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Assessing the dynamics of financial assets: MGARCH modelling of Nigerian stock returns, all share index, and crude oil prices

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Abstract

This study investigates the dynamic interrelationships and volatility spillovers among Nigerian stock returns, the All Share Index (ASI), and crude oil prices using the Multivariate GARCH-Dynamic Conditional Correlation (MGARCH-DCC) framework. The data, consisting of daily returns, were analysed multivariate generalized error distribution to capture the time-varying nature of volatility and correlations among the assets. Results from the ARCH LM tests revealed significant heteroskedasticity across all return series, justifying the application of GARCH-type models. The estimated MGARCH-DCC (1,1) parameters indicate that all series exhibit strong volatility persistence, with crude oil prices showing the highest degree of persistence ($\beta_1 = 0.866$). The DCC results further confirm significant and slowly evolving conditional correlations, suggesting that volatility shocks in one market spill over to others, particularly from the crude oil market to the stock market and ASI. These findings underscore the interconnectedness of Nigeria's financial and energy markets, implying limited diversification benefits during periods of heightened market turbulence. The study concludes that monitoring cross-market volatility dynamics is vital for effective risk management, portfolio allocation, and macroeconomic policy formulation in Nigeria.

Keywords: MGARCH-DCC, volatility spillover, Nigerian stock market, crude oil prices, dynamic correlations

Introduction

The Nigerian financial landscape is intricately shaped by the interplay between its stock market and the global oil market. As one of Africa's largest oil producers, Nigeria's economic health is closely tied to crude oil prices. The Nigerian Stock Exchange (NSE), particularly the All Share Index (ASI), serves as a barometer for investor sentiment and economic stability. Understanding the volatility of these financial assets is crucial for investors, policymakers, and economic analysts. Recent studies have employed Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models to analyze the volatility dynamics of Nigerian financial assets. For instance, Atoi (2015) ^[1] utilized various GARCH models to assess the volatility of the ASI, highlighting the significance of error distributions in enhancing model efficiency. Similarly, Yaya *et al.* (2016) ^[8] applied Beta-t-GARCH variants to model volatility in the Nigerian stock market, emphasizing the importance of accounting for jumps and asymmetry in financial time series.

The relationship between crude oil prices and stock market volatility has also been a focal point of research. Studies by Tumala *et al.* (2023) ^[6] and Okorie (2022) ^[3] have explored the impact of oil price shocks on stock market volatility in Nigeria, employing advanced GARCH-MIDAS and asymmetric models to capture the nuances of this relationship. These studies underscore the sensitivity of the Nigerian stock market to global oil price fluctuations, reflecting the country's dependence on oil revenues.

Furthermore, the application of Multivariate GARCH (MGARCH) models has provided deeper insights into the interconnectedness of financial markets. Research by Bagirov (2025) ^[2]

and Salisu & Oloko (2018) ^[4] demonstrated the effectiveness of MGARCH models in capturing volatility spillovers between crude oil prices and stock market returns, offering a comprehensive view of market dynamics.

This study aims to extend the existing literature by employing MGARCH models to assess the volatility dynamics of Nigerian stock returns, the ASI, and crude oil prices. By doing so, it seeks to provide a nuanced understanding of the interrelationships among these financial assets, thereby contributing to more informed investment and policy decisions in Nigeria's financial markets.

2. Literature Review

The relationship between crude oil prices and the Nigerian stock market has been extensively studied, with various methodologies employed to understand the dynamics of this interaction. Early research by Tule *et al.* (2017) ^[5] utilized the VARMA-AGARCH model to examine volatility spillovers between oil prices and the Nigerian sovereign bond market. Their findings indicated significant cross-market volatility transmission, highlighting the sensitivity of financial markets to oil price shocks. Similarly, Usoro and Ekong (2022) ^[7] applied MGARCH models to assess the volatility of crude oil production and price in Nigeria, revealing a bidirectional causality between production quantity and price, which underscores the interconnectedness of these variables.

In the context of the Nigerian stock market, studies have focused on the volatility dynamics of the All Share Index (ASI). Atoi (2015) ^[1] employed various GARCH models to analyse the volatility of the ASI, emphasizing the importance of error distributions in enhancing model efficiency. Yaya *et al.* (2016) ^[8] applied Beta-t-GARCH variants to model volatility in the Nigerian stock market, highlighting the significance of accounting for jumps and asymmetry in financial time series. These studies contribute to a deeper understanding of the factors influencing stock market volatility in Nigeria.

Despite the extensive research, there remains a gap in the literature regarding the simultaneous modelling of volatility spillovers between Nigerian stock returns, the ASI, and crude oil prices using Multivariate GARCH (MGARCH) models. This study aims to fill this gap by employing MGARCH models to assess the volatility dynamics of these financial assets, providing a comprehensive analysis of their interrelationships. Understanding these dynamics is crucial for investors and policymakers to make informed decisions in the face of global economic uncertainties and oil price fluctuations.

3. Methodology

3.1 The Multivariate GARCH Models

Multivariate GARCH (MGARCH) models are extensions of the univariate GARCH models, enabling the modelling of volatility dynamics across multiple interrelated time series. Unlike univariate GARCH models, which focus on modelling the conditional volatility of a single series, MGARCH models account for the conditional covariances and correlations between multiple assets or variables. A well-known MGARCH model is the Constant Conditional Correlation (CCC) model, introduced by Bollerslev in 1990. The CCC model assumes that the conditional correlations between the residuals of different series remain constant over time, while allowing for time-varying conditional variances. Despite its relative simplicity, the CCC model is effective in capturing

the co-movement and volatility structure across financial time series and other multivariate datasets.

