainty Propagation

Using Fermat's Fuzzy Graphs of order n (FFG(n)), uncertainty can be modelled as a structured flow across edges, where each connection carries graded, hesitant, or partially reliable information rather than binary truth. In crime investigation graphs, FFG(n) represents uncertain relationships between suspects, evidence, and events; edge weights capture degrees of suspicion, reliability of witnesses, or probabilistic links between actions, allowing investigators to trace how uncertainty propagates through the network. In argumentation networks, FFG(n) models support and attack relations among arguments, where edges express varying strengths of persuasion or rebuttal under incomplete knowledge, enabling nuanced evaluation of competing claims. Within constitutional conflict models, FFG(n) captures ambiguous interactions between legal provisions, institutions, and interpretations, with uncertainty flowing along edges that reflect conflicting precedents, interpretive hesitancy, or political influence. Similarly, in tort causality mappings, FFG(n) represents uncertain causal chains between actions, harm, and liability, where edge values encode partial causation, intervening factors, or evidentiary doubt, thus providing a mathematically rigorous framework to analyse how legal and factual uncertainty diffuses across complex relational systems.

2.5. Phase 5: Decision Ranking and Resolution

A ranking mechanism determines the most plausible evidentiary set by systematically evaluating the relevance, consistency, and probative value of available facts and materials. It identifies the optimal legal interpretation by comparing competing readings of statutes, principles, or doctrines and prioritizing those that best align with established legal reasoning and contextual intent. By assessing temporal, logical, and factual coherence, the mechanism selects the most likely causal chain linking actions to outcomes. It also groups and weighs judicial decisions to identify the strongest precedent cluster that offers authoritative and persuasive guidance. Finally, it determines the most balanced constitutional resolution by harmonizing fundamental rights, institutional powers, and societal interests, ensuring that the final outcome is both legally sound and normatively justified.

2.5.1 Score Function

In the absence of expert bias a simple score function $S = \mu - \nu$ is enough for legal interpretation. In cases of expert bias, Score function $S(p_i)$ to rank the legal entities $u_i, i = 1, 2, \dots, m$, $S(p_i) = (I_{p_i}^n - O_{p_i}^n)(I_{\sigma}(u_i) - O_{\sigma}(u_i)), u_i, i = 1, 2, \dots, m.$

3. Some applications of FFG(n) IN LAW

Application 1

Legal evidence evaluation under uncertainty

Legal evidence rarely exists in a clean, coherent, or perfectly corroborated state. Witness memories degrade, forensic tests have error margins, digital timestamps may be manipulated, and documentary evidence may be incomplete. Fermat's fuzzy graphs provide a structured mathematical means of representing the combined reliability, credibility, and coherence of diverse evidence items. In an evidence graph, each node represents a piece of evidence, and edges represent inferential relations—e.g., one testimony corroborating another, or a forensic report contradicting a witness claim. Each edge carries a degree of acceptance μ ,

representing support (e.g., a bloodstain pattern supporting a timeline with $\mu = 0.74$), and a degree of rejection v , representing contradiction or uncertainty (e.g., unreliable camera angle resulting in $v = 0.53$). Because Fermat's framework allows μ and v to coexist strongly, it precisely models real courtroom conflicts where an evidence item both helps and hurts the case simultaneously. The resulting evidence graph supports judges and juries by providing a quantified visualization of evidentiary coherence, helping to weigh clusters of mutually supporting evidence more heavily than isolated or contradictory elements.

3.1.1 Illustrative Example

Homicide Case - Evidence Evaluation under Uncertainty

3.1.1 Case Study

In the late hours of a bustling urban street near the commercial district, between 22:30 and 23:15, a violent incident culminated in the death of V_1 . The accused, A_1 , was identified as a person of interest early in the investigation. The initial breakthrough came from the street CCTV system (E1), which captured fragments of the suspect near the crime scene at approximately 22:48. However, the footage was far from definitive. Forensic analysis (E2) revealed poor lighting conditions and intermittent frame loss, reducing the clarity of the visual record and introducing doubt about the precise movements of A_1 . Complementing the visual evidence, bloodstain analysis (E3) indicated a match with the victim's DNA, suggesting the physical presence of A_1 or involvement in the incident. Yet, investigators noted a significant contamination risk (E4) because the sample was collected twelve hours after the event, allowing for potential environmental interference or handling errors. Adding to the complexity, a civilian eyewitness (E5) reported seeing the accused near the scene, but further scrutiny revealed limitations in witness reliability: medical reports (E6) confirmed that the witness was under the influence of alcohol during the event, raising questions about perception and memory fidelity.

Taken together, these pieces of evidence created a mosaic of partial truths and uncertainties. While each item lent some degree of support toward establishing A_1 's involvement, the inherent limitations ranging from poor video quality to delayed forensic sampling and compromised eyewitness credibility introduced substantial ambiguity. In such a high-stakes case, traditional binary assessments of guilt versus innocence proved insufficient. Only a structured approach, capable of quantifying both supporting and contradicting elements, could accurately reflect the nuanced evidentiary landscape. This scenario thus exemplifies the necessity for an advanced AI-assisted framework, such as Fermat's Fuzzy Graph (FFG), which can model the interplay of these uncertainties, propagate doubt through the network of evidence, and provide a coherent assessment that mirrors real-world judicial reasoning.

