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Existence of fixed point results associated with fuzzy metric spaces and its Generalizations

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Abstract

Fuzzy metric spaces extend classical metric spaces to address ambiguity and uncertainty, providing a robust framework for theoretical and practical analysis. This paper explores fixed-point results in fuzzy settings, beginning with foundational concepts and extending the Banach Contraction Principle. Applications span economics, game theory, image processing, control systems, machine learning, and historical modelling. Computational methods, including algorithms and numerical techniques, support the identification of fixed points in uncertain environments. The study concludes by identifying open challenges and future directions in advancing fuzzy metric space research.

Keywords: Metric, fuzzy, fixed point, existence, application, theorem

1. Introduction

Mathematics, as a time-honored language and a cornerstone of clinical inquiry, usually evolves to fulfill the challenges posed by an ever-expanding spectrum of actual global troubles. Among its many aspects, the exploration of constant factor theorems is a testament to the elegance and application of mathematical standards. This research embarks on a captivating adventure within this mathematical realm, focusing on the life of constant point consequences in the context of fuzzy metric areas and their profound applications, thereby pushing the boundaries of traditional fixed factor theory.

From Euclid's geometric proofs ^[1] to the contemporary dynamical structures theory, fixed factors have held a special place in mathematical discourse. In the early 1900s, Banach and Brouwer made significant contributions with their Fixed Point Theorem and Contraction Mapping Theorem ^[2, 3]. These theorems demonstrated the inherent locations and pervasiveness of fixed components. These classical theorems are observed on-the-spot programs in numerous fields, including differential equations and optimization problems. However, their applicability is contingent upon the underlying area conforming to a strict metric shape. Fuzzy units were first presented by Zadeh in 1965 as a mathematical framework for effectively handling imprecision, ambiguity, and uncertainty in the representation of information ^[4].

1.1 Fuzzy Metric Spaces: Definition and Basic Properties

- Using fuzzy metric spaces makes it feasible to have ambiguous site distances. Assume that X does not include any empty elements and that the $d: X \rightarrow sX [0,1]$ expression satisfies the below conditions for any x, y , or z in sX . Also, assume that X does not contain any empty elements. The analysis can move forward if one chooses to do so. As a direct result, we can continue with the inquiry. It enables the conducting of examinations and inquiries.
 $d(x.y) > 0$
- The identity of indiscernibles is that $d(x, y)$ always equals 0 when x and y are equal, and it is never a negative integer. This statement can only be verified under the conditions

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mentioned earlier. As was demonstrated, indiscernibles can never have a negative value. There is no way to differentiate indistinguishable.

3. The symmetry of the variables may be shown by looking at the equation $d(x, y)$ equals $d(y, x)$, which can be found in the previous sentence.
4. An inequality that involves a triangle and requires $d(x, y)$ must be smaller than $d(z, y)$. for the inequality to be valid. The distance between two locations may be estimated via fuzzy metric spaces using a fuzzy integer or a fuzzy set based on the range $[0, 1]$. Make use of a number that needs to be clarified. This is true even if a third location is precisely located in the middle of the two being considered, according to the findings of the study that Moussaoui carried out and A. 2022 ^[5]. “

1.2 Fixed Point Theorems in Fuzzy Metric Spaces

Principle of Banach Contraction in Fuzzy Metric Spaces”

The concept in mathematics denoted by the ordered pair (X, d) , where X is a non-empty set and d denotes a fuzzy distance function, is sometimes referred to as a fuzzy metric space. In the context of a self-mapping transformation T on a set X , it is stated that $T(x), T(y)$ is considered fuzzy and $T(y), T(x)$ is self-contracted if and only if the inequality $d(T(x), T(y)) \leq d(x, y)$ holds for every x and y in X .

In the context of fuzzy metric spaces, the fulfillment of the contraction conditions outlined in the Banach Contraction Principle guarantees the existence of a unique fixed point inside the set X , given that certain criteria for fuzzy self-contraction mapping are satisfied. The function $T(x^*)$ exhibits complete congruence with x^* when x^* assumes a specified location inside the set X .

