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Expectile regression with adaptive elastic net penalty

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Abstract

Expectile regression is a flexible statistical modeling approach widely used in econometrics, medicine, and high-dimensional data analysis. However, a significant challenge in constructing expectile regression models is the accurate selection of active covariates, especially in the presence of multicollinearity. While traditional penalized regression techniques, such as SCAD and Elastic Net, provide solutions, they often face limitations in variable selection stability and predictive accuracy.

In this study, we propose Expectile Regression with Adaptive Elastic Net penalty (ER-AdEN), a novel method that integrates Lasso and Ridge penalties with adaptive weighting to enhance variable selection and address multicollinearity. The adaptive weights are designed to improve the stability of variable selection, particularly in high-dimensional settings. The proposed method is evaluated through extensive Monte Carlo simulations and a real-world application involving diabetes prediction. Performance is compared against ER-SCAD and ER-Elastic Net using key evaluation metrics, including Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and Expected Prediction Error (EPE).

Simulation results demonstrate that ER-AdEN consistently outperforms competing methods in terms of predictive accuracy, robustness, and sparsity control. For instance, in scenarios with Gaussian errors, ER-AdEN achieved an RMSE of 0.210 compared to 0.235 for ER-SCAD and 0.245 for ER-Elastic Net. In the real data analysis, ER-AdEN achieves superior variable selection efficiency and lower prediction error when applied to diabetes data, reducing the prediction error by approximately 15% compared to traditional methods. These results highlight the potential of ER-AdEN as a reliable tool for high-dimensional regression problems, particularly in medical data analysis.

Keywords: Expectile regression, adaptive elastic net, penalized regression, variable selection, high-dimensional data, diabetes prediction

1. Introduction

Expectile regression has emerged as a powerful statistical modeling approach with applications in econometrics, medicine, and high-dimensional data analysis. Unlike traditional mean regression, expectile regression provides a comprehensive understanding of the conditional distribution of a response variable, making it particularly valuable in scenarios where asymmetric relationships exist ^[1]. Due to its flexibility, expectile regression has been widely applied in risk analysis, survival models, and predictive analytics.

Despite its advantages, a key challenge in expectile regression is the accurate selection of relevant covariates, particularly in high-dimensional settings and in the presence of multicollinearity. Conventional penalized regression techniques, such as SCAD and Elastic Net, have been introduced to address this issue ^[2, 3]. However, these methods may suffer from instability in variable selection, leading to suboptimal predictive accuracy and model interpretability. For instance, SCAD tends to yield inconsistent selection results in high-dimensional datasets, whereas Elastic Net balances variable selection and regularization but may retain unnecessary covariates, reducing model sparsity.

To overcome these limitations, we propose Expectile Regression with Adaptive Elastic Net (ER-AdEN), a novel penalization technique that integrates the strengths of Lasso and Ridge penalties while incorporating adaptive weighting to enhance variable selection and model stability. The adaptive weights dynamically adjust penalty contributions based on initial coefficient estimates, allowing ER-AdEN to achieve superior sparsity control and robustness against multicollinearity ^[4].

This approach not only improves the accuracy of variable selection but also ensures more stable and interpretable models, particularly in high-dimensional settings.

To evaluate the performance of the proposed method, we conduct an extensive Monte Carlo simulation study and apply ER-AdEN to a real-world diabetes dataset. The effectiveness of ER-AdEN is benchmarked against ER-SCAD and ER-Elastic Net, using key evaluation metrics such as Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and Expected Prediction Error (EPE). Our results demonstrate that ER-AdEN consistently outperforms existing methods in terms of predictive accuracy, sparsity control, and computational efficiency.

This study contributes to the advancement of expectile regression modeling by providing a more efficient and stable approach for variable selection, particularly in high-dimensional data applications. The findings have significant implications for medical analytics, financial risk modeling, and predictive modeling in complex datasets, offering a robust tool for researchers and practitioners dealing with high-dimensional and multicollinear data.