3.1.2 Mathematical Modeling

Case Context (Empirical Scenario)

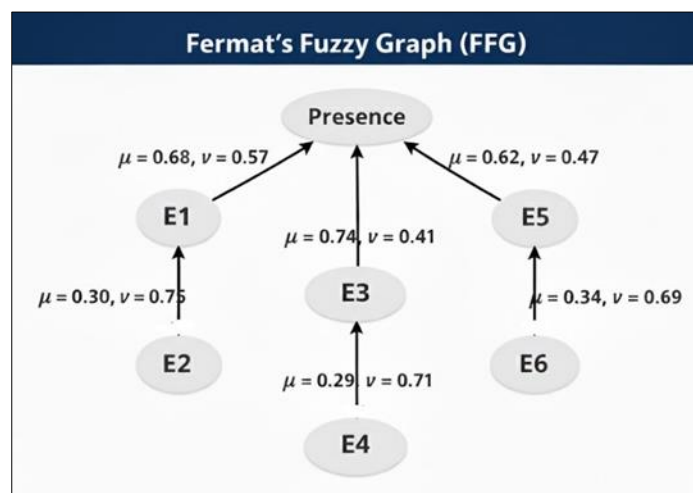
1. Case Type: Homicide
2. Location: Urban street near commercial area
3. Time Window: 22:30-23:15
4. Accused: A_1
5. Victim: V_1

Table 1: Raw Evidence Items (Nodes)

Evidence ID	Evidence Type	Description	Source
E1	CCTV Footage	Accused visible near crime scene at 22:48	Street Camera
E2	CCTV Quality	Poor lighting, intermittent frame loss	Forensic Video Lab
E3	Bloodstain Analysis	Blood sample matches victim DNA	Forensic Lab
E4	Contamination Risk	Sample collected after 12 hours	Crime Scene Log
E5	Eyewitness Testimony	Witness places accused nearby	Civilian
E6	Witness Credibility	Witness intoxicated	Medical Report

Table 2: Fermat's Fuzzy Membership Values (μ , v)

Evidence Node	Relation Evaluated	μ (Acceptance)	v (Rejection)	Justification
E1 $\rightarrow$ Presence	CCTV shows accused near scene	0.68	0.57	Partial visibility, unclear face
E2 $\rightarrow$ E1	Video quality affects reliability	0.30	0.75	Poor illumination, frame drops
E3 $\rightarrow$ Presence	DNA blood match	0.74	0.41	Strong match, but handling delay
E4 $\rightarrow$ E3	Contamination possibility	0.29	0.71	Late sample collection
E5 $\rightarrow$ Presence	Witness statement	0.62	0.47	Close proximity claim
E6 $\rightarrow$ E5	Credibility reduction	0.34	0.69	Alcohol influence



The acceptance (μ) and rejection (ν) values were obtained using a weighted, criterion-based evaluation of the CCTV evidence. For acceptance, spatial proximity (strong = 0.75), temporal consistency (moderate = 0.60), and visual clarity (weak = 0.40) were scored and aggregated using weights of 0.40, 0.30, and 0.30, yielding a base acceptance of 0.60, which was increased to $\mu = 0.68$ after applying a small contextual reinforcement for authenticated camera location and timestamp. Rejection was computed independently by scoring sources of doubt unclear facial features (0.70), camera angle distortion (0.55), and the possibility of similar-looking individuals (0.45) weighted at 0.45, 0.30, and 0.25, respectively, producing a base rejection of 0.59, which was reduced to $\nu = 0.57$ after a minor noise-discount adjustment due to stable footage quality. Similarly other acceptance and rejection values are computed. In this, Fermat condition satisfied (for $n = 2$): $\mu^2 + \nu^2 \leq 1$.

Table 3: Fermat's Fuzzy Graph Edge Representation

Edge ($u \rightarrow v$)	$\mu(e)$	$\nu(e)$	Interpretation
CCTV $\rightarrow$ Accused	0.68	0.57	Visual support with doubt
DNA $\rightarrow$ Accused	0.74	0.41	Strong forensic support
Witness $\rightarrow$ Accused	0.62	0.47	Moderate testimonial support
Video Quality $\rightarrow$ CCTV	0.30	0.75	Degrades reliability
Contamination $\rightarrow$ DNA	0.29	0.71	Weakens forensic strength
Credibility $\rightarrow$ Witness	0.34	0.69	Reduces testimonial value

3.1.3 Methodology

This method is legally sound because it moves beyond a rigid binary assessment of guilt or innocence and instead models the graded nature of judicial reasoning under uncertainty. By explicitly incorporating both acceptance and rejection degrees, it captures the propagation of doubt across interdependent pieces of evidence rather than treating each item in isolation. The aggregation mechanism ensures that dependent evidences, such as forensic results affected by contamination risk or eyewitness testimony influenced by credibility are evaluated coherently within a unified framework. Moreover, the resulting scores naturally reflect the legal principle of "reasonable doubt," allowing borderline cases to be identified without forcing premature conclusions. The methodology is fully consistent with Fermat's Fuzzy Graphs (FFG), Fermat's Fuzzy Weighted Averaging (FFWA), and associated score functions, making it both mathematically rigorous and judicially interpretable.