It may be argued that a specific point x^* in a fuzzy metric space X is considered constant if and only if the fuzzified distance between the two mapped sets, designated as $T(x)$ and $T(y)$, is less than or equal to instances of the fuzzified distance between x and y , for any value less than one. This condition is satisfied alone when the disparity between $T(x)$ and $T(y)$ is less than or equal to the overall count of instances.

2 Main Results based on Particular Theorems

2.1 Kakutani's Fixed Point Theorem for Fuzzy Metric Spaces: Kakutani, S. (1941) ^[6]

Let us consider the distance, denoted as d , in a metric space of significant size between two arbitrary points, x and y . Here, X represents the space itself, while d represents the distance between the two points. In this context, the symbol X represents the interval $[0, 1]$, denoting a range of values. Additionally, the function $d(x, y)$ represents the distance between two points x and y . The physical distance between two coordinate systems may be determined using this particular tool.

The response to the question, "What is the value of $T(x)$ for the mapping $T: X \rightarrow X$?" is "0.8 times x ."

It is necessary to establish whether or not T represents a contraction mapping with k that is less than 1.

It is the same as stating that $0.8|x - y|$ equals $|0.8x - 0.8y|$ when declaring that $d(T(x), T(y))$ equals $|0.8x - 0.8y|$.

First, to determine the most significant possible distance that X may travel between the two points, set x and y to 0 and 1, respectively.

As a direct result of this, the value of $d(T(x), T(y))$ is equivalent to 0.8 times the distance that separates x and y , divided by 1.

With a k value of 0.8, T satisfies the criteria for contraction since 0.8 is less than 1, the required threshold.

The Banach Contraction Principle states that $T(x^*)$ should be equal when just one fixed point exists. One stationary point is used for this purpose.

Calculate x^* using the formula $x^* = 0.8 * x^* - x^* = 0$ after adjusting $T(x^*)$ to equal $0.8 * x^*$."

Finding the fixed point may be aided by this.

Consequently, the sole fixed point is when $x^* = 0$.

The investigation was done by Bag and Samanta in 2007 ^[7].

2.2 Nadler's Fixed-Point Theorem for Fuzzy Metric Spaces: Nadler, S. B. Jr. (1969) ^[8]

Kakutani's theory on fixed points in fuzzy metric spaces might be seen as a simplification of Nadler's theorem. Kakutani's theorem is predicated on the antecedent assertion as a foundational assumption. The current study's extensive mapping of fuzzy measures into metric spaces uncovers a fixed point via the use of a theoretical framework, leading to this result.

In order to use Nadler's fixed point theorem in the context of the metric space (X, d) , it is necessary for the mapping $F: X \rightarrow X$ to adhere to the aforementioned prerequisites. The only need is to satisfy all of the aforementioned conditions.

The establishment of a fixed point inside the set X for the function F is accomplished by the comparison of the squares of function F with the variables x and y . This condition holds true for all instances of x and y inside the set X . The function $T(x)$ may be defined as the result of multiplying 0.8 by any real number x within the interval $[0, 1]$.

The Banach Contraction Principle demonstrates that only one fixed point is inside the range $[0, 1]$ for the variable T .

First, prove that T maps contractions.

Determine the furthest possible distance that may be put between any two points in the range $[0, 1]$: The expression " $0.8x - 0.8y$ " equals "0.8" when interpreted symbolically: The difference between x and y is one and zero is one when both of them are true.

$$x - y = 0.8 * 1 = 0.8 \quad |T(x) - T(y)| = 0.8 \quad T$$

meets the criteria for a contraction with k equal to 0.8 for any x and y in the range $[0, 1]$. This is because $|T(x) - T(y)|$ equals 0.8. This is because 0.8 is a less significant number than 1.

The Banach Contraction is the next step in the process.

“The Banach Fixed Point Theorem, alternatively referred to as the Banach Contraction Principle, posits that a mapping that satisfies the condition of contraction in a metric space can possess, at most, a solitary fixed point [9].”

The interval may represent the whole metric space $[0, 1]$.

The third stage is going to be finding the fixed point.

We start by setting $T(x^*)$ equal to 0.8 times x^* , and then we solve for x^* as follows:
 $0.8 = x^* x^* - 0.8 = 0 \quad 0.2 = 0 \quad x^* = 0$.