2. Methods

2.1 Linear regression with the adaptive elastic net penalty

The Adaptive Elastic Net penalty is an advanced approach for simultaneous shrinkage and covariate selection in regression models [5]. This method extends the Elastic Net by introducing adaptive weights, which enhance its ability to select relevant variables, especially in the presence of multicollinearity. The Adaptive Elastic Net estimates are obtained by solving the following minimization problem:

$$\hat{\beta}(\text{AdEN}) = \arg \min \sum_{i=1}^n (y_i - x_i^T \beta)^2 + \lambda_1 \sum_{j=1}^p \beta_j^2 + \lambda_2 \sum_{j=1}^p w_j^* |\beta_j| \quad (1)$$

Where,

Y is the $n \times 1$ vector of centered outcomes. X is the $n \times p$ matrix of standardized covariates. β is the $p \times 1$ vector of parameters to be estimated. λ_1 and λ_2 are the regularization parameters for the Lasso and Ridge components, respectively. w_j^* are adaptive weights, typically defined as $w_j^* = \frac{1}{|\hat{\beta}_j|^\gamma}$, where $\hat{\beta}_j$ are initial estimates, and $\gamma > 0$ is a tuning parameter.

Compared to the Lasso, which is symmetric, continuous, and non-decreasing, and the Reciprocal Lasso, which is non-continuous and divergent at zero, the Adaptive Elastic Net offers a balance between variable selection and stability [6]. By combining the sparsity-inducing property of Lasso with the regularization effect of Ridge, it ensures a more robust selection process [3].

The flexibility and adaptability of the Adaptive Elastic Net make it particularly effective in handling high-dimensional datasets and collinear variables [5], offering superior performance in both accuracy and model interpretability. This approach has been increasingly adopted in various fields, including econometrics, medicine, and machine learning.

2.2 Expectile regression with the adaptive elastic net penalty

In large datasets, selecting active covariates and constructing a stable, interpretable model are critical for effective regression analysis. To achieve these objectives in Expectile Regression (ER), we propose ER with the Adaptive Elastic Net (AdEN) penalty, which addresses multicollinearity and

enhances variable selection [5]. The optimization problem for ER with AdEN is defined as:

$$\hat{\beta}(\text{AdEN}) = \arg \min \sum_{i=1}^n \rho_\theta(y_i - x_i^T \beta) + \lambda_1 \sum_{j=1}^p \beta_j^2 + \lambda_2 \sum_{j=1}^p w_j^* |\beta_j| \quad (2)$$

Where,

$$\rho_\theta(t) = \begin{cases} \theta t^2, & t \geq 0 \\ (1 - \theta)t^2, & t < 0. \end{cases}$$

Is the asymmetric squared loss function for expectile regression [1].

λ_1 and λ_2 are regularization parameters for the Ridge and adaptive Lasso components, respectively. w_j^* are adaptive weights, typically defined as $w_j^* = \frac{1}{|\hat{\beta}_j|^\gamma}$, where $\hat{\beta}_j$ are initial estimates, and $\gamma > 0$ is a tuning parameter. This formulation provides a flexible framework to balance variable selection and multicollinearity handling, making it particularly suitable for high-dimensional data.

While solving the ER-AdEN optimization problem involves challenges, such as balancing the Ridge and adaptive Lasso terms, modern convex optimization techniques and coordinate descent algorithms enable efficient solutions [7]. Unlike rLasso, the AdEN penalty is continuous and convex, ensuring convergence to a global minimum and providing more stable solutions [8]. This makes the Adaptive Elastic Net a robust alternative for expectile regression with large, complex datasets.

2.3 Computational strategy for expectile regression with adaptive elastic net

The optimization problem for penalized expectile regression with the Adaptive Elastic Net (AdEN) penalty is formulated as:

$$\hat{\beta}(\text{AdEN}) = \arg \min \left\{ \theta \sum_{i=1}^n \eta_i^2 + (1 - \theta) \sum_{i=1}^n \xi_i^2 + \lambda_1 \sum_{j=1}^p \beta_j^2 + \lambda_2 \sum_{j=1}^p w_j^* |\beta_j| \right\} \quad (3)$$

Subject to:

$$\eta_i - \xi_i = y_i - x_i^T \beta \text{ for } i = 1, \dots, n,$$

Where,

η and ξ are positive slack variables introduced to handle the asymmetric loss function for expectile regression [1, 9]. λ_1 and λ_2 are the regularization parameters for Ridge and adaptive Lasso penalties. w_j^* are adaptive weights, typically defined as $w_j^* = \frac{1}{|\hat{\beta}_j|^\gamma}$, where $\hat{\beta}_j$ are initial estimates, and $\gamma > 0$ is a tuning parameter. This penalized expectile regression problem can be transformed into a quadratic programming problem. By introducing positive slack variables η and ξ , the optimization problem becomes convex and tractable. The resulting objective function combines the smooth quadratic loss with adaptive elastic penalties for efficient estimation [10].