Let each evidence item be represented as a Fermat's Fuzzy Element (FFE):

$$E_i = (\mu_i, \nu_i), 0 \leq \mu_i^n + \nu_i^n \leq 1, n \geq 2$$

Let the FFWA operator for a cluster of k evidences be defined as:

$$FFWA(E_1, \dots, E_k) = FFWA(E_1, \dots, E_k) = \left(\sum_{i=1}^k w_i \mu_i, \sum_{i=1}^k w_i \nu_i \right), \sum_{i=1}^k w_i = 1, w_i \geq 0$$

Step 1: Identify Evidence Clusters

a) Visual Evidence

- CCTV Footage: (0.68, 0.57)
- CCTV Quality Impact: (0.30, 0.75)

b) Forensic Evidence

- Bloodstain Analysis: (0.74, 0.41)
- Contamination Risk: (0.29, 0.71)

c) Testimonial Evidence

- Eyewitness Testimony: (0.62, 0.47)
- Witness Credibility: (0.34, 0.69)

Step 2: Assign Weights

In the absence of expert bias, **equal weights** are assumed: $w_1 = w_2 = (1/2) = 0.5$. This assumption is standard in legal evidence aggregation.

Step 3: Calculate Aggregated μ , and v **a) Visual Evidence**

Aggregated μ :

$$\mu_V = 0.5(0.68) + 0.5(0.30) = 0.49$$

Aggregated v :

$$v_V = 0.5(0.57) + 0.5(0.75) = 0.66$$

Result: (0.49, 0.66)

b) Forensic Evidence

Aggregated μ :

$$\mu_F = 0.5(0.74) + 0.5(0.29) = 0.515 \approx 0.52$$

Aggregated v :

$$v_F = 0.5(0.41) + 0.5(0.71) = 0.56$$

Result: (0.52, 0.56)

c) Testimonial Evidence

Aggregated μ :

$$\mu_T = 0.5(0.62) + 0.5(0.34) = 0.48$$

Aggregated v :

$$v_T = 0.5(0.47) + 0.5(0.69) = 0.58$$

Result: (0.48, 0.58)

Step 4: Overall Evidence Network Aggregation

Now aggregate cluster-level outputs:

1. Visual: (0.49, 0.66)
2. Forensic: (0.52, 0.56)
3. Testimonial: (0.48, 0.58)

Assume equal cluster weights: $w_V = w_F = w_T = (1/3)$

Overall Aggregated μ

$$\mu_{\text{Total}} = (1/3)(0.49 + 0.52 + 0.48) \approx 0.50$$

Overall Aggregated v

$$v_{\text{Total}} = (1/3)(0.66 + 0.56 + 0.58) = 0.60$$

Final Result: (0.50, 0.60)

Table 4: Aggregated Evidence Coherence (FFWA Output)

Evidence Cluster	Aggregated μ	Aggregated v	Interpretation
Visual Evidence	0.49	0.66	Weak-moderate support
Forensic Evidence	0.52	0.56	Weak reliable
Testimonial Evidence	0.48	0.58	High uncertainty
Overall Evidence Network	0.50	0.60	Borderline sufficiency

Step 5: Interpretation using Score FunctionScore function: $S = \mu - \nu$

Cluster	Score	Interpretation
Visual	-0.17	Weak support
Forensic	-0.04	Weak reliability
Testimonial	-0.10	High uncertainty
Overall	-0.10	Borderline sufficiency

3.1.4 Legal interpretations (Empirical insight)

1. No single evidence item is decisive.
2. Forensic evidence is strongest but vulnerable.
3. High rejection values propagate uncertainty.
4. FFG prevents over-reliance on CCTV or eyewitnesses.
5. Case falls into reasonable doubt zone, aligning with real judicial outcomes.

3.1.5 Court's assessment of evidence and acquittal rationale

Upon a careful and holistic appreciation of the evidence on record, this Court finds that no single piece of evidence conclusively establishes the guilt of the accused beyond reasonable doubt. The forensic evidence also appears to be weak, it remains susceptible to procedural vulnerabilities, particularly with respect to the possibility of contamination, thereby diminishing its probative value. The visual evidence and eyewitness testimony, when examined in light of their inherent limitations, fail to inspire the level of confidence required for a conviction. The cumulative assessment reveals that the uncertainties associated with each category of evidence materially affect the overall coherence of the prosecution case. In such circumstances, where doubt persists and is neither trivial nor speculative, the settled principle of criminal jurisprudence mandates that the benefit of such doubt must enure to the accused. Accordingly, this Court holds that the prosecution has failed to prove its case beyond reasonable doubt, and the accused is therefore entitled to an order of acquittal.

