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Solving triangular intuitionistic fuzzy transportation problem: A new approach

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Abstract

As traditional transportation models often rely on crisp, deterministic data, which is frequently unavailable or unrealistic, fuzzy numbers are used instead to model several critical types of uncertainty. Fuzzy numbers are not just widely used but are essential tools for making transportation and logistics models applicable to real-world scenarios. By explicitly quantifying uncertainty in supply, demand, cost, and time, they enable planners and managers to develop more robust, flexible, and realistic strategies. In this study, the classical fully fuzzy transportation problem, which is a comprehensive fuzzy extension of the transportation model, has been extended to a fully intuitionistic fuzzy transportation problem to better handle imprecise and hesitant information in real-world logistics scenarios. The model is formulated by representing transportation costs, supply, and demand parameters as Triangular Intuitionistic Fuzzy Numbers (TIFNs), which offer a more comprehensive framework by incorporating both membership and non-membership degrees. Furthermore, a new method has been developed to derive the initial basic feasible solution and to obtain an optimal solution. The proposed method enhances the process of decision-making under uncertainty by capturing both ambiguity and hesitation inherent in transportation data. A detailed numerical example is introduced to illustrate the applicability and effectiveness of the proposed method.

Keywords: TIFN, IFTP, IBFS, optimal solution, transportation problem, membership function, non-membership function

1. Introduction

The set theory established by Cantor, founded on the principle of element membership, and has become one of the fundamental frameworks of modern mathematics. It has provided a robust foundation for modelling processes and for advancing the study of various scientific disciplines. Nevertheless, the classical interpretation of membership in a set is strictly binary: an element either belongs to the set (assigned the value 1) or does not (assigned the value 0). This rigid dichotomy excludes intermediate possibilities of membership, which have since been explored within the broader domain of logical models. In recent decades, the transportation problem has more often than not garnered significant attention in the fields of operations research and logistics due to its wide applicability in optimizing supply chain operations. Traditional transportation models, which assume precise and deterministic values for supply, demand, and transportation costs, often fail to reflect the inherent uncertainty and vagueness that are present in real-world environment scenarios. To address these limitations and challenges, fuzzy set theory originally introduced by Zadeh ^[1] has been extensively applied to transportation problems, enabling decision-makers to handle imprecise information more effectively.

However, while fuzzy transportation models provide a framework for capturing uncertainty through degrees of membership, they fall short when it comes to handling hesitation or dual uncertainty. In traditional fuzzy sets, an element is described only by the degree of its inclusion: that is, the extent to which it belongs to a set, represented by a value between 0 and 1. However, this approach overlooks the parallel notion of exclusion, which reflects the extent to which the same element does not belong to the set.

In many real-world decision-making scenarios, especially in transportation problems, decision-makers face situations where both inclusion and exclusion are uncertain. For example, the cost associated with transporting goods may not only be vaguely defined (uncertain inclusion) but may also involve uncertainty about rejecting or ruling out certain cost values (uncertain exclusion). This duality creates a form of hesitation, where neither inclusion nor exclusion can be expressed precisely. To overcome this shortcoming, Atanassov [2] proposed the concept of Intuitionistic Fuzzy Sets (IFSs), which extend fuzzy sets by incorporating both membership and non-membership functions. This dual representation offers a richer and more flexible and realistic structure for modeling uncertainty, particularly in complex decision-making scenarios such as in transportation models. Building on these developments, this study introduces a novel approach to solving the Triangular Intuitionistic Fuzzy Transportation Problem (TIFTP). The proposed model enhances the classical fully fuzzy transportation framework by incorporating *Triangular Intuitionistic Fuzzy Numbers* (TIFNs) for all relevant parameters namely, transportation costs, supply, and demand. TIFNs not only account for ambiguity in data but also represent the degree of hesitation, offering a more comprehensive means of dealing with real-world logistics uncertainties. To effectively solve the TIFTP, the proposed intuitionistic fuzzy method is designed to derive the initial basic feasible solution. Additionally, an optimization strategy specifically tailored to intuitionistic fuzzy environments is developed to determine the optimal transportation plan.

The importance of this study lies in its contribution to the growing body of work in the field of fuzzy optimization by addressing scenarios of uncertainty and hesitation in transportation models. It provides a robust decision-support method for logistics managers and researchers aiming to improve reliability and adaptability in uncertain and vague environments.