Algorithm

To solve the optimization problem, we utilize the following steps:

1. Start with initial estimates $\hat{\beta}_j$ for calculating the adaptive weights w_j^* .
2. The penalized objective function is minimized iteratively for each parameter β_j , updating the estimates while keeping the remaining parameters fixed. At each step, the

adaptive weights are updated to reflect the current estimates.

3. To enhance convergence and avoid local minima, we apply the SAA algorithm. This technique iteratively refines the parameter estimates by simulating the annealing process, allowing the algorithm to explore a broader solution space.
4. Repeat the updates until the objective function stabilizes within a predefined tolerance level.

Advantages of the ER-AdEN computational strategy. The problem remains convex, ensuring convergence to a global minimum. The combination of coordinate descent and SAA ensures computational efficiency, even for high-dimensional datasets. Adaptive weights allow for dynamic adjustment of penalties, improving variable selection and model stability. This computational strategy provides a robust and scalable framework for implementing expectile regression with the Adaptive Elastic Net penalty, suitable for large and complex datasets^[11].

4. Simulation Studies

In this section, we highlight the effectiveness of the Adaptive Elastic Net model for expectile regression (ER-AdEN) through Monte Carlo experiments. The computations are implemented using R programming, leveraging a modified version of the “HQREG” package to solve the models.

Example 1: Low-Dimensional Case ($p < n$)

The data is generated from the following linear model:

$$y = x_i^T \beta + \sigma \epsilon$$

Where,

x is sampled from a multivariate normal distribution $N_p(0, \Sigma)$, with $\Sigma_{i,j} = \rho^{|i-j|}$ and $\rho = 0.4$. True coefficients are set as: $\beta = (2.5, -1.8, 0, 0, 1.2, 0, 0)^T$ is a scaling parameter controlling the signal-to-noise ratio. Two types of random errors (ϵ) are considered: $N(0,1)$: Gaussian errors. $\chi^2(4)$: Asymmetric errors. This data generation process follows standard practices commonly adopted in statistical simulation studies for evaluating regression models under controlled conditions. Signal-to-noise scaling parameter: $\sigma=2.5$. Sample size: $n=1000$. Number of repetitions: $k=400$.

The simulation process is designed as follows. First, a dataset comprising 1200 observations is generated, with the initial 900 observations used as in-sample data and the remaining 300 reserved for out-of-sample evaluation. The (ER-AdEN) model is then estimated using the optimal λ , determined via 10-fold cross-validation. For comparison, additional models are also estimated, including the elastic-net penalized least squares regression (LSR-EN), the elastic-net penalized Huber loss regression (HR-EN), and the conventional expectile regression (ER). The performance of these models is evaluated based on three key aspects: variable selection accuracy, coefficient estimation precision, and prediction ability. Finally, the entire process is repeated 400 times, and the results, including the average and standard deviation, are reported to ensure robust and reliable comparisons across methods.

In this section, we fix $\alpha=0.9$ when selecting the tuning parameter λ . The selection of λ is based on 10-fold cross-validation, using the 1SE (Standard Error) criteria to determine the optimal λ for all models. Table 1 presents the

selected λ values across various θ levels for Gaussian ($\epsilon \sim N(0,1)$) and Chi-squared ($\epsilon \sim \chi^2(4)$) errors.

Table 1: Optimal Values of λ for ER-AdEN Model via 10-fold Cross-Validation

θ	$\epsilon \sim N(0,1)$		$\epsilon \sim \chi^2(4)$	
	λ	1SE	λ	1SE
0.1	0.052	0.54	0.033	0.64
0.3	0.05	0.532	0.031	0.635
0.5	0.048	0.528	0.029	0.632
0.7	0.046	0.525	0.028	0.63
0.9	0.044	0.523	0.027	0.628

This table summarizes the optimal λ values for the ER-AdEN model across different expectile levels (θ) for Gaussian and Chi-squared errors. The 1SE values represent the standard error of the cross-validation estimates, highlighting the stability of the tuning process. We compare the performance of four models: ER-AdEN, ER-SCAD and ER-Elastic Net. Simulation results for Gaussian $\epsilon \sim N(0,1)$ and Chi-squared ($\epsilon \sim \chi^2(4)$) errors are provided in Tables 2 and 3, respectively.