Another important MGARCH model is the Dynamic Conditional Correlation (DCC) model, proposed by Engle in 2002. Unlike the CCC model, the DCC model allows for time-varying conditional correlations between the residuals of different series, offering greater flexibility in modelling dynamic interdependencies. This feature makes the DCC model particularly suitable for capturing changes in the strength and direction of correlations between assets over time. Additionally, there are further variations and extensions of MGARCH models, such as the Vector Autoregressive Moving Average GARCH (VARMA-GARCH) model and the factor MGARCH model. These models incorporate additional structures, like vector autoregressive dynamics or common latent factors, which drive the volatility across multiple series. MGARCH models are crucial in capturing the complex dependencies and volatility transmission across multivariate time series. They are widely used in applications such as risk management, portfolio optimization, and forecasting in finance, economics, and macroeconomics, as they allow for the analysis of the conditional variance-covariance structure and the interrelationships among multiple time series simultaneously. The MGARCH models handles multiple time series. It is used to analyse and forecast the conditional volatility and correlation between multiple financial time series. MGARCH models can handle multiple time series, allowing for the analysis of relationships between different financial instruments.

In MGARCH models, the conditional variance and covariance matrices evolve over time, depending on past values of the time series. The model assumes that the data follows a multivariate distribution and that conditional variances and covariances are functions of lagged innovations (shocks or residuals) and past conditional variances. The general form of an MGARCH model can be represented as:

$$r_t = \mu_t + \varepsilon_t \quad (1)$$

$$\varepsilon_t = H_t^{\frac{1}{2}} Z_t \quad (2)$$

Where;

- $r_t = n \times 1$ vector of log returns of n assets at time t .
- μ_t is the vector of mean returns (or a constant)
- $\varepsilon_t = n \times 1$ is the vector of residuals (errors) at time t ,
- $E(\varepsilon_t) = 0$ $Cov(a_t) = H_t$.
- $\mu_t = n \times 1$ vector of the expected value of the conditional R_t .
- $H_t = n \times n$ matrix of conditional variances at time t .
- $H_t^{\frac{1}{2}} =$ Any $n \times n$ matrix at time t such that H_t is the conditional variance matrix of a_t . $H_t^{\frac{1}{2}}$ could be gotten by Cholesky factorization of H_t
- $\varepsilon_t = H_t^{\frac{1}{2}} Z_t$ where Z_t is a vector of independent and identically distributed errors (often assumed to be normal or t-distributed), and H_t is the conditional covariance matrix at time t .
- Z_t is an $n \times 1$ vector of iid errors such that $E(Z_t) = 0$ and $E(Z_t Z_t') = 1$

Just like in the single-variable case, the residuals (a_t) appear to be uncorrelated over time. However, this doesn't imply a complete lack of serial dependence; rather, any existing

dependence is non-linear in nature MGARCH models are used to analyse the conditional variance and covariance structure of multivariate time series data. There are numerous ways to specify the conditional covariance matrix H_t . However, as the number of variables (dimension of a_t) becomes larger, the number of parameters in H_t increases exponentially, making estimation computationally challenging, especially for large datasets. To overcome this, MGARCH models aim to strike a balance between parsimony and flexibility. Additionally, ensuring that the conditional covariance matrix remains positive definite is crucial, as this guarantees that the matrix is valid and can be inverted efficiently. The conditional covariance matrix H_t in an MGARCH model depends on past residuals and past covariance terms, and it models how the volatility and correlation evolve over time.

There are different types of MGARCH models, such as: CCC-GARCH (Constant Conditional Correlation GARCH) which assumes constant correlations between assets while allowing variances to change over time; DCC-GARCH (Dynamic Conditional Correlation GARCH) which allows time-varying correlations between the assets, providing greater flexibility to capture dynamic interdependencies; The VARMA-GARCH which incorporates vector autoregressive moving average dynamics into the volatility modelling.

This study utilized the Dynamic Conditional Correlation (DCC) model which is particularly suitable for analysing the volatility and interdependence of multiple financial assets because it allows for time-varying correlations. Unlike constant-correlation multivariate models, the DCC model captures the evolving relationship between asset returns, such as Nigerian stock returns, the All Share Index (ASI), and crude oil prices, over time. This is crucial in financial markets that are highly sensitive to shocks from both domestic and global factors, including oil price fluctuations and economic policy changes. Additionally, the DCC model efficiently accommodates large datasets and provides more accurate estimates of volatility spillovers and co-movements compared to simpler multivariate GARCH specifications. Its flexibility in modeling conditional correlations dynamically enables researchers and investors to monitor changing risk structures, optimize portfolio allocation, and implement more informed hedging strategies. Consequently, employing the DCC model provides a robust framework for understanding the interconnectedness of Nigerian financial markets and the impact of exogenous shocks on asset volatility.

3.2 Dynamic Conditional Correlation (DCC) Model

(DCC-) GARCH belongs to the class of models that take models of conditional variances and correlations. It was introduced by Engle and Sheppard in 2001. The Dynamic Conditional Correlation (DCC-) GARCH model is a popular econometric model used to estimate time-varying correlations between multiple financial time series. It is an extension of the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model, which itself models time-varying volatility. Models in this category are based on the approach of modelling conditional variances and correlations, rather than directly focusing on the conditional covariance matrix. The conditional covariance matrix is instead expressed as a combination of conditional standard deviations and a correlation matrix, as follows:

$$H_t = D_t R_t D_t \quad (3)$$

Where;

- R_t is the time-varying correlation matrix
- $D_t = \text{diag} \left[h_{1t}^{\frac{1}{2}}, h_{2t}^{\frac{1}{2}}, h_{3t}^{\frac{1}{2}}, \dots, h_{nt}^{\frac{1}{2}} \right]$ is the conditional standard deviation

Models in this class can be classified in two groups; those with a constant correlation matrix and those when the correlation matrix is time-varying. The DCC-GARCH model combines univariate GARCH models for individual series with a dynamic correlation structure. Each financial time series is modelled using a univariate GARCH model to capture its own time-varying volatility:

$$\sigma_{it}^2 = \omega_t + \alpha_i \epsilon_{i,t-1} + \beta_i \sigma_{i,t-1}^2 \quad (4)$$

Equation (3.18) is the variance equation For each series i

Where;

- σ_{it}^2 is the Conditional variance of series i at time t
- $\epsilon_{i,t-1}$ is the Innovation (shock) from the previous period
- ω_t, α_i and β_i are the Model parameters