2. Literature Review

The classical Transportation Problem (TP) has evolved significantly to accommodate the increasing complexity and uncertainty inherent in real-world logistical systems. In this context, the fuzzy set theory and its extensions, particularly Intuitionistic Fuzzy Sets (IFSs), have emerged as robust tools to handle imprecise and hesitant data. This literature review presents a detailed overview of contributions in the area of intuitionistic fuzzy transportation problems (IFTPs), focusing on key methodologies, solution techniques, and emerging trends.

The brilliant concept of Intuitionistic Fuzzy Sets (IFS) was introduced by Atanassov [2], which extended traditional fuzzy sets by incorporating he called a degree of non-membership along with the membership function. This formulation ideally captures not only ambiguity but also hesitation, offering a more nuanced representation of uncertainty. This foundational work has underpinned numerous applications in optimization and decision-making problems, including the transportation problem. Antony, Savarimuthu, and Pathinathan [4] extended this theory to formulate a transportation model based on Triangular Intuitionistic Fuzzy Numbers (TIFNs). Their method provided a systematic approach to identify/determine the initial basic feasible solution (IBFS) and showcased the practical applicability of IFS in solving logistics problems. Ranking intuitionistic fuzzy numbers are crucial for comparing alternatives and solving optimization problems.

Nagoorgani and Abbas [5] developed a direct algorithm for solving such Intuitionistic Fuzzy Transportation Problems (IFTPs) where supplies and demands are triangular intuitionistic fuzzy numbers. Their central contribution is a novel ranking function based on the Graded Mean Integration Representation, which converts the fuzzy parameters into crisp values, enabling the use of a deterministic allocation algorithm. Hussain and Kumar [6] developed an intuitionistic fuzzy zero point method which operates directly on triangular intuitionistic fuzzy supplies and demands. A distinguishing feature of their work is that the allocations and the final total cost are obtained as intuitionistic fuzzy numbers themselves, through the application of fuzzy arithmetic operations during the allocation process. This contrasts with ranking-based methods that defuzzify the problem at the outset, as it provides a solution that fully captures the propagated uncertainty. Singh and Yadav [7, 8] advanced fuzzy transportation research by applying Triangular Intuitionistic Fuzzy Numbers (TIFNs) to capture both uncertainty and hesitation. For Type-1 IFTP, they modeled fuzzy supply and demand with crisp costs, introduced an accuracy-based ranking method, and developed intuitionistic fuzzy versions of NWCM, LCM, VAM, and MODIM. Later, in Type-2 IFTP, they extended the model by treating costs as TIFNs while supplies and demands remained crisp, better reflecting real-world fluctuations. Together, these works show how TIFNs and improved ranking functions enhance the accuracy and realism of transportation problem solutions under dual uncertainty. Kumar and Hussain [9] addressed real-life transportation problems characterized by uncertainty and hesitation in supply, demand, and costs. To capture this dual uncertainty, they employed Triangular Intuitionistic Fuzzy Numbers (TrIFNs). The authors proposed a new PSK method that directly provides an intuitionistic fuzzy optimal solution in a single stage, unlike earlier multi-stage approaches. They also introduced a new multiplication operation on TrIFNs, enabling the calculation of the optimal objective value in fuzzy terms. Pathade *et al.* [10] proposed the Intuitionistic Fuzzy Transportation Problem Method (IFTPM) using Triangular Intuitionistic Fuzzy Numbers (TrIFNs) to model uncertainty in supply, demand, and costs. The method is computationally simple, reduces solution time, and effectively incorporates both membership and non-membership functions to capture hesitation. Aggarwal and Gupta [11] generalized the modeling capability of IFTPs by introducing the use of Generalized Trapezoidal Intuitionistic Fuzzy Numbers (GTRIFNs), which offer greater flexibility in capturing asymmetric uncertainty and hesitation. Their primary contribution is a novel centroid-based ranking method specifically designed to order these complex GTRIFNs, addressing a gap in existing literature. Ebrahimnejad and Verdegay [12] introduced a novel method for solving fully intuitionistic fuzzy transportation problems (IFTPs), where all parameters transportation costs, supplies, and demands are represented by trapezoidal intuitionistic fuzzy numbers (TrIFNs). Unlike earlier approaches that rely heavily on fuzzy arithmetic and repeated comparisons of TIFNs using score and accuracy functions, their method reduces complexity by performing arithmetic on real numbers and limiting fuzzy comparisons to a few simple steps. Bharati & Singh [13] extended the transportation problem (TP) into an interval-valued intuitionistic fuzzy (IVIF) environment to handle more complex and realistic uncertainty where membership and non-membership degrees are themselves intervals rather than fixed values. The authors argued that conventional fuzzy or