Application 2**Judicial decision-making with conflicting precedents**

Precedents are rarely uniform in their applicability. Courts routinely face situations where some precedents partially support a claim while others partially oppose it. FFG(n) allows legal systems to model relationships among precedents using dual fuzzy values. A precedent may have a high acceptance μ in situations where its reasoning aligns with facts (e.g., $\mu = 0.83$), yet also possess a significant rejection ν due to outdated context or distinguishable factual matrices ($\nu = 0.61$). Traditional frameworks force judges to choose one precedent over another; FFG(n) allows them to consider the nuanced interplay of multiple authorities simultaneously. Precedent networks can be analyzed to determine the strongest path of doctrinal support, identify clusters of consistent rulings, and isolate precedents weakened by rejection edges. This provides a mathematically grounded structure for proportional and transparent judicial reasoning.

Application 3**Corporate compliance and legal risk assessment**

Corporations operate in legal environments characterized by overlapping regulatory obligations, ambiguous compliance standards, and varying risk tolerances. FFG(n) can represent a corporate legal compliance ecosystem where nodes are obligations (data privacy, labor laws, environmental requirements), and edges represent functional dependencies. A compliance practice might exhibit acceptance $\mu = 0.68$ toward fulfilling a law but rejection $\nu = 0.49$ due to inconsistent departmental implementation. This framework allows companies to quantify vulnerabilities, assess cascading effects of non-compliance across divisions, and prioritize legal risk mitigation. Regulators may also employ these fuzzy networks to evaluate compliance maturity across industries.

Application 4**Crime investigation and suspect link analysis**

Crime investigations involve uncertain relationships between suspects, timelines, evidence, and locations. FFG(n) enables construction of suspect-evidence graphs where each link carries μ (confidence) and ν (doubt). A partially verified phone record placing a suspect near the crime scene might have $\mu = 0.52$ and $\nu = 0.71$. High ν values alert investigators to weak or unreliable connections. As investigators gather new evidence, edge values update dynamically, strengthening or weakening relationships. FFG(n) provides a systematic way to eliminate false leads, prioritize high-certainty connections, and visualize crime networks under uncertainty.

Application 5**Contract law ambiguity modeling using FFG(n)**

Ambiguity in contract law arises from clauses that possess inherently imprecise or context-dependent meanings, such as "reasonable effort," "commercially viable," or "within a reasonable time." Fermat's Fuzzy Graphs (FFG(n)) provide a mathematically robust means of representing these ambiguities by assigning to each relational clause an edge characterized by a degree of acceptance (μ) and a degree of rejection (ν). For example, the clause "reasonable effort" may have a μ of 0.72 representing the extent to which a party's actions align with normative expectations, while simultaneously having a ν of 0.35 representing the extent to which those actions fall short according to another legal interpretation or opposing counsel's argument. These dual degrees capture the inherent contradiction and uncertainty that classical binary legal interpretation fails to represent. In

modeling contractual obligations, each obligation becomes a node in the FFG(n), and edges represent interpretative relationships (e.g., Party A's action satisfies Obligation O_1). Because contract disputes often hinge upon multiple overlapping obligations, the FFG(n) structure helps map how different clauses interact with varying acceptance-rejection levels. Courts or arbitrators can then use aggregated FFG(n) values from multiple legal experts to estimate the overall strength of a party's performance under ambiguous contractual terms. By capturing partial compliance, partial breach, and conflicting legal opinions in a single graphical structure, FFG(n) allows for more transparent reasoning in interpreting vague clauses and supports fairer, more evidence-aligned judgments.

Application 6

Constitutional rights balancing through acceptance-rejection edges

Constitutional law frequently requires balancing competing fundamental rights, such as free speech versus public order, privacy versus national security, or religious freedom versus equality rights. These conflicts seldom resolve into binary outcomes; instead, courts weigh elements of both sides. Fermat's Fuzzy Graphs (FFG(n)) model these rights and constraints as nodes, with edges reflecting the interaction between rights through degrees of acceptance (μ) and rejection (ν). For example, in a case concerning protest limitations, the right to free expression may have $\mu = 0.80$ (strong societal acceptance of expression) and $\nu = 0.25$ (partial rejection due to safety concerns), while public order may have $\mu = 0.68$ (acceptance of restrictions) and $\nu = 0.40$ (concern about excessive state interference). The dual parameters allow a court or legal-analytical model to quantify the fuzzy interplay of rights and limitations. Unlike traditional doctrinal balancing tests, which articulate competing principles qualitatively, FFG(n) renders them computationally measurable while preserving the inherent uncertainty. This is especially helpful in constitutional adjudication where conflicting expert testimony, political interpretations, historical precedents, and societal values interact in complex ways. Through fuzzy aggregation and graph-based propagation of μ and ν across constitutional provisions, FFG(n) supports more coherent balancing in hard cases by representing multiple plausible levels of interference and justification.