Suppose we have returns, a_t , from n assets with expected value 0 and covariance matrix H_t . Then the mean equation for the Dynamic Conditional Correlation (DCC-) GARCH model is defined as:

$$r_t = \mu_t + \varepsilon_t \quad (5)$$

$$\varepsilon_t = H_t^{-\frac{1}{2}} Z_t \text{ and } H_t = D_t R_t D_t$$

Where;

- $r_t = n \times 1$ vector of log returns of n assets at time t .
- ε_t = Residuals at time t . $E(\varepsilon_t) = 0$
- $\text{Cov}(\varepsilon_t) = H_t$.
- $\mu_t = n \times 1$ vector of the expected value of the conditional R_t .
- $H_t = n \times n$ matrix of conditional variances of at time t . or Time-varying covariance matrix.
- $H_t^{-\frac{1}{2}} =$ Any $n \times n$ matrix at time t such that H_t is the conditional variance matrix of a_t . $H_t^{-\frac{1}{2}}$ could be gotten by Cholesky factorization of H_t
- Z_t is an $n \times 1$ vector of iid errors such that $E(Z_t) = 0$ and $E(Z_t Z_t') = 1$
- Z_t could also be defined as a Vector of standardized residuals. $Z_t \sim N(0,1)$

The idea of the models in this class is that the covariance matrix, $H_t =$, can be decomposed into conditional standard deviations, D_t , and a correlation matrix, R_t . The elements in the diagonal matrix D_t are standard deviations from univariate GARCH models.

$$D_t = \begin{bmatrix} \sqrt{\sigma_{1t}^2} & 0 & 0 & \dots & 0 \\ 0 & \sqrt{\sigma_{2t}^2} & 0 & \dots & 0 \\ 0 & 0 & \sqrt{\sigma_{3t}^2} & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & 0 & \sqrt{\sigma_{nt}^2} \end{bmatrix} \quad (6)$$

$$D_t = \text{diag}(\sigma_{1t}, \sigma_{2t}, \sigma_{3t}, \dots, \sigma_{nt})$$

Where σ_{1t} is the time-varying conditional standard deviation (square root of variance) of the i th time series at time t , typically obtained from the univariate GARCH models.

The overall conditional covariance matrix H_t is constructed as $H_t = D_t R_t D_t$

Where: R_t is the time-varying correlation matrix (from the DCC part of the model). In the DCC-GARCH model, the matrix R_t is the time-varying conditional correlation matrix. It represents the dynamic correlations between the standardized residuals of the time series at time t . The residuals $Z_t = D_t^{-1} \varepsilon_t$ are standardized with unit variances, which are used to estimate R_t .

Since R_t is a correlation matrix it is symmetric, thus we have:

$$R_t = \begin{bmatrix} 1 & \rho_{12,t} & \rho_{13,t} & \dots & \rho_{1n,t} \\ \rho_{12,t} & 1 & \rho_{23,t} & \dots & \rho_{2n,t} \\ \rho_{13,t} & \rho_{23,t} & 1 & \dots & \rho_{3n,t} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_{1n,t} & \rho_{2n,t} & \dots & \rho_{n-1n,t} & 1 \end{bmatrix} \quad (7)$$

After estimating the conditional variances, the model estimates the conditional correlation matrix R_t , which is dynamic over time. H_t has to be positive definite because it is a covariance matrix. To ensure H_t to be positive definite, R_t has to be positive definite (D_t is positive definite since all the diagonal elements are positive). All the elements in the correlation matrix R_t have to be equal to or less than one. The standardized residuals Z_t are used to estimate the dynamic correlation matrix R_t . To ensure both of these requirements in the DCC-GARCH model, R_t is decomposed into:

$$R_t = Q_t^{*-1} \bar{Q} Q_t^{*-1} \quad (8)$$

$$Q_t = (1 - a - b) \bar{Q} + a Z_{t-1} Z'_{t-1} + b Q_{t-1} \quad (9)$$

Where;

- Q_t : Dynamic covariance matrix of standardized residuals.
- $\bar{Q} = \text{Cov}(Z_{t-1} Z'_{t-1}) = E(Z_{t-1} Z'_{t-1})$ is the Unconditional covariance of the standardized errors, Z_t .
- a, b are the DCC parameters.
- Z_{t-1} is a Vector of standardized residuals at time $t - 1$.
- Q_t^* is a diagonal matrix with the square root of the diagonal elements of Q_t at the diagonal:
- $\bar{Q}$ and Q_t^* are estimated as follows:

$$\bar{Q} = \frac{1}{T} \sum_{t=1}^T Z_{t-1} Z'_{t-1} \quad (10)$$

$$Q_t^* = \begin{bmatrix} \sqrt{q_{11t}} & 0 & 0 & \dots & 0 \\ 0 & \sqrt{q_{22t}} & 0 & \dots & 0 \\ 0 & 0 & \sqrt{q_{33t}} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & \sqrt{q_{nnt}} \end{bmatrix} \quad (11)$$

Q_t^* rescales the elements in Q_t to ensure the second requirement

$$|\rho_{i,j}| = \left| \frac{q_{ijt}}{\sqrt{q_{iit} q_{jjt}}} \right| \leq 1$$

Further Q_t has to be positive definite to ensure R_t to be positive definite. There are some conditions imposed on the parameters a and b to guarantee H_t to be positive definite. In addition to the conditions for the univariate GARCH model to ensure positive unconditional variances the scalars a and b must satisfy: $a \geq 0, b \geq 0$ and $a + b < 1$.