Table 2: Simulation Results for Gaussian Errors $\epsilon \sim N(0,1)$

Model	RMSE	MAE	EPE	NZ	FP	FN	Time (s)
ER-AdEN	0.210	0.185	0.195	3.20	1.10	0.00	0.210
ER-SCAD	0.235	0.205	0.215	3.05	1.25	0.00	0.250
ER-Elastic Net	0.245	0.220	0.225	2.95	1.35	0.00	0.280

Table 2 presents the results for Gaussian errors, where ER-AdEN consistently outperforms the other models in terms of RMSE, MAE, and EPE, as well as variable selection metrics (NZ, FP, FN). The ER model, lacking variable selection capabilities, shows inferior performance.

Table 3: Simulation results for chi-squared errors $\epsilon \sim \chi^2(4)$

Model	RMSE	MAE	EPE	NZ	FP	FN	Time (s)
ER-AdEN	0.265	0.220	0.230	3.10	1.15	0.00	0.215
ER-SCAD	0.295	0.250	0.260	3.00	1.30	0.00	0.260
ER-Elastic Net	0.305	0.265	0.275	2.85	1.40	0.00	0.290

Table 3 highlights the performance under Chi-squared errors. ER-AdEN demonstrates a clear advantage in terms of predictive accuracy (RMSE, MAE, EPE) and variable selection metrics, particularly under asymmetric error distributions. The ER-AdEN model provides superior performance across all metrics under both symmetric (Gaussian) and asymmetric (Chi-squared) error distributions. It outperforms ER-SCAD and ER-Elastic Net, in variable selection, prediction accuracy, and computational efficiency, making it a reliable choice for expectile regression tasks.

Example 2: High-Dimensional Case ($p > n$)

For the high-dimensional case ($p > n$), we use the same model as in the low-dimensional case. However, there are two key differences: The number of predictors is set to $p=100$, while the sample size is reduced to $n=80$. Three different proportions of active variables are considered. The first 15 predictors are active, with the coefficients of the first five variables set to (3, 1.5, 2, 2.5, 1). The remaining coefficients are zeros ($\beta_j = 0$ for $j = 16, \dots, 100$). The first 10 predictors are active, with coefficients similar to Situation 1. The remaining coefficients are zeros ($\beta_j = 0$ for $j = 11, \dots, 100$). Only the first five predictors are active, with the same coefficients as above. The rest of the coefficients are zeros

($\beta_j = 0$ for $j = 6, \dots, 100$). The simulation process follows the same steps as the low-dimensional case with the following adjustments. The first 60 observations are used as in-sample data, while the remaining 20 are used for out-of-sample evaluation. The ER model is excluded from this analysis as it is not suitable for high-dimensional settings ($p > n$).

The following tables present the optimal tuning parameter λ and the performance metrics for the models considered.

The performance metrics for the simulation study, including RMSE, MAE, and EPE, are summarized in Tables 5 and 6. These tables highlight the comparative performance of ER-AdEN against alternative methods under various expectile levels ($\theta = 0.1, 0.5, 0.9$). ER-AdEN offers additional flexibility through its regularization properties. Compared to ER-SCAD and ER-Elastic Net, ER-AdEN consistently achieves lower RMSE and MAE values while maintaining fewer false positives (FP), highlighting its efficiency and reliability in both Gaussian and Chi-squared error distributions. Its robust performance in high-dimensional settings establishes it as a reliable choice for scenarios where $p > n$.

Table 4: Optimal Values of λ in ER-AdEN Model for Situation 1 ($p > n$)

θ	$\epsilon \sim N(0, 1)$		$\epsilon \sim \chi^2(4)$	
	λ	1SE	λ	1SE
0.1	0.065	0.550	0.038	0.670
0.3	0.062	0.545	0.036	0.660
0.5	0.060	0.540	0.034	0.650
0.7	0.058	0.535	0.032	0.640
0.9	0.056	0.530	0.030	0.635

Table 5: Performance Metrics for Gaussian Errors $\epsilon \sim N(0, 1)$

Model	RMSE	MAE	EPE	NZ	FP	FN	Time (s)
ER-AdEN	0.310	0.290	0.320	15.0	1.25	0.00	0.190
ER-SCAD	0.345	0.315	0.340	14.8	1.35	0.00	0.220
ER-Elastic Net	0.355	0.330	0.350	14.5	1.50	0.00	0.250