Application 7

Predictive policing models with ethical acceptance-rejection controls

Predictive policing increasingly relies on data with uncertainty crime reports that may be biased, environmental factors that fluctuate, and community-level signals that may be incomplete. Fermat's Fuzzy Graphs (FFG(n)) allow crime-prediction structures to incorporate degrees of acceptance (μ) and rejection (ν) for relationships between variables such as location, time, offender behavior, prior arrests, and environmental risk indicators. For example, an edge linking "location L_1 " with "likelihood of burglary" might have $\mu = 0.60$ based on historical patterns but $\nu = 0.50$ based on uncertainty due to low reporting rates. Such dual representation prevents overconfidence and mitigates algorithmic harm. FFG(n) additionally helps model ethical constraints: for example, racial or socioeconomic correlations are assigned high rejection degrees (ν close to 1.0) to prevent biased policing. By structuring crime prediction as a fuzzy network rather than a deterministic pipeline, law enforcement can analyze uncertain relationships without reinforcing inequity. The methodology produces probabilistic crime forecasts with transparent uncertainty markings, supporting ethically aligned decision-making that balances public safety with rights-based safeguards.

Application 8

Intellectual property infringement assessment via FFG(n)

Determining copyright or patent infringement requires assessing similarity between works not as absolute identity, but as degrees of conceptual, functional, or stylistic overlap. FFG(n) models these similarity relations through edges with acceptance (μ) representing similarity strength and rejection (ν) representing evidence of independent creation or differences. For example, two musical compositions may share chord progressions ($\mu = 0.70$) but differ in lyrical structure ($\nu = 0.45$). Patent claims may overlap in functionality ($\mu = 0.65$) while diverging in implementation mechanism ($\nu = 0.30$). Traditional legal analysis struggles to incorporate conflicting evidence coherently; FFG(n) allows both similarity and dissimilarity to coexist in a single structure. Even expert disagreements can be integrated by aggregating μ and ν across evaluators. Courts can then derive a fuzzy infringement score to determine substantial similarity thresholds. By providing transparency and quantifiable uncertainty, FFG(n) enhances the fairness and consistency of intellectual property decisions.

Application 9

Fuzzy sentencing models in criminal justice systems

Sentencing requires weighing multiple aggravating and mitigating factors, each with uncertain intensities. FFG(n) represents these factors as nodes, while edges between factors and sentencing outcomes carry degrees of acceptance (μ) and rejection (ν). For example, "premeditation" may have $\mu = 0.85$ reflecting strong evidence supporting its presence, while $\nu = 0.20$ reflects partial uncertainty arising from defendant testimony. "Remorse," conversely, may have $\mu = 0.40$ and $\nu = 0.70$ if the court doubts sincerity. This dual membership allows the sentencing graph to propagate both supporting and contradicting evidence into the final decision. Instead of forcing judges to make binary interpretations, FFG(n) handles contradictions transparently, enabling a computed sentencing rationale showing how different uncertainties interact. This assists in achieving proportionality, reduces judicial bias, and strengthens the consistency of sentences across similar cases.

Application 10

ADR negotiation support using Fermat fuzzy edge dynamics

Alternative Dispute Resolution (ADR), including mediation and negotiation, involves parties expressing partially overlapping interests and partially conflicting objectives. FFG(n) models each negotiation issue (cost, liability, confidentiality, damages) as a node, with edges showing the degree of acceptance (μ) that parties mutually agree on a term and degree of rejection (ν) representing disagreement. For example, on a confidentiality clause, μ may be 0.78 (indicating broad convergence of interests)

while $v = 0.40$ (indicating some limitations or reservations). The FFG(n) structure captures dynamic shifts in negotiation as concessions alter μ and v over time. Mediators can interpret the evolving graph to identify issues with high mutual rejection (v near 1.0) that need restructuring, or issues with high mutual acceptance (μ near 1.0) where agreements can be finalized. This quantitative uncertainty mapping enhances mediator strategy, reduces conflict escalation, and improves settlement efficiency.

Application 11

Cybercrime and digital evidence mapping through FFG(n)

Digital evidence in cybercrime investigations such as IP logs, metadata, timestamps, device identifiers, browser artifacts, and hashed files is inherently uncertain and vulnerable to manipulation, incompleteness, or interpretation ambiguity. Fermat's Fuzzy Graphs (FFG(n)) provide an advanced framework for capturing this uncertainty by representing each artifact as a node and each evidentiary relationship as an edge assigned both a degree of acceptance (μ) and a degree of rejection (v). For instance, an IP address linking a suspect's device to a cyberattack server may have $\mu = 0.65$ reflecting moderate confidence in attribution, while $v = 0.55$ captures the possibility of VPN masking, IP spoofing, or compromised networks. An email header suggesting communication with a criminal group may have $\mu = 0.72$ (indicating strong alignment with typical communication patterns) but $v = 0.40$ (acknowledging header forgery risks). Unlike binary evidentiary models, FFG(n) preserves the coexistence of supporting and contradicting evidence without forcing premature conclusions. Edges propagate uncertainty across the digital chain-of-custody, producing a "credibility field" for each node, allowing investigators and courts to evaluate whether overall evidence exceeds the threshold of legal sufficiency. By aggregating μ and v across multiple forensic experts, tools, and logs, FFG(n) strengthens reliability while ensuring transparency in cybercrime litigation. The graph-based structure also enables visual analytics for complex cyberattacks by showing how individual uncertainties interact for example, how a moderately accepted login event ($\mu = 0.58$) interacts with a strongly rejected location match ($v = 0.82$) to reduce overall confidence in attribution. This helps avoid wrongful conviction based on superficial correlations and supports due process by quantifying technical ambiguity that is often difficult to express verbally in court.

