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Structure of the Equal Bi-Quadratic Graph of Finite Groups

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Abstract

In this research, we introduce the new graph Equal Bi-Quadratic graph. The Equal Bi-Quadratic graph of a finite group G denoted as $EBQ(G)$ is an undirected simple graph having pairs of different vertices u and v adjacent iff $u^4 = v^4$. This manuscript explores the representation of Equal Bi-Quadratic graph structure for finite cyclic and dihedral groups.

Keywords: Equal Bi-Quadratic graph, cyclic group, dihedral group

Introduction

Numerous graphs have been introduced and examined in association with algebraic structures such as square element graph [10, 11], k^{th} -power graph [9], square power graph [3-5], equal-square graph [1, 2], cubic power graph [6-8] etc. Various characteristics like completeness, traversability, girth, diameter, chromatic number and bipartiteness of line graph are examined in [12].

Authors have examined and introduced graphical representation of Equal-Square graphs having finite groups as their vertex sets in [1]. This research extends the concept of Equal-Square graph to Equal Bi-Quadratic graph of a finite group. Equal Bi-Quadratic of a finite group G , is an undirected simple graph with different vertices pairs x and y having edge iff $x^4 = y^4$.

In Section 2, connectedness and structure of Equal Bi-Quadratic graph of cyclic groups are examined where as in section 3, for dihedral group.

Equal Bi-Quadratic Graph of Finite Cyclic Group

Theorem 2.1 The Equal Bi-Quadratic graph of finite group G of order m is an empty graph iff $(m, 4) = 1$.

Proof. Let $EBQ(G)$ be the Equal Bi-Quadratic graph of finite group G of order m .

For $(m, 4) = 1$, m is of form $m = 4n + 1$ or $4n + 3$.

Let us assume that different pair of adjacent vertices u and v in $EBQ(G)$.

Case (i) $m = 4n + 1$

As $(u, v) \in E(EBQ(G))$, so $u^4 = v^4 \rightarrow (u^4)^{3n+1} = (v^4)^{3n+1} \rightarrow u^{12n+4} = v^{12n+4} \rightarrow u^{4n+1}u^{4n+1}u^{4n+1}u = v^{4n+1}v^{4n+1}v^{4n+1}v \rightarrow u = v$. Thus, there is no pair of distinct vertices adjacent in $EBQ(G)$.

Case (ii) $m = 4n + 3$

As $(u, v) \in E(EBQ(G))$, so $u^4 = v^4 \rightarrow (u^4)^{n+1} = (v^4)^{n+1} \rightarrow u^{4n+4} = v^{4n+4} \rightarrow u^{4n+3}u = v^{4n+3}v \rightarrow u = v$. Thus, there is no pair of distinct vertices adjacent in $EBQ(G)$.

Hence $EBQ(G)$ is an empty graph for $(m, 4) = 1$.

Theorem 2.2 Let $EBQ(G)$ be the Equal Bi-Quadratic graph of a finite cyclic group of order m , then $EBQ(G) =$

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$$\begin{cases} \phi, & \text{if } (m, 4) = 1, \\ \frac{m}{2}K_2, & \text{if } (m, 4) = 2 \text{ and} \\ \frac{m}{4}K_4, & \text{if } (m, 4) = 4. \end{cases}$$

Proof. Let $EBQ(G)$ be the Equal Bi-Quadratic graph of a finite cyclic group G of order m where $G = \{e, x, x^2, \dots, x^{m-1}\}$. Let us assume different adjacent vertices pair $u = x^i$ and $v = x^j$ in $EBQ(G)$.

Case (i) $(m, 4) = 1$

Using Theorem 2.1, we have the required result.

In below figure structure of Equal Bi-Quadratic Graph of $\mathbb{Z}_5 = \{e, x, x^2, x^3, x^4\}$ is shown.