intuitionistic fuzzy sets are insufficient when experts cannot assign exact acceptance and rejection degrees. Giri *et al.* [14] introduced a generalized non-linear intuitionistic fuzzy number (GNIFN), which extends traditional linear intuitionistic fuzzy numbers by allowing non-linear membership and non-membership functions. This is motivated by real-world scenarios where linear functions fail to capture rapid or biased changes in uncertain parameters, such as fluctuating costs or demand. Nasir and Beenu [15] introduced the concept of a Generalized Odd Intuitionistic Fuzzy Number (GOIFN), which extends traditional intuitionistic fuzzy numbers by allowing an odd number of reference points in its membership and non-membership functions. This structure provides greater flexibility in modeling complex uncertainty patterns. Nathiya *et al.* [16] addressed the limitations of traditional transportation problems, which assume precise cost, supply, and demand values, by introducing a fuzzy optimization approach using Decagonal Intuitionistic Fuzzy Numbers (DIFNs) to handle real-world uncertainty. Roy, Ebrahimnejad & Verdegay [17] addressed a multi-objective transportation problem (MOTP) under an intuitionistic fuzzy (IF) environment, where transportation costs, supply, and demand are all represented as triangular intuitionistic fuzzy numbers (IFNs) to capture real-world uncertainty more effectively than traditional fuzzy sets. In [18] Ghosh *et al.* dealt with multi-objective fixed-charge solid transportation problem (MOFCSTP) in a fully intuitionistic fuzzy (IF) environment, where all parameters including transportation costs, fixed charges, supply, demand, conveyance capacity, and decision variables are represented as triangular intuitionistic fuzzy numbers (TIFNs). The model considers three objectives: minimizing total transportation cost with fixed charge, deterioration rate, and transportation time. Singh and Yadav [19] presented a solution approach for intuitionistic fuzzy multi-objective linear programming problems (IFMOLPP) where all parameters objective function coefficients, constraint coefficients, and right-hand-side values are represented as triangular intuitionistic fuzzy numbers (TIFNs). Recently, Joshi *et al.* [20] introduced a fully triangular intuitionistic fuzzy number (FTTrIFN) model to address uncertainty and hesitation in multi-objective transportation problems (MOTP). The model simultaneously optimizes conflicting objectives such as cost, time, and risk under uncertain supply, demand, and transportation parameters.

3. Definitions

Definition 3.1

Let X be a universal set. Then a fuzzy set $\tilde{A}$ in X is mathematically defined by

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\} \text{ Zadeh [1]}$$

Where,

$\mu_{\tilde{A}}(x) : X \rightarrow [0, 1]$ is called the membership function”.

Definition 3.2

Let X be a universe of discourse. An IFS $\tilde{A}^I$ on X is mathematically defined by a set of ordered triples of the form:

$$\tilde{A}^I = \{ \langle x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x) \rangle : x \in X \}$$

Where $\mu_{\tilde{A}^I}, \nu_{\tilde{A}^I} : X \rightarrow [0, 1]$ are said to be functions such that $0 \leq \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \leq 1, x \in X$. So the value $\mu_{\tilde{A}^I}(x)$ signifies the

degree of the membership and $\nu_{\tilde{A}^I}(x)$ signifies the degree of the non-membership of the element $x \in X$ being in $\tilde{A}^I$.” Atanassov [2].