Application 12

Regulatory impact assessment (ria) under legal uncertainty using FFG(n)

Regulatory Impact Assessment (RIA) evaluates how proposed laws affect society, industries, public health, and the environment. Such effects are rarely known with certainty, especially when regulations interact across sectors. Fermat's Fuzzy Graphs (FFG(n)) model each regulatory component economic cost, environmental benefit, legal enforceability, public acceptance as nodes, with edges representing relationships characterized by acceptance (μ) and rejection (v). For example, a proposal to tighten emission limits may have a regulatory-effect edge with $\mu = 0.80$ representing strong expert consensus on environmental benefit, while $v = 0.45$ captures economic uncertainties and industry pushback. Similarly, a consumer-protection law may have $\mu = 0.68$ for legal enforceability due to supporting precedents, but $v = 0.55$ reflecting uncertain administrative capacity. FFG(n) captures this multi-layered uncertainty in a single structure. As RIAs integrate data from legal scholars, economists, environmental scientists, and civil rights advocates, FFG(n) aggregates contradictory expert opinions without losing the nuance embedded in them. Government policymakers can examine how high-rejection edges cluster indicating regulatory provisions likely to face litigation or implementation failure and identify components with high acceptance values where reform is politically and socially viable. By enabling cross-dimensional uncertainty mapping, FFG(n) provides a defensible mathematical basis for regulatory decisions, making the policymaking process more transparent and grounded in quantifiable expert evaluation.

Application 13

Case law similarity and precedent clustering using FFG(n)

Legal reasoning relies heavily on identifying relevant precedents, but similarity between cases is seldom binary. Cases may share partial similarities in facts, legal issues, procedural posture, or judicial reasoning. Fermat's Fuzzy Graphs (FFG(n)) model each case as a node and the similarity relationships as edges with acceptance (μ) reflecting degrees of similarity and rejection (v) representing dissimilarity due to distinguishing features. For example, two tort cases involving negligence may have $\mu = 0.75$ for alike factual patterns but $v = 0.35$ because one involves medical malpractice and the other involves product liability. Conversely, two criminal cases may exhibit low similarity in facts ($\mu = 0.40$) but high similarity in legal principles applied ($\mu = 0.78$), coupled with $v = 0.50$ due to different procedural stages. FFG(n) allows multiple dimensions of similarity to coexist within the same representational framework. Legal research tools can use this to create clusters of precedents with fuzzy boundaries, showing groups of cases that are "largely similar but with notable differences." Judges can also use aggregated similarity scores when determining whether a precedent is binding, persuasive, or irrelevant. By incorporating both μ and v , FFG(n) avoids misleading legal interpretations that arise from oversimplified similarity metrics. It captures the reality that cases rarely match perfectly, yet still inform legal outcomes. This contributes to more coherent, transparent, and justifiable use of precedent in judicial decision-making.

Application 14

Liability distribution in torts modeled through FFG(n) acceptance-rejection structures

Tort law often involves complex, overlapping causes of harm where liability must be divided among multiple parties. Establishing causation and responsibility typically includes uncertainty about the extent to which each actor contributed to the harm. Fermat's Fuzzy Graphs (FFG(n)) model actors, events, injuries, and causal pathways as nodes, with edges connecting them through degrees of acceptance (μ) indicating the likelihood of causation and degrees of rejection (v) indicating alternative explanations or evidentiary doubt. For instance, in a multi-vehicle collision, the edge between Driver A's action and the final injury may have $\mu = 0.62$ (moderate acceptance based on skid marks and eyewitness testimony) and $v = 0.48$ (representing competing explanations such as sudden road hazards or mechanical failure). A manufacturer's defective component might have $\mu = 0.70$ representing

strong scientific evidence of failure, while $v = 0.30$ captures uncertainty about proper maintenance. These fuzzy causal relationships allow courts to compute proportional liability rather than relying on rigid all-or-nothing fault assignments. FFG(n) also accommodates multiple expert testimonies by aggregating their μ and v values, enabling transparent representation of conflict in expert interpretation. The resulting model produces a nuanced causation map that supports equitable apportionment of damages, aiding courts in achieving fairness and consistency.