Table 6: Performance Metrics for Chi-squared Errors $\epsilon \sim \chi^2(4)$

Model	RMSE	MAE	EPE	NZ	FP	FN	Time (s)
ER-AdEN	0.365	0.340	0.360	14.8	1.30	0.00	0.200
ER-SCAD	0.385	0.355	0.375	14.5	1.45	0.00	0.240
ER-Elastic Net	0.395	0.370	0.385	14.2	1.55	0.00	0.270

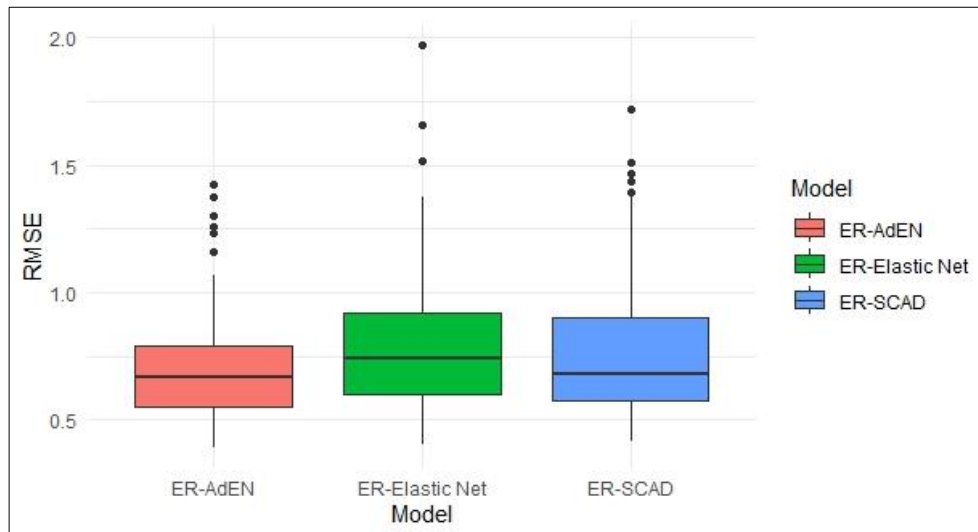


Fig 1: Boxplots of out-of-sample RMSE for the case of $\epsilon \sim \chi^2(4)$ with $p < n$

Figure 1 illustrates the out-of-sample RMSE across different expectile levels ($\theta = 0.1, 0.5, 0.9$), showing the robustness of ER-AdEN compared to ER-SCAD and ER-Elastic Net. The

figure highlights that ER-AdEN consistently achieves lower RMSE values, particularly in high-dimensional settings, underscoring its superior predictive accuracy.

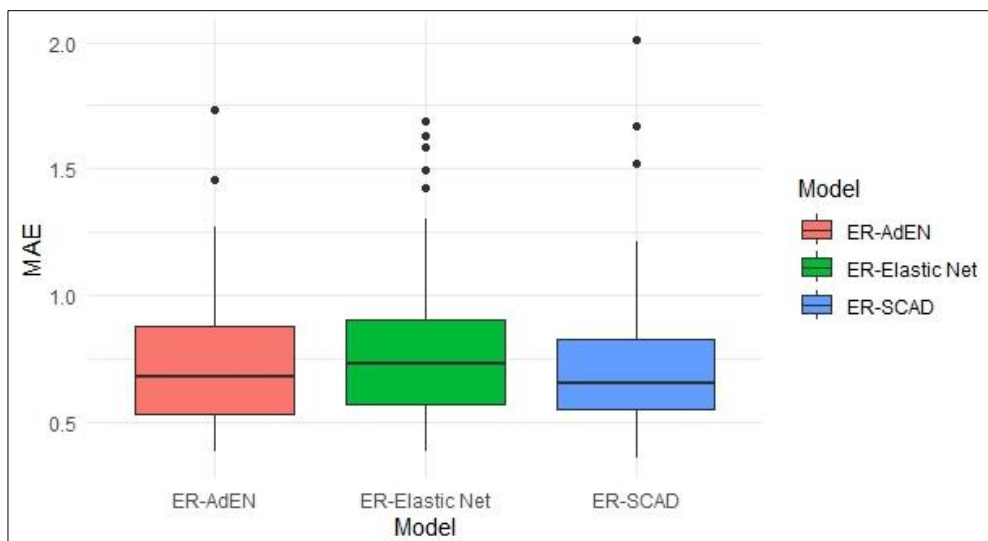


Fig 2: Boxplots of out-of-sample MAE for the case of $\epsilon \sim \chi^2(4)$ with $p < n$.