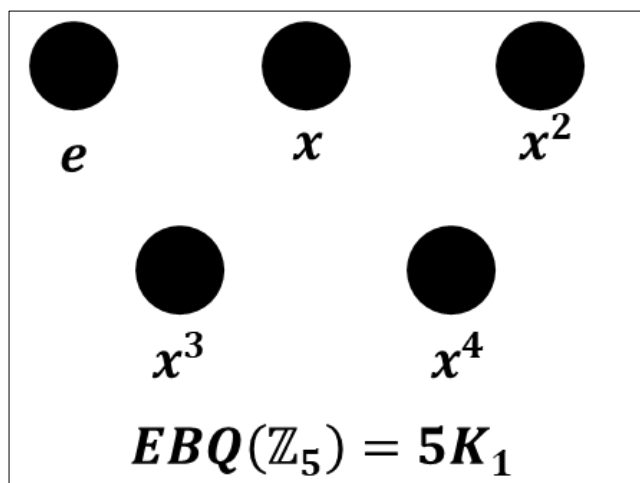


Fig 1: Equal Bi-Quadratic Graph of $\mathbb{Z}_5$

Case (ii) $(m, 4) = 2$

We have $(m, 4) = 2$, so $m = 4n + 2$. Then, $u^4 = v^4 \rightarrow (u^4)^{n+1} = (v^4)^{n+1} \rightarrow u^{4n+4} = v^{4n+4} \rightarrow u^{4n+2}u^2 = v^{4n+2}v^2 \rightarrow u^2 = v^2$. As $(x^i)^2 = (x^{i+\frac{m}{2}})^2$ for all $1 \leq i \leq \frac{m}{2}$. There are distinct $\frac{m}{2}$ pairs of elements $(x^i, x^{\frac{m}{2}+i})$ for $1 \leq i \leq \frac{m}{2}$ in G for which $u^2 = v^2$. Thus, $EBQ(G) = \frac{m}{2}K_2$.

In below figure structure of Equal Bi-Quadratic Graph of $\mathbb{Z}_{10} = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8, x^9\}$ is shown.

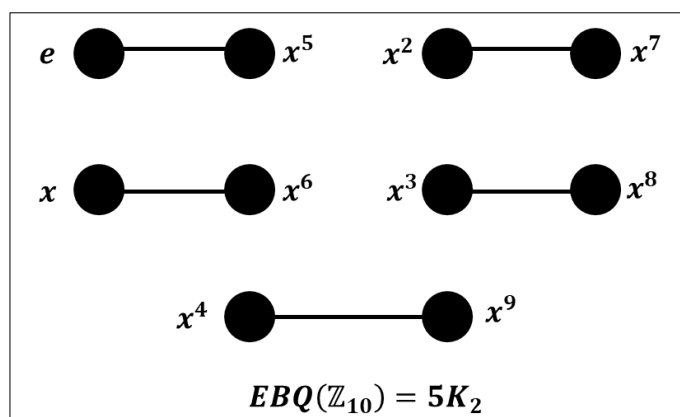
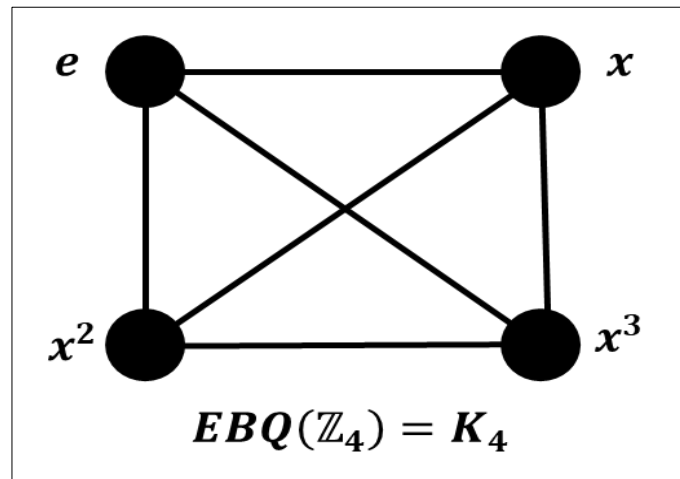