The correlation structure can be extended to the general DCC (P, M)-GARCH model:

$$Q_t = \left(1 - \sum_{i=1}^P a_i - \sum_{j=1}^M b_j \right) \bar{Q} + \sum_{i=1}^P a_i Z_{t-1} Z'_{t-1} + \sum_{j=1}^M b_j Q_{t-1}$$

3.3 Parameter Estimation of DCC-GARCH

For the DCC model GARCH, the estimation of the suitably specified log-likelihood is difficult, and hence the DC model is designed to allow for two stage estimation. In the first stage the parameters of the univariate GARCH models are estimated for each asset series. The likelihood used in the first stage results in replacing R_t with an identity matrix. In the second stage, the parameters of the DCC model are estimated using the log-likelihood. The parameter estimation for the DCC-GARCH model involves deriving the log-likelihood function and using it to estimate the model parameters via numerical optimization. The DCC-GARCH estimation involves:

1. Estimating GARCH parameters for conditional variances (D_t).
2. Using standardized residuals to estimate DCC parameters (a, b).
3. Maximizing the combined likelihood function numerically.

3.4 Likelihood Function for $\epsilon_{i,t}$

The conditional distribution of the residuals $\epsilon_{i,t}$ is: $\epsilon_{i,t} / \mathcal{F}_{t-1} \sim N(0, \sigma_{i,t}^2)$, where $\mathcal{F}_{t-1}$ signifies the information set available up to time $t-1$. The probability density function (PDF) for a normal random variable $\epsilon_{i,t}$ with mean 0 and variance $\sigma_{i,t}^2$ is given by:

$$f(\epsilon_{i,t}) = \frac{1}{\sqrt{2\pi\sigma_{i,t}^2}} \exp \left[-\frac{\epsilon_{i,t}^2}{2\sigma_{i,t}^2} \right] \quad (14)$$

The log-likelihood for a single observation is the natural logarithm of the PDF:

$$L_{i,t} = \text{Log} \left(f(\epsilon_{i,t}) \right) = -\frac{1}{2} \log(2\pi) - \frac{1}{2} \log(\sigma_{i,t}^2) - \frac{\epsilon_{i,t}^2}{2\sigma_{i,t}^2}$$

For N observations, the total log-likelihood is:

$$L_{garch} = -\frac{1}{2} \sum_{i=1}^N \left[\log(2\pi) + \log(\sigma_{i,t}^2) + \frac{\epsilon_{i,t}^2}{\sigma_{i,t}^2} \right]$$

Where;

$\epsilon_{i,t} = y_{i,t} - \mu_{i,t}$ are the residuals

3.5 DCC Likelihood

The joint likelihood of the multivariate DCC model is based on the standardized residuals

$$Z_t = (z_{1,t}, z_{2,t}, z_{3,t}, \dots, z_{N,t})^T.$$

The log-likelihood for the DCC part is:

$$L_{DCC} = -\frac{1}{2} \sum_{i=1}^N [\text{Log}(\det(R_t)) + Z_t^T R_t^{-1} Z_t]$$

3.6 Combined Log-Likelihood Function

The full log-likelihood of the DCC-GARCH model is:

Let φ be the parameter vector all parameters, including GARCH parameters (ω_i , α_i and β_i) and DCC parameters (a,b). Then

$$L(\varphi) = \frac{N}{2} \text{Log}(2\pi) - \frac{1}{2} \sum_{i=1}^N \{ \log|H_t| + \epsilon_i^T H_t^{-1} \epsilon_i \} \quad (17)$$

Where;

$$\varphi = [\omega_i, \alpha_i, \beta_i, a, b]$$

Prove

From equation (5) $H_t = D_t R_t D_t$, Where $D_t =$

$\text{diag} \left[h_{1t}^2, h_{2t}^2, h_{3t}^2, \dots, h_{nt}^2 \right]$ and R_t is the time-varying

correlation matrix. The determinant of H_t can be calculated using the following properties of determinants:

Determinant of a Product: For any two conformable square matrices A and B, $|AB| = |A| \cdot |B|$. Thus

$$|H_t| = |D_t| \cdot |R_t| \cdot |D_t| \quad (18)$$

Determinant of a Diagonal Matrix: For a diagonal matrix $D_t = \text{diag}[d_1, d_2, d_3, \dots, d_N]$

$$|D_t| = \prod_{i=1}^N d_i \quad (19)$$

Equation (18) could also be written as

$$|H_t| = |D_t|^2 \cdot |R_t| \quad (20)$$

$$\text{Log}|H_t| = 2\text{Log}|D_t| + \text{Log}|R_t| \quad (21)$$

$$H_t^{-1} = D_t^{-1} R_t^{-1} D_t^{-1} \quad (22)$$

By substituting equations (20), (21) and (22) in equation (17)

$$\text{Log}(L(\varphi)) = \frac{N}{2} \text{Log}(2\pi) - \frac{1}{2} \sum_{i=1}^N \{ 2\text{Log}|D_t| + \text{Log}|R_t| + \epsilon_i^T D_t^{-1} R_t^{-1} D_t^{-1} \epsilon_i \}$$

3.7 Source of Data

This study employed secondary data comprising daily price observations spanning the period from March 3, 2017, to March 12, 2025. The data was sourced from the financial market database available on www.investing.com, a globally recognized platform for comprehensive financial market information. The dataset includes the following variables, Nigerian All-Share Index (ASI), Crude Oil Price (Brent) reflecting global crude oil market trends, and Stock price. The choice of secondary data is justified by its reliability, accessibility, and the historical coverage required for time-series modelling. Using data from a reputable source like investing.com ensures accuracy and consistency, which are critical for volatility modelling and forecasting. Before analysis, the raw price series were cleaned and transformed into continuously compounded returns using the logarithmic difference formula:

$$\text{Return} = \text{Log} \left(\frac{P_t}{P_{t-1}} \right) * 100 \quad (24)$$

Where;

P_t is the price at time t

P_{t-1} is the price at time, $t - 1$

4 Results

Table 1: Descriptive Statistics of the Daily Return Series of Nigeria Stock Prices Brent Crude Oil Price and All Share.