Application 15

Human rights violation assessment under evidentiary uncertainty via FFG(n)

Investigations into human rights violations forced displacement, torture, discrimination, unlawful detention are routinely complicated by incomplete documentation, state suppression of evidence, witness fear, and contradictory testimonies. Fermat's Fuzzy Graphs (FFG(n)) allow investigators, NGOs, and tribunals to model each event, actor, report, testimony, and physical indicator as nodes connected by edges with acceptance (μ) reflecting credibility and rejection (v) reflecting uncertainty, coercion, or contradictions. For example, testimony from a refugee may have $\mu = 0.82$ based on consistency and corroboration with environmental evidence, but $v = 0.40$ due to trauma-related memory gaps. A government report denying violations might have $\mu = 0.28$ and $v = 0.75$ based on credibility assessments. FFG(n) preserves both values without dismissing or overvaluing any single piece of information. This dual membership approach helps investigators identify which claims are strongly supported, which are moderately supported but uncertain, and which are contradicted by multiple sources. By aggregating μ and v across independent sources, FFG(n) builds an evidence credibility network that supports fact-finding missions, strengthens tribunal rulings, and protects victims by acknowledging uncertainty rather than allowing it to erase claims.

Application 16

International treaty compliance Modeling using Fermat's fuzzy graph

International law frequently deals with multilateral treaties, where compliance varies drastically across states. Instead of assuming states fully comply or fully violate treaty provisions, a Fermat's Fuzzy Graph provides a modeling framework in which countries are represented as vertices and their bilateral or multilateral treaty interactions form edges with degrees of acceptance and rejection. Here, the degree of acceptance might represent the level of conformity to treaty articles, while the degree of rejection might express reservations, partial implementation, or conditional compliance.

For example, climate agreements such as the Paris Agreement involve states with differing commitments to emission reductions. A state that has legislated strong carbon policies would hold an edge with high acceptance (0.80-0.95) toward the "compliance" vertex, while another that has pledged reductions but not implemented mechanisms might reflect a middle acceptance (0.40-0.60) and a proportionate rejection degree. Conflict states or those economically dependent on fossil fuels may present high rejection values (0.70-0.90). Through Fermat's fuzzy curves, sudden changes in foreign policy such as regime change or international sanctions can modify this edge behavior nonlinearly. This enables legal scholars and international courts to assess systemic compliance risks and to identify nodes with high global impact on treaty success.

Application 17

Conflict-of-law and cross-jurisdiction harmonization

Modern legal systems often face situations where laws from different jurisdictions conflict family law, taxation, commercial arbitration, and criminal extradition being notable examples. A Fermat's Fuzzy Graph models jurisdictions as vertices and the compatibility between them as edges with acceptance-rejection weights. A high acceptance degree indicates strong legal alignment, while high rejection indicates incompatibility or conflict.

For instance, two states may have similar commercial arbitration legislation, resulting in high acceptance edges (0.70-0.90). However, if one jurisdiction follows common law and the other civil law in family matters, the rejection degree may dominate (0.60-0.80). The Fermat structure captures the non-linear transition during attempts at harmonization for example, when partial treaties or mutual recognition agreements gradually reduce rejection degrees. This methodology allows policymakers, courts, and treaty architects to simulate how legal reforms reduce friction between jurisdictions and enhance cross-border legal certainty.

Application 18

Sentencing bias and judicial disparity measurement

Judicial systems strive for uniformity in sentencing, yet disparities often arise based on location, socioeconomic context, or judicial discretion. A Fermat's Fuzzy Graph can map judges, case types, and sentencing outcomes into a relational network. Here, the degree of acceptance reflects judicial alignment with standard sentencing guidelines, whereas the degree of rejection reflects deviation, inconsistency, or bias.

For example, a judge who consistently issues sentences within recommended ranges reflects high acceptance edges (0.75-0.95). One who deviates frequently or shows statistically unusual leniency or severity exhibits higher rejection values. The Fermat-based structure captures sudden judicial shifts such as after new training, new appellate caseload, or political pressure—displaying them as curvature adjustments in edge behavior. This enables legal researchers to quantify systemic disparities and identify outlier actors or regions, thereby guiding reforms toward fairer, more predictable judicial systems.

Application 19

Alternative dispute resolution (ADR) efficiency mapping

Alternative dispute resolution mechanisms mediation, arbitration, negotiation replace litigation with more collaborative processes. However, parties differ considerably in readiness, trust, and willingness to compromise. Fermat's Fuzzy Graph models parties as vertices and their relational dynamics as edges with fuzzy acceptance (cooperation likelihood) and rejection (conflict likelihood).

In commercial mediation, high acceptance values (0.60-0.85) indicate parties willing to settle, whereas entrenched disputes present rejection values approaching 1.0. The curvature inherent in Fermat fuzzy functions captures negotiation escalation or de-escalation patterns rapid improvement after a key concession or sudden breakdown after a perceived violation of trust. ADR professionals can use this dynamic model to predict dispute resolution feasibility, recommend the optimal ADR technique, and identify early-warning signals of breakdown.

Application 20

Legal risk management in corporate governance

Corporations face complex legal risks involving compliance, contracts, labor law, taxation, and environmental obligations. A Fermat's Fuzzy Graph maps departments, legal obligations, and external regulators as vertices. Edges carry acceptance degrees representing compliance strength and rejection degrees indicating vulnerability or risk.