Definition 3.3

An Intuitionistic Fuzzy (IF) subset $\tilde{A}^I = \{ \langle x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x) \rangle : x \in X \}$ of the real line R is called an intuitionistic fuzzy number (IFN) if the following hold:

- There exists $m \in R$ such that $\mu_{\tilde{A}^I}(m) = 1$ and $\nu_{\tilde{A}^I}(m) = 0$ (m is known to be the mean value of $\tilde{A}^I$)
- $\mu_{\tilde{A}^I}$ and $\nu_{\tilde{A}^I}$ are piecewise continuous mappings/functions from R into the closed interval $[0, 1]$ and

$$0 \leq \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \leq 1, x \in R, \text{” Atanassov [3]}$$

Where,

$$\mu_{\tilde{A}^I}(x) = \begin{cases} g_1(x), m - a \leq x < m, \\ 1, x = m, \\ h_1(x), m < x \leq m + b, \\ 0, \text{otherwise.} \end{cases}$$

And

$$\nu_{\tilde{A}^I}(x) = \begin{cases} g_2(x), m - \hat{a} \leq x \leq m; 0 \leq g_1(x) + g_2(x) \leq 1, \\ 0, x = m \\ h_2(x), m \leq x \leq m + \hat{b}; 0 \leq h_1(x) + h_2(x) \leq 1, \\ 1, \text{otherwise.} \end{cases}$$

In the above equations, m is said to be the mean value of $\tilde{A}^I$; while a and b are considered to be the left and right spreads of membership function $\mu_{\tilde{A}^I}$ respectively; and a' and b' are the left and right spreads of non-membership function $\nu_{\tilde{A}^I}$ respectively. When it comes to g_1 and h_1 , they are defined as piecewise continuous functions, with g_1 being strictly increasing on the interval $[m - a, m]$ and h_1 being strictly decreasing on $[m, m + b]$. Similarly, the functions g_2 and h_2 are also piecewise continuous, where g_2 is strictly decreasing on $[m - \hat{a}, m]$ and h_2 is strictly increasing on $[m, m + \hat{b}]$. The IFN $\tilde{A}^I$ is thus represented by

$$\tilde{A}^I = (m; a, b; \hat{a}, \hat{b}).$$

Definition 3.4

Mahapatra & Roy [21] “A TIFN $\tilde{A}^I$ is an IFN with a membership function $\mu_{\tilde{A}^I}$ and a non-membership function $\nu_{\tilde{A}^I}$ given by:

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, \text{ if } a_1 \leq x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2} \text{ if } a_2 \leq x \leq a_3 \\ 0, \text{otherwise.} \end{cases}$$

And

$$\nu_{\tilde{A}^I}(x) = \begin{cases} \frac{a_2 - x}{a_2 - \hat{a}_1}, \text{ if } \hat{a}_1 \leq x \leq a_2 \\ \frac{x - a_2}{\hat{a}_3 - a_2}, \text{ if } a_2 \leq x \leq \hat{a}_3 \\ 1, \text{otherwise.} \end{cases}$$

Where $\hat{a}_1 \leq a_1 < a_2 < a_3 \leq \hat{a}_3$. This TIFN is denoted by $\tilde{A}^I = (a_1, a_2, a_3; \hat{a}_1, a_2, \hat{a}_3)$.”

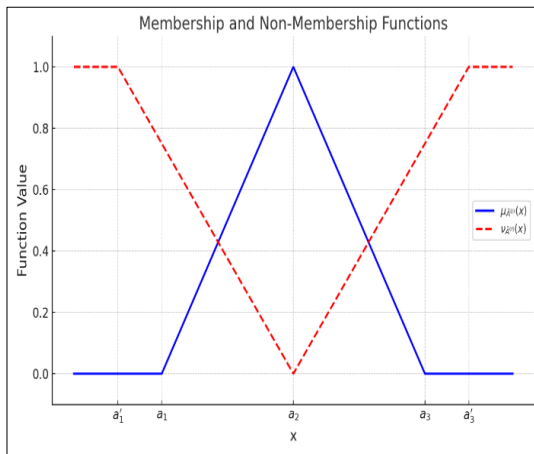


Fig 1: Membership and Non-Membership Functions

Arithmetic Operations on TIFNs

Let $\tilde{A}^I = (a_1, a_2, a_3; \hat{a}_1, a_2, \hat{a}_3)$ and $\tilde{B}^I = (b_1, b_2, b_3; \hat{b}_1, b_2, \hat{b}_3)$.