Fig 2: Equal Bi-Quadratic Graph of $\mathbb{Z}_{10}$

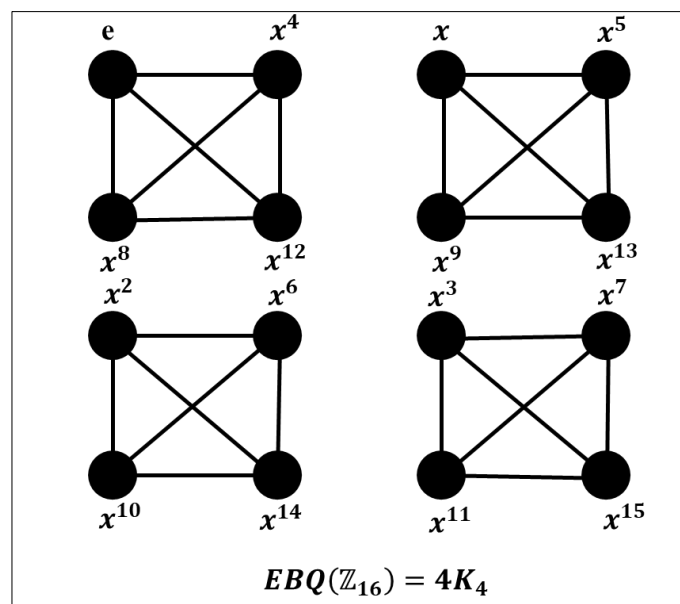
Case (iii) $(m, 4) = 4$

We have $(m, 4) = 4$, so $m = 4n$. For $i \neq j$, and $0 \leq i, j \leq m - 1$, $(x^i)^4 = (x^j)^4$ iff $i \equiv 0 \pmod{\frac{m}{4}}$ and $j \equiv 0 \pmod{\frac{m}{4}}$ or $i \equiv 1 \pmod{\frac{m}{4}}$ and $j \equiv 1 \pmod{\frac{m}{4}}$ or $i \equiv 2 \pmod{\frac{m}{4}}$ and $j \equiv 2 \pmod{\frac{m}{4}}$ or ... or $i \equiv (\frac{m}{4} - 1) \pmod{\frac{m}{4}}$ and $j \equiv (\frac{m}{4} - 1) \pmod{\frac{m}{4}}$. So, there are disjoint $\frac{m}{4}$ sets of different elements from G , each set containing four elements satisfying one of the above congruent condition. Thus, $EBQ(G) = \frac{m}{4}K_4$.

In below figure structure of Equal Bi-Quadratic Graph of $\mathbb{Z}_4 = \{e, x, x^2, x^3\}$ is shown.

Fig 3: Equal Bi-Quadratic Graph of $\mathbb{Z}_4$

In below figure structure of Equal Bi-Quadratic Graph of $\mathbb{Z}_{16} = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8, x^9, x^{10}, x^{11}, x^{12}, x^{13}, x^{14}, x^{15}\}$ is shown.

Fig 4: Equal Bi-Quadratic Graph of $\mathbb{Z}_{16}$

Equal Bi-Quadratic Graph of Dihedral Group

Theorem 3.1 Let $EBQ(D_m)$ be the Equal Bi-Quadratic graph of a dihedral group D_m of order $2m$, then

$$EBQ(D_m) = \begin{cases} K_{m+1} \cup (m-1)K_1 & \text{if } (m, 4) = 1, \\ K_{m+2} \cup \left(\frac{m-2}{2}\right)K_2 & \text{if } (m, 4) = 2 \text{ and } \\ K_{m+4} \cup \left(\frac{m-4}{4}\right)K_4 & \text{if } (m, 4) = 4. \end{cases}$$