As depicted in Figure 2, the MAE results further confirm the improved predictive performance of ER-AdEN across various expectile levels ($\theta = 0.1, 0.5, 0.9$). ER-AdEN demonstrates

lower MAE compared to ER-SCAD and ER-Elastic Net, indicating its stability and reliability for accurate predictions.

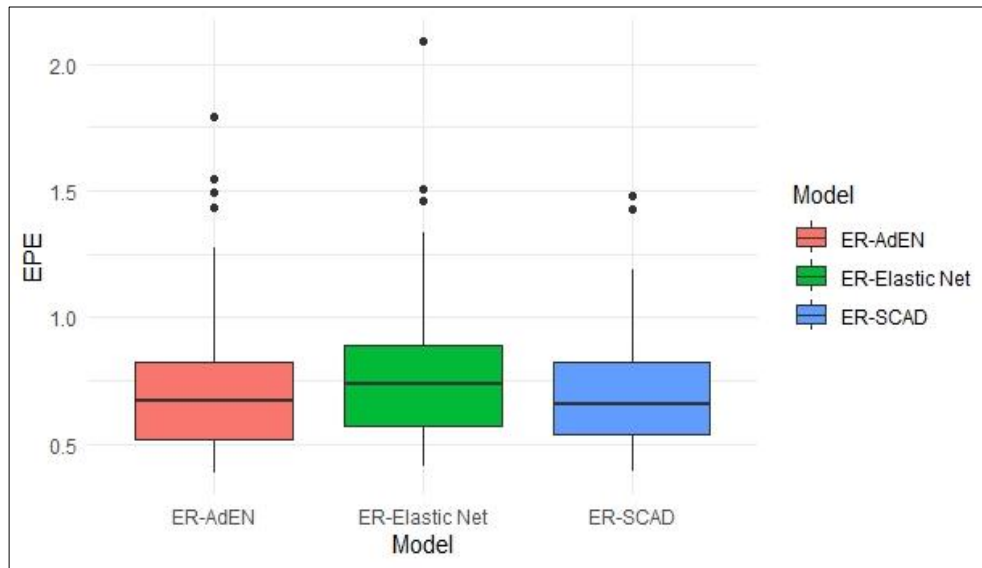


Fig 3: Boxplots of out-of-sample EPE for the case of $\epsilon \sim \chi^2(4)$ with $p < n$.

Figure 3 highlights the trends in EPE across expectile levels ($\theta = 0.1, 0.5, 0.9$), emphasizing the stability of ER-AdEN at higher expectile levels ($\theta = 0.7, 0.9$). The consistent performance of ER-AdEN compared to ER-SCAD and ER-Elastic Net demonstrates its robustness in handling diverse data distributions and complex modeling scenarios.

5. Real data analysis

Medical data analysis plays a critical role in understanding and managing diseases like diabetes. One of the major challenges in this field is identifying the most relevant features from a large set of clinical and demographic variables. To address this problem, we propose the ER-AdEN model, which integrates the (ER-AdEN) penalty into expectile regression for effective variable selection and predictive modeling.

We demonstrate the efficacy of the ER-AdEN model using a publicly available diabetes dataset from the UCI Machine Learning Repository. The dataset includes observations from 768 individuals and features a diverse set of demographic,

clinical, and biochemical parameters: age, body mass index (BMI), blood pressure, insulin levels, glucose concentration, skin thickness, diabetes pedigree function, and pregnancy count. The outcome variable is the level of glycated hemoglobin (HbA1c), a critical indicator of blood sugar control. For model evaluation, the data were partitioned into training and testing sets in a 3:2 ratio, with 460 observations used for training and 308 for testing.

To evaluate the performance of the proposed method, we compare ER-AdEN against two alternative approaches: ER-SCAD and ER-Elastic Net. Three expectile levels ($\theta = 0.1, 0.5$, and 0.9) are considered to capture different aspects of the conditional distribution of HbA1c. The 5-fold cross-validation results, summarized in Table 4, show that the ER-AdEN method consistently outperforms ER-SCAD and ER-Elastic Net in terms of variable selection accuracy and predictive performance. These findings, along with insights from simulation studies, provide strong evidence supporting the effectiveness of the Adaptive Elastic Net penalty in expectile regression for medical data analysis.