Then

$$\tilde{A}^I \oplus \tilde{B}^I = (a_1 + b_1, a_2 + b_2, a_3 + b_3; \hat{a}_1 + \hat{b}_1, a_2 + b_2, \hat{a}_3 + \hat{b}_3)$$

$$\tilde{A}^I \ominus \tilde{B}^I = (a_1 - b_3, a_2 - b_2, a_3 - b_1; \hat{a}_1 - \hat{b}_3, a_2 - b_2, \hat{a}_3 - \hat{b}_1)$$

$$\tilde{A}^I \otimes \tilde{B}^I = (l_1, l_2, l_3; \hat{l}_1, l_2, \hat{l}_3)$$

Where

$$l_1 = \min \{a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3\}, \quad l_2 = a_2 b_2, \quad l_3 = \max \{a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3\}$$

$$\hat{l}_1 = \min \{\hat{a}_1 \hat{b}_1, \hat{a}_1 \hat{b}_3, \hat{a}_3 \hat{b}_1, \hat{a}_3 \hat{b}_3\}, \quad \hat{l}_3 = \max \{\hat{a}_1 \hat{b}_1, \hat{a}_1 \hat{b}_3, \hat{a}_3 \hat{b}_1, \hat{a}_3 \hat{b}_3\}$$

Scalar multiplication

$$k\tilde{A}^I = (ka_1, ka_2, ka_3; k\hat{a}_1, ka_2, k\hat{a}_3); k > 0$$

$$k\tilde{A}^I = (ka_3, ka_2, ka_1; k\hat{a}_3, ka_2, k\hat{a}_1); k < 0.$$

Definition 3.5

Ye [22] "Let $\tilde{A}^I = (a_1, a_2, a_3; \hat{a}_1, a_2, \hat{a}_3)$ be the TIFN. The score function corresponding to its membership function $\mu_{\tilde{A}^I}$ is denoted as $S(\mu_{\tilde{A}^I})$ and mathematically is defined by

$$S(\mu_{\tilde{A}^I}) = \frac{a_1 + 2a_2 + a_3}{4}.$$

The score function for the non-membership function $v_{\tilde{A}^I}$ is denoted as $S(v_{\tilde{A}^I})$ and is mathematically defined by

$$S(v_{\tilde{A}^I}) = \frac{\hat{a}_1 + 2a_2 + \hat{a}_3}{4}.$$

The accuracy function of $\tilde{A}^I$ is denoted as $f(\tilde{A}^I)$ and mathematically defined by

$$f(\tilde{A}^I) = \frac{S(\mu_{\tilde{A}^I}) + S(v_{\tilde{A}^I})}{2} = \frac{(a_1 + 2a_2 + a_3) + (\hat{a}_1 + 2a_2 + \hat{a}_3)}{8}.$$

Definition 3.6

A TIFN $\tilde{A}^I = (a_1, a_2, a_3; \hat{a}_1, a_2, \hat{a}_3)$ is said to be zero TIFN if $f(\tilde{A}^I) = 0$.

Definition 3.7

A TIFN $\tilde{A}^I = (a_1, a_2, a_3; \hat{a}_1, a_2, \hat{a}_3)$ is said to be non-negative TIFN if $f(\tilde{A}^I) \geq 0$.

Definition 3.8

A TIFN $\tilde{A}^I = (a_1, a_2, a_3; \hat{a}_1, a_2, \hat{a}_3)$ is said to be positive TIFN if $f(\tilde{A}^I) > 0$.

Definition 3.9

Let $\tilde{A}^I = (a_1, a_2, a_3; \hat{a}_1, a_2, \hat{a}_3)$ and $\tilde{B}^I = (b_1, b_2, b_3; \hat{b}_1, b_2, \hat{b}_3)$ be two TIFNs.

Then,

- $\tilde{A}^I \geq \tilde{B}^I$ if $f(\tilde{A}^I) \geq f(\tilde{B}^I)$
- $\tilde{A}^I \leq \tilde{B}^I$ if $f(\tilde{A}^I) \leq f(\tilde{B}^I)$
- $\tilde{A}^I = \tilde{B}^I$ if $f(\tilde{A}^I) = f(\tilde{B}^I)$
- $\min(\tilde{A}^I, \tilde{B}^I) = \tilde{A}^I$ if $\tilde{A}^I \leq \tilde{B}^I$ or $\tilde{B}^I \geq \tilde{A}^I$
- $\max(\tilde{A}^I, \tilde{B}^I) = \tilde{A}^I$ if $\tilde{A}^I \geq \tilde{B}^I$ or $\tilde{B}^I \leq \tilde{A}^I$.