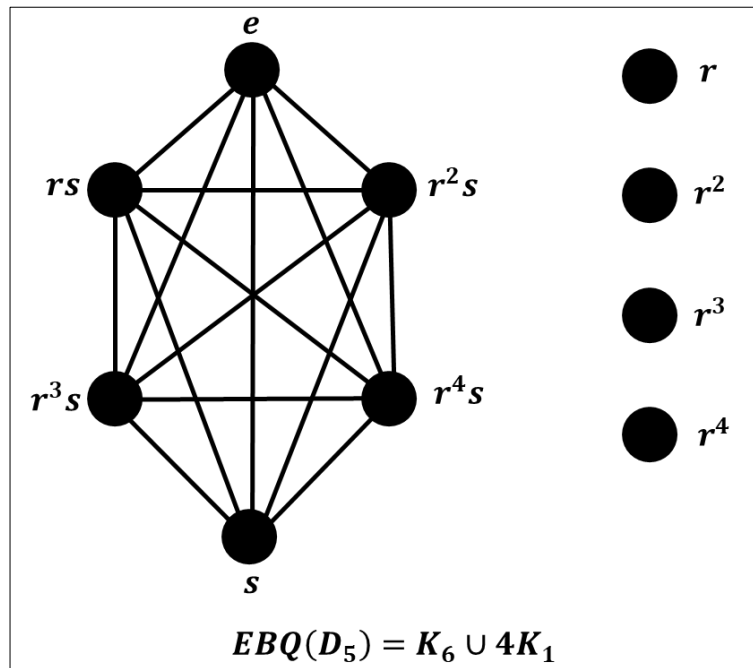
Proof. Let $EBQ(D_m)$ be the Equal Bi-Quadratic graph of a dihedral group $D_m = \{e, r, r^2, \dots, r^{m-1}, s, rs, r^2s, \dots, r^{m-1}s\}$ of order $2m$.

For reflection elements of D_n , $(r^i s)^4 = e \forall 0 \leq i \leq m-1$ and for.

Case (i) $(m, 4) = 1$

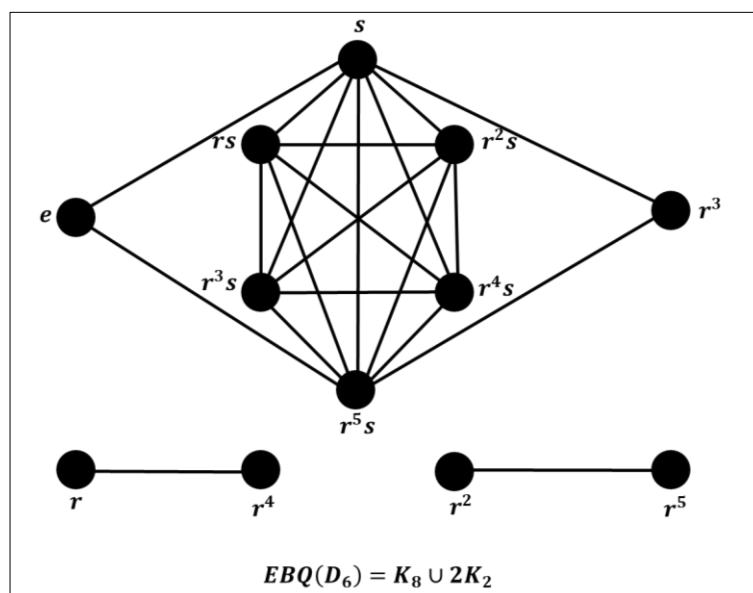
In this case, $(r^i)^4 \neq (r^j)^4$ for all $1 \leq i, j \leq m-1$ and $(r^i s)^4 = e$ for all $0 \leq i \leq m-1$. Thus, $m-1$ rotation elements of D_n forms the $(m-1)K_1$ disjoint component of $EBQ(D_n)$ and identity element & m reflection elements forms the K_{m+1} component of $EBQ(D_n)$. Thus, $EBQ(D_n) = K_{m+1} \cup (m-1)K_1$.

In below figure structure of Equal Bi-Quadratic Graph of $D_5 = \{e, r, r^2, r^3, r^4, s, rs, r^2s, r^3s, r^4s\}$ is shown.

Fig 5: Equal Bi-Quadratic Graph of D_5 **Case (ii)** $(m, 4) = 2$

In this case, $(r^i)^4 = (r^{i+\frac{m}{2}})^4 \neq e$ for all $1 \leq i, j \leq \frac{m}{2} - 1$ and $(r^i s)^4 = e$ for all $0 \leq i \leq m - 1$ & $(r^{\frac{m}{2}})^4 = e$. Thus, $m - 2$ rotation elements of D_n forms the $\left(\frac{m-2}{2}\right) K_2$ disjoint component of $EBQ(D_n)$ and identity element, $r^{\frac{m}{2}}$ & m reflection elements forms the K_{m+2} component of $EBQ(D_n)$. Thus, $EBQ(D_n) = K_{m+2} \cup \left(\frac{m-2}{2}\right) K_2$.