In the fourth step, check whether the x^* issue has been fixed.

$$T(0) = 0.8 * 0 = 0$$

Therefore, the single fixed point that T has in the range $[0, 1]$ is represented by the equation $x^* = 0$.

The use of Nadler's fixed point theorem may enhance the effectiveness of satisfying Kakutani's convexity requirement by imposing restrictions on the form of a contraction. When mapping F is used, the point distances are closer together because of the contraction factor. Kakutani's theorem (Miheţ, D. 2007) demonstrates that even when the restrictions are stretched to their farthest limits, a stable point may be found [10].

2.3 The Fixed Point Theorem of Suzuki for Fuzzy Metric Spaces by Suzuki, T. (2008) [11]

Several methods, such as the Suzuki fixed point theorem, can provide evidence of fixed points in fuzzy metric fields. The Suzuki fixed point theorem, a seminal result in fixed point theory, allows such occurrences to occur. Suzuki is credited with developing this tactic.

In the setting of a fuzzy metric space (X, d) , Suzuki's fixed point theorem states that for a function $F: X$ defined on X , there exists a fixed point F in X . If the fixed point theorem is valid, it may be reasonably inferred that the previously indicated criterion has been met. The applicability of the fixed point theorem is limited to cases when the function F is explicitly defined in relation to the independent variable X . This particular instance is the only scenario in which the aforementioned assertion holds true. The inclusion of a mapping using the letter X may enough to satisfy the requirements outlined in the previous paragraph. In the event that your hypothesis is valid, it may be inferred that X has a stationary point situated at the coordinates denoted as F . The fuzzy distance, represented by the notation $d(F(x), x)$, must be shown to be a lower semicontinuous fuzzy set for all x values. The symbol " F " denotes the fixed point of the variable " X ". The only prerequisite is that it is necessary for the given circumstance. In order to assert the existence of a fixed point F for X , it is imperative that this condition be satisfied. The singular demand is essential for the given circumstance. In order to assert the existence of a fixed point F for X , it is imperative that this condition be satisfied.

$T: [0, 1]$ is the notation for this function, which can also be written as. $T(x)$ is defined as $0.5 * x$ for all x 's in the range $[0, 1]$, and this definition is applied to all x 's.

Suppose there is only one fixed point for the variable T . In that case, it must be in the interval $[0, 1]$, as the Banach Contraction Principle proves. This is what will happen. This will clarify that no more fixed points may be introduced.

Initially, this effort will succeed in providing compelling proof that T is a mapping of contractions.

To subtract $T(x)$ from $T(y)$, it is found that the outcome is the same as expressing $[0.5x - 0.5y]$ as $[0.5 |x - y|]$. For all values of x and y outside the interval $[0, 1]$, it is evident that the statement is true. Because of this condition, the absolute value of the difference between the values of the function T at the locations x and y , denoted by $|T(x) - T(y)|$, is equal to 0.5. In addition, the absolute difference between x and y , represented by $|x - y|$, is 0.5; therefore, multiplying 0.5 by 1 gives us a value of 0.5. This connection is genuine for any x and y outside the interval $[0, 1]$. When both x and y are set to 1, it is shown that the most significant possible separation between any two points in the interval $[0, 1]$ is 1. This assertion is true regardless of how the data points are laid out in space.

The fact that 0.5 is less than one directly affects the value of T 's contraction constant, which is set at 0.5.

The Banach Contraction is the next step in the process.

The Banach Contraction Principle states that only one fixed point in a metric space may be used for mapping throughout the contraction process.

The interval may represent the whole metric space $[0, 1]$.

The third stage is going to be finding the fixed point.

You may calculate x^* by using the following formula once you have set $T(x^*)$ to equal 0.5 times x^* . When you take 0.5 times x^* away from x^* , you end up with x^* minus x^* , which equals 0.

In the fourth step, check whether the x^* issue has been fixed.

$$T(0) = 0.5 * 0 = 0$$

Therefore, the single fixed point that T has in the range $[0, 1]$ is represented by the equation $x^* = 0$.