4. Proposed Method

To address the Triangular Intuitionistic Fuzzy Transportation Problem (TIFTP), we propose a novel algorithm that efficiently deals with scenarios of uncertainty and hesitation. The method extends the classical penalty approach into the intuitionistic fuzzy framework by explicitly incorporating triangular intuitionistic fuzzy numbers (TIFNs) in cost, supply, and demand parameters. Unlike existing ranking-based approaches, which defuzzify fuzzy numbers prematurely, the proposed method preserves both membership and non-membership functions throughout the solution process, thereby retaining hesitation and dual uncertainty. Let the supplies, demands, and costs be represented as triangular intuitionistic fuzzy numbers (TIFNs) $\tilde{a}_i^I, \tilde{b}_j^I$ and $\tilde{c}_{ij}^I$ respectively. The steps of the proposed method are as follows:

Step 1. Penalty Computation

For each row i , determine the difference between the largest and smallest cost entries by using definition 9, then multiply this difference by the corresponding fuzzy supply $\tilde{a}_i^I$. This value is denoted as the row penalty.

$$P_i = \tilde{a}_i^I \otimes (\max_j \tilde{c}_{ij}^I \ominus \min_j \tilde{c}_{ij}^I), \quad i=1, 2, 3, \dots, m.$$

Similarly, for each column j , compute the column penalty as:

$$Q_j = \tilde{b}_j^I \otimes (\max_i \tilde{c}_{ij}^I \ominus \min_i \tilde{c}_{ij}^I), \quad j=1, 2, 3, \dots, n.$$

Step 2: Cell Selection and Allocation

Identify the row or column with the highest penalty using definition 9. In the corresponding row (or column), choose the cell with the minimum cost $\tilde{c}_{ij}^I$.

The fuzzy allocation for that cell is given by:

$$\tilde{X}_{ij}^I = \text{Min}(\tilde{a}_i^I, \tilde{b}_j^I)$$

Now, one of the following cases arises:-

Case 1: If $\text{Min}(\tilde{a}_i^I, \tilde{b}_j^I) = \tilde{a}_i^I$, then allocate $\tilde{X}_{ij}^I = \tilde{a}_i^I$ in the $(i, j)^{\text{th}}$ cell of $m \times n$

FIFBTPT. Eliminate the i^{th} row to form a reduced table of order $(m-1) \times n$. Then update the demand $\tilde{b}_j^I$ by $\tilde{b}_j^I \ominus \tilde{a}_i^I$ and move on to the next step.

Case 2: If $\text{Min}(\tilde{a}_{ij}^l, \tilde{b}_j^l) = \tilde{b}_j^l$, then allocate $\tilde{X}_{ij}^l = \tilde{b}_j^l$ in the $(i, j)^{\text{th}}$ cell of $m \times n$. Eliminate the j^{th} column to form a reduced table of order $m \times (n-1)$. Then update $\tilde{a}_i^l$ by $\tilde{a}_i^l \ominus \tilde{b}_j^l$ and move on to the next step.

Case 3: If $\tilde{a}_i^l = \tilde{b}_j^l$, then arbitrarily eliminate either i row or j column, but not both simultaneously.

Step 3: Iteration

Recompute the penalties for the reduced table obtained in Step 2. Repeat the allocation process until all supplies and demands are exhausted. If a row and a column are

simultaneously satisfied, suppress only one and assign the other a zero supply (or demand).

Step 4: Solution and Cost Evaluation

The Initial Basic Feasible Solution (IBFS) is given by the allocation matrix $\tilde{X}_{ij}^l$. The corresponding Intuitionistic Fuzzy Transportation Cost (IFTC) is:

$$\text{IFTC} = \sum \sum \tilde{c}_{ij}^l \otimes \tilde{X}_{ij}^l \text{ for } i = 1, 2, 3, \dots, m \text{ and } j = 1, 2, 3, \dots, n.$$

5. Numerical Example: Consider the FIFBTP given in table 1. In this table costs, availabilities and demands all are TIFNs.