Descriptive Measure	Stock Prices	Bret Crude Oil Price	All share
Mean	0.032358	69.53431	0.042973
Median	-0.001857	68.71000	0.006547
Maximum	9.360020	127.9800	7.984833
Minimum	-6.254517	19.33000	-5.032939
Std. Dev.	1.027891	20.18082	0.972248
Skewness	0.532741	0.311519	0.371539
Kurtosis	10.82226	2.665146	8.639190
Jarque-Bera	7611.194	61.12036	3951.065
Probability	0.000000	0.000000	0.000000
Sum	94.84164	203874.6	
Sum Sq. Dev.	3095.720	1193695.	125.9543
Observations	2931	2931	2769.628

Table 1 presents the descriptive statistics of daily return series for three economic indicators: Nigeria Stock Prices, Brent Crude Oil Prices, and the All Share Index (ASI). The mean returns for Stock Prices and the ASI are 0.032358 and

0.042973 respectively. In contrast, the Brent Crude Oil Price has a much higher average value (69.53431). The mean returns for Stock Prices and All Share are slightly positive, while the mean price for Brent Crude Oil is relatively high at

69.53. The median values for the series are close to zero for returns and slightly lower than the mean for Brent Crude Oil Price. The maximum and minimum values indicate a significant range of values for all series. The maximum and minimum values suggest high volatility, particularly in the Stock Prices and the ASI. Stock Prices have a wide range (from -6.25% to +9.36%), and similarly, the ASI ranges from -5.03% to +7.98%. Brent crude oil prices vary significantly between 19.33 and 127.98, reflecting global oil market volatility. Standard deviations reinforce this volatility: Stock Prices and the ASI have relatively high standard deviations (1.03 and 0.97, respectively), while Brent Oil Prices have a very large standard deviation (20.18), the skewness values for all three series are positive but modest, indicating slight asymmetry with more frequent small negative returns and a few large positive returns. Kurtosis values are particularly high for Stock Prices (10.82) and the ASI (8.64), implying heavy tails and a higher probability of extreme values compared to a normal distribution. Brent Oil Prices have a

kurtosis closer to normal (2.67), suggesting a more moderate distribution.

The standard deviations of the series are relatively high compared to their means, indicating substantial variability. The skewness values are positive for all series, suggesting that the distributions are asymmetric with a longer tail on the right side. The kurtosis values are high for Stock Prices and All Share, indicating leptokurtic (fat-tailed) distributions, while Brent Crude Oil Price has a relatively lower kurtosis value. These results suggest that the series exhibit non-normality and may require robust statistical methods for analysis.

The Jarque-Bera test confirms the non-normality of the series, rejecting the null hypothesis of normality for all three series. This implies that traditional statistical methods assuming normality may not be suitable for these series. Overall, the results highlight the importance of considering the unique characteristics of each series when selecting statistical models or methods for analysis. By acknowledging and addressing these characteristics, researchers can develop more accurate and reliable models that capture the complexities of the data.

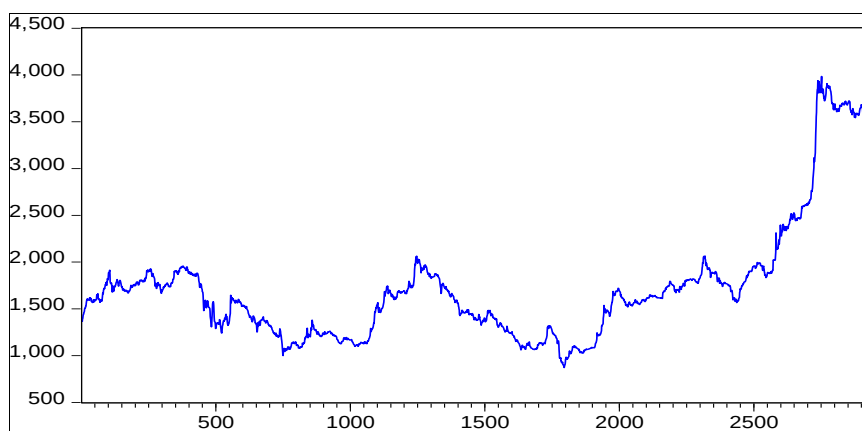


Fig 1: Time Plot of the Nigeria Daily Stock Prices from 2017-2025

Figure 1 shows the time plot of the Nigeria Daily Stock Prices over the period from March 2017 to May 2025. The series shows substantial fluctuations over time, with a generally upward trend beginning around early 2021 and continuing through early 2024. This indicates a period of increasing value or performance in the underlying asset or index after a period of relative stagnation or decline. From 2017 to late

2020, the graph exhibits moderate volatility with several local peaks and troughs. A significant decline is visible between 2018 and 2020, possibly corresponding to economic challenges or global events (e.g., COVID-19 impact). Post-2020, volatility continues but is accompanied by a stronger upward trend.

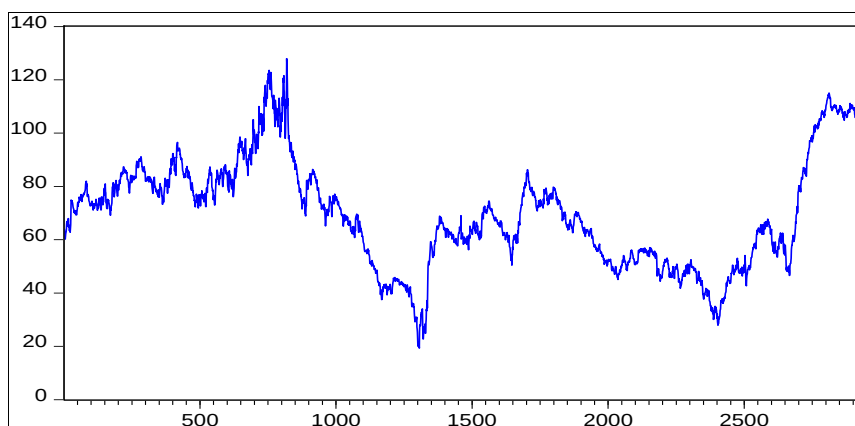


Fig 2: Time Plot of Bent Crude Oil Price from 2017-2025

Figure 2 shows the time series plot of Bent Crude Oil Price from 2017-2025. The series from shows significant fluctuations, reflecting global supply-demand dynamics, geopolitical events, and market shocks. Initially, prices

fluctuate between \$60-80 per barrel until mid-2019. There is a sharp surge around late 2018 to early 2019, reaching above \$120. A dramatic crash is visible in early 2020, with prices falling sharply. This corresponds with the global economic

shutdown due to the COVID-19 pandemic, where oil demand plummeted, causing one of the steepest drops in oil prices in modern history. From late 2020 through 2023, Brent prices gradually recover, oscillating mostly between \$60 and \$90, showing sensitivity to pandemic recovery, OPEC+ policies,

inflation, and global energy demands. The fluctuations reflect the market's ongoing adjustment to post-COVID economic conditions, and possibly the Russia-Ukraine conflict starting in 2022.