This method mitigates the implications of Kakutani's and Nadler's theorems. This is accomplished as a direct consequence of the fact that both Kakutani's and Nadler's theorems are demonstrated to be accurate. According to Saha *et al*, (2013), the fuzzy distance possesses a property referred to as lower semicontinuity, making it feasible for a fixed point in existence. In other words, the fuzzy distance makes it conceivable for there to be a fixed point [12].

2.4 Gregori and Sapena's Fixed Point Theorem for Fuzzy Metric Spaces: Gregori, V., & Sapena, A. (2002) ^[13]

The Gregori and Sapena, Fixed Point Theorem guarantees a fixed point in a fuzzy metric space for each continuous self-mapping. Gregori and Sapena are the authors of this concept.

Theorems, fundamental mathematical ideas, will be explained in this talk. The proposal focuses on fuzzy metric space X in the range $[0, 1]$. Think about the factorial $x!$ of X while analyzing X mathematically. For each x , define $T(x)$ as a fuzzy subset of X . We can demonstrate that X maps T continuously and is fuzzy.

For all values of x outside of X , we discovered no difference between the converted values $Tn(x)$ and $Tn(x + 1)$. The outcome of applying T to x an n th time is $Tn(x)$. The time interval T is equal to $d(Tn, Tn + 1)$.

X is a stationary point in a fuzzy set $T(x^*)$.

Consider the fuzzy metric space denoted by (X, d) , wherein X represents the closed interval $[0, 1]$ and d symbolizes the function $d(x, y) = |x - y|$, where x and y are constrained to lie within the interval as mentioned earlier.

Each x possesses a fuzzy self-mapping T comparable to 0.8 times its value, and it is written as follows: $XX! [0, 1]$.

The Fixed Point Theorem developed by Gregori and Sapena states that the value of $d(Tn(x), Tn + 1(x))$ must always equal 0, regardless of the value that $x!$ X may take.

The result of the first iteration is that $T(x) = 0.8 * x$. The following are the outcomes of the second set of calculations for this problem: $T2(x)$ equals $T(T(x))$, which may be written as $0.8 * (0.8 * x) = 0.82 * x$. The following equation is the product of the third iteration of the process: $T3(x)$ equals $T(T2(x))$, which equals 0.83 times 0.82 times x multiplied by 0.8 times 0.82 times x .

$Tn(x) = 0.8n * x$ is the formula for the iteration number n . 0.8 times $(0.8n \text{ times}) x$ equals 0.8 times $(0.8n + 1 \text{ times}) x$ in the iteration that comes after $n + 1$.

Determine the amount of time that elapses between each successive repetition by computing:

$$d(Tn(x), Tn(x + 1)) = 0.8n * x - 0.8n + 1 * x = |x| * 0.2 |x| 1$$

where $0.8n$ is a constant that decreases as n increases.

$0.8n$ approaches zero. This establishes Gregori and Sapena's Fixed Point Theorem since $d(Tn(x), Tn + 1(x))$ approaches zero.

The theorem provides evidence that x^* is included in $T(x^*)$ by demonstrating that it can be shown to include x^* at least once. Then, to get the solution for x^* , apply the following formula: $0.2 * x^*$ equals 0; x^* also equals 0; $0.8 * x^*$ equals x^* ; x^* minus $0.8 * x^*$ equals 0. We can get the fixed point instantly if we set $T(x^*)$ to equal $0.8 * x^*$.

Therefore, the position $x^* = 0$ represents T 's fixed point.

2.5 Cerić's Fixed Point Theorem for Fuzzy Metric Spaces:Ćirić, L. B. (2008) ^[14-15]

Finding fixed points for translations in total fuzzy metric spaces is made easy by the Cerić Fixed Point Theorem. The theorem allows for this to be achievable.

Fuzzy metric space XX is assumed in the theorem. The user entered $[0, 1]$ concisely and numerically. This number range is employed in many disciplines. Rephrase the user's " $XX!$ ": This user shared their ideas or opinions with " $XX!$ ".

$D(x, y)$ must be positive for $d(T(x, y))$ to rise.

$T(x^*)$ is a fuzzy set with a fixed point, X .

This manuscript was entirely digital. Cerić's theorem inspired the concept.

The formula " $x - y$ " resembles the variables X, d , and $d(X, Y)$ in a fuzzy metric space when x and y are restricted to the closed interval $[0, 1]$.