Table 1: FIFBTP

	D_1	D_2	D_3	$\tilde{a}_i^l$
S_1	(2, 3, 4; 1, 3, 5)	(1, 4, 5; -1, 4, 5)	(3, 4, 6; 2, 4, 7)	(2, 5, 7; 1, 5, 7)
S_2	(3, 6, 8; 2, 6, 8)	(4, 7, 8; 3, 7, 9)	(1, 2, 3; 1, 2, 4)	(5, 7, 8; 4, 7, 9)
S_3	(3, 5, 6; 2, 5, 7)	(5, 6, 7; 4, 6, 8)	(0, 1, 3; -1, 1, 4)	(2, 5, 8; 1, 5, 9)
$\tilde{b}_j^l$	(4, 6, 9; 3, 6, 9)	(3, 7, 9; 2, 7, 10)	(2, 4, 5; 1, 4, 6)	(9, 17, 23; 6, 17, 25)

Now to find the starting BFS we can apply proposed method discussed in above. The obtained BFS is given in Table 2.

Table 2: Solution by using proposed method

	D_1	D_2	D_3	
S_1	(2, 3, 4; 1, 3, 5)	(1, 4, 5; -1, 4, 5) (2, 5, 7; 1, 5, 7)	(3, 4, 6; 2, 4, 7)	[-7, 5, 28; -21, 5, 42]
S_2	(3, 6, 8; 2, 6, 8) (-4, 1, 7; -6, 1, 8)	(4, 7, 8; 3, 7, 9) (-4, 2, 7; -5, 2, 9)	(1, 2, 3; 1, 2, 4) (2, 4, 5; 1, 4, 6)	[5, 35, 56; -9, 35, 72]
S_3	(3, 5, 6; 2, 5, 7) (2, 5, 8; 1, 5, 9)	(5, 6, 7; 4, 6, 8)	(0, 1, 3; -1, 1, 4)	[4, 25, 56; 0, 25, 81]
	[-9, 18, 54; -27, 18, 63]	[-9, 21, 63; -20, 21, 100]	[0, 12, 30; -12, 12, 48]	

OR

	D_1	D_2	D_3	
S_1	(2, 3, 4; 1, 3, 5)	(1, 4, 5; -1, 4, 5) (2, 5, 7; 1, 5, 7)	(3, 4, 6; 2, 4, 7)	[-7, 5, 28; -21, 5, 42]
S_2	(3, 6, 8; 2, 6, 8) (-4, 1, 7; -6, 1, 8)	(4, 7, 8; 3, 7, 9) (-7, 2, 10; -10, 2, 14)	(1, 2, 3; 1, 2, 4) (2, 4, 5; 1, 4, 6)	[5, 35, 56; -9, 35, 72]
S_3	(3, 5, 6; 2, 5, 7) (2, 5, 8; 1, 5, 9)	(5, 6, 7; 4, 6, 8)	(0, 1, 3; -1, 1, 4)	[4, 25, 56; 0, 25, 81]
	[-9, 18, 54; -27, 18, 63]	[-9, 21, 63; -20, 21, 100]	[0, 12, 30; -12, 12, 48]	

The optimal solution is

$$\tilde{X}_{12}^l = (2, 5, 7; 1, 5, 7), \tilde{X}_{21}^l = (-4, 1, 7; -6, 1, 8), \tilde{X}_{22}^l = (-4, 2, 7; -5, 2, 9), \text{ OR } \tilde{X}_{22}^l = (-7, 2, 10; -10, 2, 14), \tilde{X}_{23}^l = (2, 4, 5; 1, 4, 6), \tilde{X}_{31}^l = (2, 5, 8; 1, 5, 9).$$

The IF transportation cost $Z^l =$

$$(1, 4, 5; -1, 4, 5) \otimes (2, 5, 7; 1, 5, 7) \oplus (3, 6, 8; 2, 6, 8) \otimes (-4, 1, 7; -6, 1, 8) \oplus (4, 7, 8; 3, 7, 9) \otimes (-4, 2, 7; -5, 2, 9) \oplus (1, 2, 3; 1, 2, 4) \otimes (2, 4, 5; 1, 4, 6) \oplus (3, 5, 6; 2, 5, 7) \otimes (2, 5, 8; 1, 5, 9)$$

$$= (2, 20, 35; -7, 20, 35) \oplus (-32, 6, 56; -48, 6, 64) \oplus (-32, 14, 56; -45, 14, 81) \oplus (2, 8, 15; 1, 8, 24) \oplus (6, 25, 48; 2, 25, 63)$$

$$= (-54, 73, 210; -97, 73, 267).$$

6. Results and Discussion

The proposed method was applied to the Triangular Intuitionistic Fuzzy Transportation Problem (TIFTP). In this formulation, all parameters transportation costs, supply, and demand were represented as Triangular Intuitionistic Fuzzy Numbers (TIFNs), enabling the model to capture both ambiguity and hesitation in decision-making. Following the