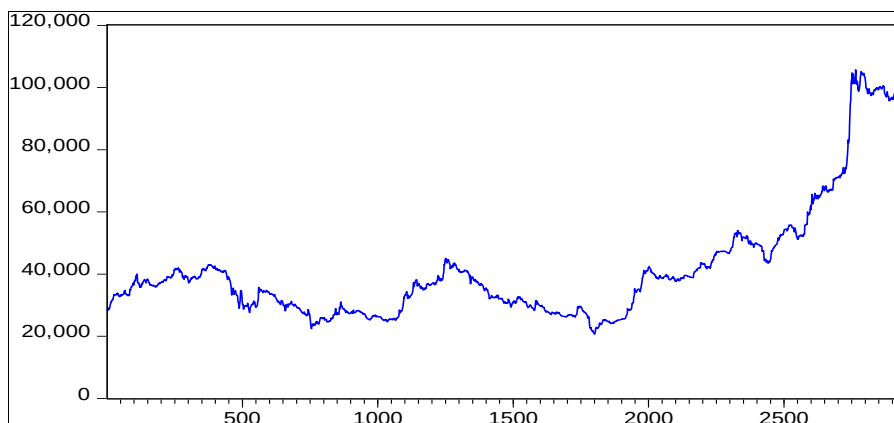


Fig 3: Time Plot of Time All Share Index (ASI) from 2017-2025

Figure 3 shows the time series plot of the Nigerian All Share Index from 2017-2025. The graph shows while the price level does not shift drastically overall, the graph displays short bursts of increased variability (e.g., late 2018 and early 2020), followed by relatively quieter periods. These visible fluctuations suggest the presence of short-term volatility clustering. The early 2020 segment, during the COVID-19 pandemic onset, shows a sudden sharp decline followed by a

quick rebound. This is a classic example of volatility clustering a high-volatility period (market crash and recovery) concentrated in a short time span. From 2021 to early 2024, the ASI trends upward sharply, but not smooth. There are noticeable stretches where prices rise rapidly followed by abrupt pullbacks (e.g., sharp corrections in late 2022 and mid-2023), again indicating periods of intensified volatility grouped together.

Table 2: Unit Root Test for the Nigeria Daily Stock Price, Stock Prices, Brent Crude Oil Price and All Share

Null Hypothesis: Price has a Unit Root				
		Stock Prices	Brent Crude Oil Price	All share
Augmented Dickey-Fuller test statistic		0.964348	-1.574216	2.091968
Test critical values:	1% level	-3.432391	-3.432391	-3.432391
	5% level	-2.862328	-2.862327	-2.862328
	10% level	-2.567234	-2.567233	-2.567234
	p-value	0.9963	0.4957	0.9999

Table 2 shows the unit root test for the Nigeria daily stock price, Brent crude oil price and all share. The Augmented Dickey-Fuller (ADF) test statistic for Nigerian stock prices is 0.964348, which is significantly higher than all the critical values at the 1%, 5%, and 10% levels (i.e., -3.432, -2.862, -2.567, respectively). Furthermore, the p-value is 0.9963, far above the critical 0.05 value. Therefore, the stock price series is non-stationary in levels thus, must be differenced (usually converted to returns) to achieve stationarity before any further modelling.

The ADF test statistic for Brent crude oil price is -1.574216, still less negative than any of the critical values. The associated p-value of 0.4957 also exceeds levels of significance. Again, we fail to reject the null hypothesis of a unit root, implying that the Brent crude oil price series is also non-stationary. Like the stock prices, it requires transformation typically taking log differences or returns to

stabilize its statistical properties for reliable volatility modelling and forecasting using time series techniques. For the All Share Index, the ADF test statistic is 2.091968, which is not only higher than all critical values but also positive, which is unusual for a stationary series. The p-value of 0.9999 confirms that there is overwhelming evidence against stationarity. This result further supports the conclusion that the ASI is non-stationary in its current form. Thus, just like the other two series, the ASI must be differenced or converted to returns to ensure stationarity prior to fitting MGARCH models or any other statistical or econometric analysis.

Clearly, none of the three series is stationary in levels, as all their ADF test statistics are insufficiently negative and their p-values are far above standard significance thresholds. Stationarity is a prerequisite for MGARCH modelling, so all three must be transformed into returns to proceed with meaningful volatility and correlation analysis.

Table 3: Unit Root Test for the Return on Daily Stock Price

Null Hypothesis: PRICE has a Unit Root				
		Stock Prices	BrentCrude Oil Price	All share
ADF Test Statistic		-39.18259	-53.50654	-39.74618
Test critical values:	1% level	-3.432391	-3.432391	-3.432391
	5% level	-2.862328	-2.862328	-2.862328
	10% level	-2.567234	-2.567234	-2.567234
	p-value	0.0000	0.0001	0.0000

Footnote: * = significant at 5%

Table 3 shows the unit root test results for the return on three series used in this study. The Augmented Dickey-Fuller (ADF) test for the All-Share Index returns series yields a test statistic of -39.18259 which is below all the corresponding critical values at the 1%, 5%, and 10% significance levels (-3.432391, -2.862328, and -2.567234, respectively). Additionally, the p-value is 0.0000, which is less than the level of significance (0.05). These results strongly reject the null hypothesis that the return series for the All-Share Index has a unit root. Therefore, the return series is stationary, meaning it has a constant mean and variance over time and is suitable for time series modelling.