$T: X \rightarrow X$, the fuzzy self-mapping, may be used to find the reciprocal of any element x in X . The user specified $[0, 1]$.

Probability theory, among other fields, uses this range.

To prove that the difference between $T(x, y)$ and 0 is more extensive than zero for every x and y , we need to use Cerić's Fixed Point Theorem. The formula for $d(T(x, y))$ holds when d is in the positive direction: $0.5 * d(x, y) > 0$. Not y .

The function $T(x^*)$ must have at least one fixed point in the interval $[0, 1]$, as this determines if Cerić's Fixed Point Theorem is true.

The fixed point may be obtained by setting $T(x^*)$ to $0.5 * x^*$ and then following the procedures below to solve for x^* . x^* equals 0 when subtracted from $0.5 * x^*$.

The fixed point of T may be found at $x^* = 0$.

"Rhoades abruptly stopped the delight. The present theorem says that fuzzy metric spaces have uniform fixed point arrangements under constructive limits. Fuzzy mapping requires the above component.

The inequality $(X > X > X)$ defines the fuzzy metric space point (X, d) . The idea has gained support from recent empirical findings ^[16]. The input range in this study was $[0, 1]$. Mathematicians know that real numbers can only be represented by values within a finite range. A mapping T is a fuzzy self-mapping if and only if the image of a given element, x , under T , $T(x)$, can be described as a fuzzy subset of the set X , as established by the relevant literature. The common-sense intuition that $T(x)$ represents the output of applying the mapping T to the element x underpins this assertion.

To evaluate the expression, replace integer values into the equation $d(T(x, y)) * \maxed(x, x), d(y, y \text{ for all } x, y \in X)$, where there is a constant in $[0, 1]$.

Rhoades abruptly stopped the delight. The present theorem claims that all constructively constrained fuzzy metric spaces have the same fuzzy self-mapping fixed point distribution. In fuzzy mapping, this element is crucial.

Fuzzy metric space X, d is discussed below. The input interval in this study was $[0, 1]$. Lower and upper boundaries commonly describe separate ranges. Scholarly consensus is that a mapping X is a fuzzy self-mapping if its image, $T(x)$, is a fuzzy subset of X .

The empirical contraction constant was 0.6.

Before continuing the study, the distance between $T(x)$ and $T(y)$ must be determined.

The reformulated equation is more concise. The equation $|0.6 * x - 0.6 * y|$ may be rewritten as $0.6 * d(x, y)$, where d is the distance between the variables. The distance between 0.6 times x and 0.6 times y may be stated simply using this transformation. In the following equation, d is $0.6d$, the degree difference between $T(x)$ and $T(y)$. $WD(x, y)$ is 0.6 times the Euclidean distance $d(x, y)$ for any two points x and y . Academics often use weighted distance to compare two objects. The mathematical function $d(x, y)$ represents the distance or dissimilarity between two entities. This function is crucial to many fields, including mathematics and computers. Consider the $T(x, y)$ function's $d(x, y)$ derivatives. $T(x, y)$ and d produce the equation $0.6(d, x, y)$, which has been mathematically proven. Validity can be verified if x and y fall inside variable X 's range.

Find the individual with the most significant physical distance.

Research the immense feasible value of the distance metrics $d(x, x)$ and $d(y, y)$ for any pairs of values x and y smaller than X . The distance between two points, x and y , is represented by the functions $d(x, x)$ and $d(y, y)$. The most significant distance allowed within limits must be established. From now on, x and y values below X are unimportant. Examine all permutations within the range to find the maximum values of $d(x, x)$ and $d(y, y)$. Since just the shortest distance is evaluated, all distances are positive.

For every x less than X , the maximum term is zero. Fuzzy measurements support this claim: According to metric spaces, the absolute value of the difference between an element x and itself, $d(x, x)$, reduces to zero. The absolute value of the difference between two elements is zero; hence, $d(y, y)$ equals the distance between element y and itself.

In the third stage, sickness may rise.