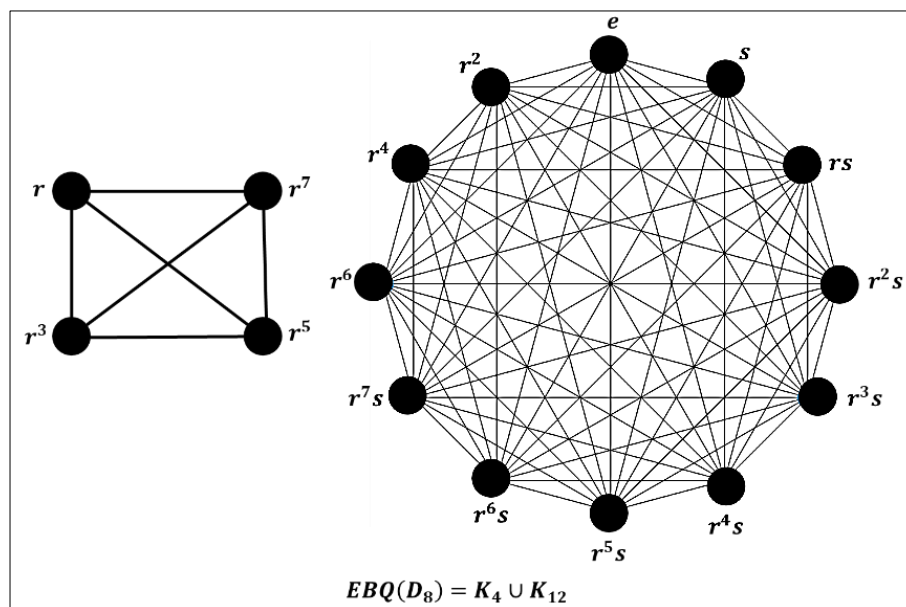
In below figure structure of Equal Bi-Quadratic Graph of $D_6 = \{e, r, r^2, r^3, r^4, r^5, s, rs, r^2s, r^3s, r^4s, r^5s\}$ is shown.

Fig 6: Equal Bi-Quadratic Graph of D_6 **Case (iii)** $(m, 4) = 4$

For $i \neq j$, and $0 \leq i, j \leq m - 1$, $(r^i)^4 = (r^j)^4$ iff $i \equiv 0 \pmod{\frac{m}{4}}$ and $j \equiv 0 \pmod{\frac{m}{4}}$ or $i \equiv 1 \pmod{\frac{m}{4}}$ and $j \equiv 1 \pmod{\frac{m}{4}}$ or $i \equiv 2 \pmod{\frac{m}{4}}$ and $j \equiv 2 \pmod{\frac{m}{4}}$ or $i \equiv 3 \pmod{\frac{m}{4}}$ and $j \equiv 3 \pmod{\frac{m}{4}}$ or $i \equiv (\frac{m}{4} - 1) \pmod{\frac{m}{4}}$ and $j \equiv (\frac{m}{4} - 1) \pmod{\frac{m}{4}}$. Also for $i \equiv 0 \pmod{\frac{m}{4}}$ and $j \equiv 0 \pmod{\frac{m}{4}}$, we have $(r^i)^4 = (r^j)^4 = e$ and $(r^i s)^4 = e$ for all $0 \leq i \leq m - 1$.

Thus, $EBQ(D_n) = K_{m+4} \cup \left(\frac{m-4}{4}\right) K_4$.

In below figure structure of Equal Bi-Quadratic Graph of $D_8 = \{e, r, r^2, r^3, r^4, r^5, r^6, r^7, s, rs, r^2s, r^3s, r^4s, r^5s, r^6s, r^7s\}$ is shown.

Fig 7: Equal Bi-Quadratic Graph of D_8

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