For the Brent Crude Oil Price return series, the ADF test statistic is -53.50654, which is also substantially lower than the critical values at all significance levels. The corresponding p-value of 0.0001 supports the rejection of the null hypothesis

of a unit root. This shows that the return series of Brent Crude Oil Price is also stationary. The high magnitude of the test statistic suggests strong evidence against non-stationarity, making this series appropriate for econometric modelling and forecasting.

Also, the Stock Prices return series shows an ADF test statistic of -39.74618. Like the other variables, this value is much lower than the critical thresholds at the 1%, 5%, and 10% levels. With a p-value of 0.0000, thus, null hypothesis that the series has a unit root is rejected. This indicates that the return series for Stock Prices is stationary. Overall, all three variables exhibit stationarity, which is an essential condition for accurate time series analysis and modelling of returns. The plots of the returns on the series (volatility clustering) are shown in the figures below.

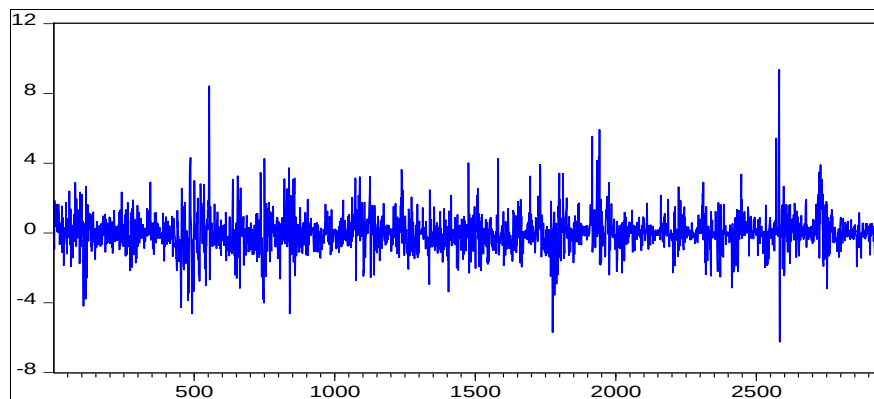


Fig 4: Time Plot of the Return Series for the Nigeria Daily Stock Prices

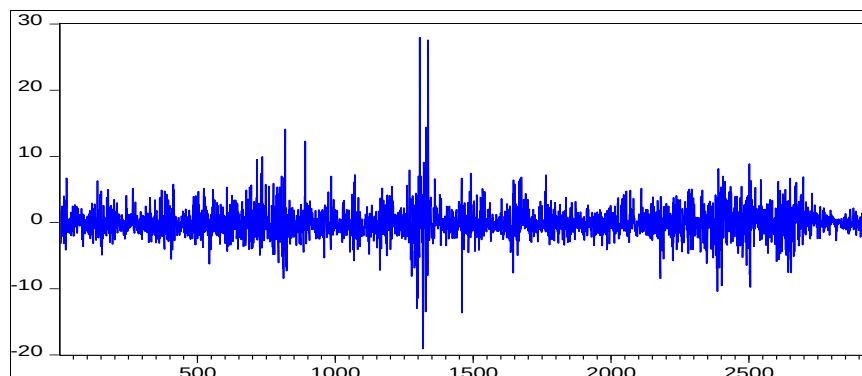


Fig 5: Time Plot of the Return Series for Nigeria Daily Brent Crude Oil Price

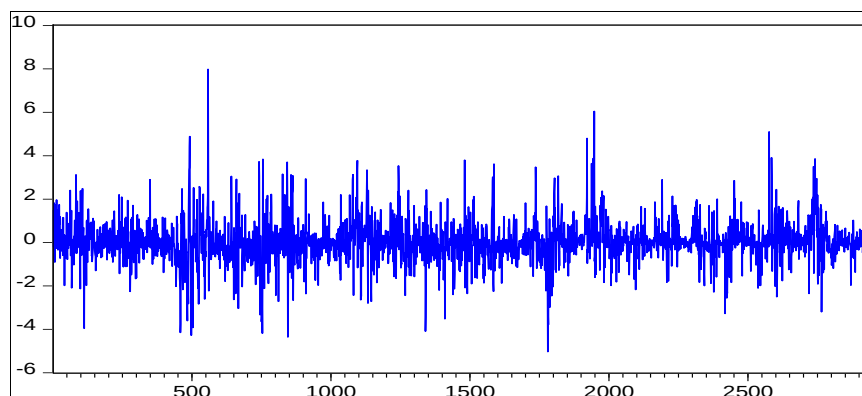


Fig 6: Time Plot of the Return Series for Nigeria Daily All Share Price

The time plot in Figure 4-6 illustrates the daily return series for Nigerian stock prices, Crude oil prices and all share Index from March 2017 to mid-2025. The returns fluctuate around a mean of zero, indicating that the series has been mean-

adjusted. Notably, the graph displays periods of high and low volatility a phenomenon known as volatility clustering where large movements (both positive and negative) tend to cluster together over time, reflecting changes in market uncertainty

or external shocks. The presence of volatility clustering suggests that the mean is stable and that the GARCH-type models could be applied to capture time-varying volatility in the financial return series.

Testing for ARCH Effects: Before estimating a GARCH-type model, it is important first to identify whether there is

substantial evidence of heteroskedasticity (ARCH effects), which determine whether or not it is necessary to apply the ARCH estimation methods instead of the OLS. In order to test for the presence of ARCH effects in the return series, this study employed Lagrange Multiplier Test, Correlogram and Ljung-Box Statistics methods to check the correlation in residuals.