Our investigation will center on these terms. The formula for $T(x, y)$: The mathematical derivation of the x – and y –dependent function follows. T : $T(x, y)$ is the mathematical sum of two terms. The first term is 0.6 multiplied by $\max(x, x)$, the immense value between x and x . Take the product of d and the highest value between y and y , $\max(y, y)$, then multiply by 0.6 to get the second term. It is $\max(a, b)$ returns the greater of a and b . Returning the bigger of two numbers is its purpose. Both phrases employ the coefficient 0.6, indicating its relevance in the computation. I believe "d" scales the second term relating to the dependent variable "y." More study is needed to understand this factor's features and usefulness.

The contraction constant must be monitored throughout the last period.

In $[0, 1]$, the contraction constant is 0.6.

We will call the theoretical framework "Rhoades' Fixed Point." [17] Only under tight conditions does $T(x^*)$ have at least one fixed point in the closed interval $[0, 1]$.

Many x^* values meet $T(x^*) = 0.6x^*$. The debate suggests several approaches. Set $T(x^*)$ to $0.6xx^*$ to accelerate convergence to the fixed point.

Thus, transformation T 's fixed point is $x^* = 0$.

3. Various Other Important Fixed-Point Theorems in Fuzzy Metric Spaces

3.1 Brouwer Fixed Point Theorem: Brouwer, L. E. J. (1911) [18-19]

Moving point for continuous mappings on a closed and limited subset of Euclidean space, the Brouwer theorem ensures a fixed point.

Theorem The closed and infinite subset D of the Euclidean space R^n has been demonstrated to be closed. Thesis that has been validated: Between D and D , there is no interruption in the continuity. As a result, $f(x^*)$ must always equal x^* in at least one situation.

An illustration in the format of a digital file Points That Stay Put the Broom Moving Point The theorem makes the following assertion:

The range $[0, 1]$ denotes the function $f \cdot f(x)$ equals x^2 for any value of x within the range $[0, 1]$.

The Fixed-Point Theorem of Brouwer demonstrates that the range $[0, 1]$ contains the value of f at its fixed point.

First, f must be continuous on $[0, 1]$.

$x^2 = f(x)$ represents the polynomial $f(x)$ as a continuous function over $[0, 1]$. F is continuous on $[0, 1]$.

Next, verify that the area R bordered by $[0, 1]$ is closed.

Since it includes both terminal locations, $[0, 1]$ is closed. The constraint $0 \leq x \leq 1$ applies to any value of x inside $[0, 1]$.

The Brouwer Fixed Point Theorem is where this approach ends.

A fixed point exists within the closed interval $[0, 1]$, according to the Brouwer Fixed Point Theorem, a key conclusion in topology. The continuity of the function under examination throughout this interval is required for this theorem to hold. As a result, it may be inferred that a value x^* exists inside the interval, as mentioned earlier, and is maintained under the function's mapping.

Find the stationary item.

We find the fixed point where $x^2 = x^*$ is the first step in solving the equation $f(x^*) = x^*$.

Both $x^2 - x^* = 0$ and $x^*(x^* - 1) = 0$ hold when x^* is solved for.

It may be fixed by setting x^* equal to 0 or 1.

Verify whether x^* has been modified.

$$f(0) = 0^2 = 0 \quad f(1) = 1^2 = 1$$

The values $x^* = 0$ and $x^* = 1$ are regarded to be fixed points of the mapping function since they are included inside that interval.

4. Conclusion

Theorems regarding fixed points make locating fixed points for mappings in various contexts possible. This makes finding fixed points a desirable goal. The findings of this research may call for the development of innovative hypotheses. The conditions that led to the creation of the fixed-point existence theorems by Kakutani, Nadler, and Suzuki are discussed in this article, as are the ramifications of those theorems. In addition to utilizing fuzzy logic, implementing fixed-point solutions is required for optimization and financial modeling activities. Future research on fuzzy metric space fixed-point solutions may focus on developing new theorems that relax constraints and broaden applicability, integrating machine learning, AI, and data mining for managing ambiguous data, and adopting advanced convergence techniques to enhance computational efficiency. Hybrid approaches combining fuzzy metric spaces with interval analysis, rough sets, and possibility theory could further strengthen models of uncertainty, with real-world applications supporting the significance highlighted by Huang (2007) ^[20].

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