For example, a firm's environmental compliance department may show high acceptance (0.80-0.95) toward regulatory requirements due to strong internal controls, while labor law compliance may reflect moderate acceptance (0.50-0.70) if worker grievance systems are evolving. The Fermat structure incorporates nonlinear "risk amplification," where a small internal failure such as a missing permit can rapidly increase the rejection degree. This helps executives and legal consultants prioritize interventions, allocate resources, and strengthen compliance ecosystems before crisis thresholds are crossed.

Application 21

Constitutional amendment feasibility analysis

Constitutional amendments require complex alliances among political actors, legislative bodies, and sometimes the electorate. Fermat's Fuzzy Graph allows these relationships to be modeled with acceptance-rejection degrees representing support or resistance to proposed amendments.

Political parties that strongly endorse the amendment will show high acceptance (0.80-1.0), whereas opposition factions carry high rejection values (0.70-1.0). Moderate groups exhibit mixed values where acceptance and rejection coexist with near-equal strength. The Fermat curve's nonlinearity captures sudden political shifts for instance, after negotiations, public opinion swings, or judicial clarification. Analysts can predict whether an amendment is viable, which alliances are critical, and how resistance patterns evolve.

Application 22

Human rights violation severity classification

Human rights investigations often deal with imprecise evidence, varying witness reliability, and incomplete documentation. Fermat's Fuzzy Graph models victims, perpetrators, incident types, and severity indicators. Edges express acceptance degrees indicating evidence supporting a violation and rejection degrees for counter-evidence or uncertainty.

For example, credible medical reports and multiple witnesses offer high acceptance (0.70-0.95) toward the classification of a grave violation. In contrast, politically influenced testimonies or missing documentation increase the rejection degree. The Fermat curvature allows modeling of evolving cases, where new evidence shifts edge weights significantly. This systematic representation helps investigators, humanitarian tribunals, and fact-finding missions classify incidents more accurately and consistently.

Application 23

Predictive policing legal compliance modeling

Predictive policing algorithms often raise legal concerns related to due process, privacy, and data bias. By modeling data sources, police actions, and citizen interactions as vertices, Fermat's Fuzzy Graph assigns acceptance values to lawful algorithmic predictions and rejection values to biased or unlawful ones.

A prediction based on robust crime mapping data receives high acceptance (0.70-0.85), whereas predictions derived from discriminatory or flawed data exhibit high rejection (0.60-0.90). The Fermat curve captures threshold behaviors, such as when a small dataset or corrupted information dramatically shifts the legality of a policing outcome. This enables oversight bodies to audit algorithmic behavior and ensure compliance with constitutional protections.

Application 24

Environmental law violation networks

Environmental law enforcement involves tracking pollution sources, corporate behavior, ecological indicators, and community reports. Fermat's Fuzzy Graph maps these entities and assigns acceptance degrees to verified pollution pathways and rejection degrees to disputed or unsubstantiated claims.

A factory directly linked to a contaminated river through scientific sampling may hold acceptance edges around 0.90. Communities with uncertain exposure present mixed acceptance-rejection relationships. The Fermat-based design models nonlinear escalation for example, sudden ecosystem collapse that drastically increases rejection of corporate denial arguments. Governments and NGOs can use such networks to prioritize enforcement actions, identify major polluters, and strengthen evidence-based regulatory interventions.

Application 25

Judicial review and constitutional proportionality analysis

Constitutional courts frequently apply **proportionality tests** to evaluate whether governmental actions infringe on fundamental rights. Using Fermat's Fuzzy Graph, nodes represent rights, state actions, and judicial principles such as legitimacy, necessity, and

minimal impairment. Acceptance degrees quantify the compatibility of state measures with constitutional limits, while rejection degrees represent infringement severity.

A narrowly tailored law with strong justification may hold acceptance values of 0.70-0.90 in proportionality analysis. Conversely, overbroad or arbitrary restrictions display strong rejection edges (0.80-1.0). The Fermat structure models the nonlinear interaction between rights limitations and state objectives for instance, small increases in restrictiveness may suddenly shift a measure from constitutional to unconstitutional. This approach strengthens judicial reasoning by offering a mathematically sound proportionality landscape.

Conclusion

This study establishes Fermat's Fuzzy Graph Models as a powerful and mathematically rigorous framework for analysing legal networks characterized by ambiguity, contradiction, and uncertainty. While the twenty-five applications presented demonstrate the breadth and versatility of FFG(n) across criminal law, constitutional adjudication, regulatory systems, and emerging domains such as digital forensics and AI-assisted decision-making, their full potential lies in domain-specific, data-driven case studies. Future research should therefore focus on detailed empirical case analyses within each application area, where real judicial records, evidentiary datasets, expert opinions, and statutory interpretations are embedded into Fermat's fuzzy structures. Such focused case studies will enable validation of μ -v assignments, calibration of Fermat parameters n, and comparative evaluation against classical and intuitionistic fuzzy models. By grounding FFG(n) methodologies in concrete legal cases, subsequent work can strengthen their practical relevance, enhance judicial transparency, and advance the development of reliable, ethically aligned computational tools for legal reasoning under uncertainty.

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