Table 4: ARCH Effect Tests for the Return on Daily Stock Price, All Share Index (ASI) and Crude Oil Prices (OIL)

Series	Chi-squared Statistic	Degrees of Freedom (df)	p-value	Conclusion
Return on Stock Prices (RSP)	349.62	12	0.0000**	Significant ARCH effects (reject H_0)
Return on All Share Index (RASI)	375.02	12	0.0000**	Significant ARCH effects (reject H_0)
Return on Crude Oil Prices (ROIL)	365.97	12	0.0000**	Significant ARCH effects (reject H_0)

Footnote: ** = significant at 5%

The results of the ARCH LM tests for stock returns, the All Share Index (ASI), and crude oil prices all reveal highly significant ARCH effects, with Chi-squared statistics of 349.62, 375.02, and 365.97 respectively, each having p-values below 0.0000. These findings strongly reject the null hypothesis of no ARCH effects, indicating that the conditional variances of the return series are time-varying and

exhibit volatility clustering—a common feature of financial time series. This means that periods of high volatility tend to follow high volatility, while calm periods follow calm periods. Therefore, the presence of significant heteroskedasticity across all three return series justifies the use of ARCH or GARCH-type models to appropriately capture and model their dynamic volatility behaviour.

Table 5: Estimated Parameters of MGARCH-DCC (1,1) Models Using Multivariate Generalized Error Distributions

Parameter	Estimate	Std. Error	p-Value	Significance
Stock Returns (RSP)				
μ (Mean)	-0.004	0.016	0.779	-
ω (Omega)	0.083	0.030	0.006	*
α_1 (Alpha1)	0.220	0.047	0.000	*
β_1 (Beta1)	0.712	0.063	0.000	*
All Share Index Returns (RASI)				
μ (Mean)	-0.001	0.015	0.931	-
ω (Omega)	0.091	0.040	0.024	*
α_1 (Alpha1)	0.211	0.051	0.000	*
β_1 (Beta1)	0.700	0.085	0.000	*
Crude Oil Returns (ROIL)				
μ (Mean)	-0.020	0.033	0.546	-
ω (Omega)	0.094	0.030	0.002	*
α_1 (Alpha1)	0.122	0.018	0.000	*
β_1 (Beta1)	0.866	0.018	0.000	*
Shape	-	-	-	-
Skew	-	-	-	-
Dynamic Conditional Correlation (DCC) Parameters				
DCC α_1 (dcc α_1)	0.013	0.009	0.168	-
DCC β_1 (dcc β_1)	0.679	0.107	0.000	*

Footnote: * = significant at 5% level.

The results from the MGARCH-DCC (1,1) model indicate that all three financial return series stock returns (RSP), All Share Index returns (RASI), and crude oil returns (ROIL) exhibit significant volatility persistence. For the RSP series, both the ARCH ($\alpha_1 = 0.220$, $p < 0.01$) and GARCH ($\beta_1 = 0.712$, $p < 0.01$) coefficients are positive and statistically significant, suggesting that shocks to stock return volatility have a lasting impact over time. Similarly, the RASI series shows significant ARCH ($\alpha_1 = 0.211$) and GARCH ($\beta_1 = 0.700$) effects, confirming that periods of high volatility are likely to be followed by more volatility, consistent with volatility clustering typically observed in financial markets. These findings validate the suitability of GARCH-type models in capturing the dynamic nature of Nigerian stock market volatility.

For the crude oil returns (ROIL), the results reveal strong evidence of volatility persistence as well, with $\alpha_1 = 0.122$ and $\beta_1 = 0.866$, both statistically significant at the 1% level. The relatively higher β_1 value indicates that crude oil price

volatility is more persistent compared to stock and ASI returns. This suggests that shocks in the crude oil market tend to have longer-lasting effects, possibly due to the influence of global oil supply-demand imbalances, geopolitical tensions, and macroeconomic policy changes. The result highlights the critical role of crude oil price movements in shaping the volatility dynamics of Nigeria's financial system, given the country's heavy dependence on oil exports.

The Dynamic Conditional Correlation (DCC) parameters further reveal important insights into the co-movement of the three financial assets. The DCC α_1 parameter (0.013) is not statistically significant, implying that immediate responses to new shocks in correlations are relatively weak. However, the DCC β_1 coefficient (0.679, $p < 0.01$) is significant, indicating strong persistence in the conditional correlations among stock returns, the ASI, and crude oil prices. This suggests that correlations among these markets evolve slowly over time and are influenced by long-term common factors. Overall, the significant volatility persistence and dynamic

correlations confirm that the Nigerian stock market and crude oil market are interlinked, implying that portfolio diversification opportunities may be limited during periods of market turbulence.

6 Conclusion

The spillover effects from the MGARCH-DCC (1,1) estimation reflect the extent to which volatility in one financial market transmits to others in this case, between Nigerian stock returns (RSP), the All Share Index (RASI), and crude oil prices (ROIL). The significant ARCH and GARCH coefficients across all series indicate that volatility shocks are not isolated within individual markets but tend to propagate through the financial system. Specifically, the persistence of volatility in crude oil returns ($\beta_1 = 0.866$) implies that fluctuations in the oil market driven by global supply-demand imbalances or geopolitical developments can spill over into the Nigerian stock market, influencing both stock returns and the ASI. This interconnected behavior suggests that investors and policymakers should monitor the oil market closely, as shocks originating there can affect the overall financial stability of the Nigerian economy.

The DCC parameters further support the presence of time-varying and persistent correlations among these assets, which provide evidence of volatility spillovers. The significant DCC β_1 value (0.679, $p < 0.01$) indicates that correlations among crude oil, stock returns, and the ASI are not constant but evolve gradually over time. This means that periods of high volatility in the oil market are likely to coincide with heightened volatility in the Nigerian stock market. Such co-movements are particularly pronounced during global financial or commodity market disturbances, implying that shocks in one asset class tend to amplify risk levels in others. This persistence of dynamic correlations underscores the integrated nature of Nigeria's financial and commodity markets.

Conclusively, the observed spillover effects highlight the vulnerability of the Nigerian stock market to external shocks, particularly those emanating from crude oil price volatility. As the economy remains heavily reliant on oil revenue, disturbances in the global oil market can transmit rapidly to domestic financial assets, influencing investor sentiment and market performance. These results emphasize the importance of diversification and the implementation of sound macroeconomic and risk management policies to cushion the Nigerian economy against adverse volatility spillovers from the oil sector into the broader financial market.

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