stepwise procedure, an Initial Basic Feasible Solution (IBFS) was obtained, and subsequently the Intuitionistic Fuzzy Transportation Cost (IFTC) was derived. The IFTC is expressed through piecewise-defined membership and non-membership functions, which describe the degree of belief and disbelief in the transportation cost values. Specifically, the functions are:

$$\mu_{\tilde{z}^I}(c) = \begin{cases} \frac{c+54}{127}, & -54 < c \leq 73 \\ \frac{210-c}{137}, & 73 \leq c < 210 \text{ and} \\ 0, & \text{otherwise} \end{cases}$$

$$v_{\tilde{z}^I}(c) = \begin{cases} \frac{73-c}{170}, & -97 < c \leq 73 \\ \frac{c-73}{194}, & 73 \leq c < 267 \\ 1, & \text{otherwise} \end{cases}$$

These equations represent the degree of membership $\mu_{\tilde{z}^I}(c)$ and non-membership $v_{\tilde{z}^I}(c)$ of an element within an intuitionistic fuzzy set, offering dual perspectives on the data's certainty as shown in Fig. no. 2. Each function is

defined over specific intervals with linear segments and is zero or one outside those intervals, reflecting intuitionistic fuzzy number behaviour.

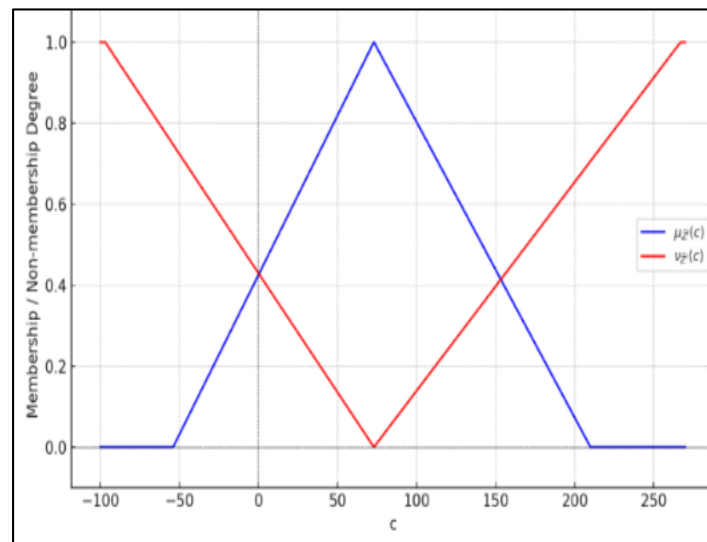


Fig 2: IF transportation cost

As shown in Figure 2, the membership function (blue curve) represents the extent of belief in the transportation cost values within the triangular range, while the non-membership function (red curve) represents the degree of disbelief. The intersection between these curves captures the hesitation component, which is absent in classical fuzzy models. This dual characterization provides a richer description of uncertainty, validating the use of intuitionistic fuzzy sets as a more realistic and comprehensive framework for solving transportation problems under imprecise and hesitant conditions.

7. Conclusions

The study presents a novel approach to solving the Triangular Intuitionistic Fuzzy Transportation Problem (TIFTP) by integrating triangular intuitionistic fuzzy numbers (TIFNs) to model transportation costs, supply, and demand. This methodology effectively captures both ambiguity and hesitation inherent in real-world logistics scenarios, offering a more comprehensive framework than traditional fuzzy models. The proposed method, which includes deriving an initial basic feasible solution and optimizing the transportation plan, demonstrates its practicality and efficiency through a detailed numerical example. The results highlight the flexibility and robustness of the approach in handling higher-order uncertainty, as illustrated by the membership and non-membership functions of the intuitionistic fuzzy transportation cost. This article significantly advances fuzzy optimization techniques by addressing dual uncertainty in transportation problems,

providing valuable insights for logistics managers and researchers. Future work could explore extensions to multi-objective scenarios or hybrid models, further enhancing the applicability of intuitionistic fuzzy sets in complex decision-making environments.

8. Conflict of Interest Statement

On behalf of all authors, the corresponding author states that there is no conflict of interest regarding the publication of this paper.

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