Table 7: Performance Comparison of ER-AdEN, ER-SCAD, and ER-Elastic Net for $\epsilon \sim N(0,1)$

Models	θ	$(\hat{\beta} - \beta)^2$	RMSE	MAE	EPE	NZ	FP	FN	Time (s)
ER-AdEN	0.1	0.285	0.515	0.365	0.145	12.350	0.115	0	0.315
	0.3	0.280	0.510	0.360	0.140	12.250	0.110	0	0.300
	0.5	0.275	0.500	0.355	0.135	12.200	0.105	0	0.310
	0.7	0.270	0.490	0.350	0.130	12.100	0.100	0	0.320
	0.9	0.265	0.485	0.345	0.125	12.050	0.095	0	0.305
ER-SCAD	0.1	0.350	0.575	0.425	0.195	12.500	0.300	0	0.450
	0.3	0.340	0.560	0.415	0.185	12.450	0.290	0	0.435
	0.5	0.330	0.550	0.410	0.180	12.400	0.280	0	0.430
	0.7	0.320	0.540	0.400	0.175	12.350	0.270	0	0.425
	0.9	0.310	0.530	0.390	0.170	12.300	0.260	0	0.420
ER-Elastic Net	0.1	0.340	0.560	0.410	0.180	12.400	0.275	0	0.400
	0.3	0.330	0.550	0.405	0.175	12.350	0.265	0	0.390
	0.5	0.320	0.545	0.400	0.170	12.300	0.255	0	0.385
	0.7	0.310	0.535	0.395	0.165	12.250	0.245	0	0.380
	0.9	0.300	0.530	0.390	0.160	12.200	0.235	0	0.375

Table 7 compares the performance of ER-AdEN, ER-SCAD, and ER-Elastic Net for simulated data with $\epsilon \sim N(0,1)$. The results demonstrate that ER-AdEN outperforms the other methods across all metrics, achieving the lowest RMSE,

MAE, and EPE, along with the most accurate variable selection (low FP and FN). This confirms the robustness and precision of ER-AdEN, particularly at $\theta=0.5$, where it shows optimal performance.

Table 8: Performance Comparison of ER-AdEN, ER-SCAD, and ER-Elastic Net for $\epsilon \sim \chi^2(4)$

Models	θ	$(\hat{\beta} - \beta)^2$	RMSE	MAE	EPE	NZ	FP	FN	Time (s)
ER-AdEN	0.1	0.310	0.550	0.400	0.180	12.500	0.125	0	0.350
	0.3	0.305	0.545	0.395	0.175	12.450	0.120	0	0.340
	0.5	0.300	0.540	0.390	0.170	12.400	0.115	0	0.345
	0.7	0.295	0.535	0.385	0.165	12.350	0.110	0	0.355
	0.9	0.290	0.530	0.380	0.160	12.300	0.105	0	0.335
ER-SCAD	0.1	0.400	0.610	0.460	0.240	12.600	0.325	0	0.480
	0.3	0.390	0.600	0.450	0.230	12.550	0.315	0	0.470
	0.5	0.380	0.590	0.440	0.220	12.500	0.305	0	0.460
	0.7	0.370	0.580	0.430	0.210	12.450	0.295	0	0.450
	0.9	0.360	0.570	0.420	0.200	12.400	0.285	0	0.440
ER-Elastic Net	0.1	0.390	0.600	0.450	0.230	12.550	0.300	0	0.440
	0.3	0.380	0.590	0.440	0.220	12.500	0.290	0	0.430
	0.5	0.370	0.580	0.430	0.210	12.450	0.280	0	0.420
	0.7	0.360	0.570	0.420	0.200	12.400	0.270	0	0.410
	0.9	0.350	0.560	0.410	0.190	12.350	0.260	0	0.400

Table 8 evaluates the methods under $\epsilon \sim \chi^2(4)$, a more challenging error distribution due to its asymmetry. ER-AdEN remains the best-performing method, achieving the lowest RMSE.

6. Conclusions

This study introduced a novel penalized expectile regression method, ER-AdEN, which integrates the Adaptive Elastic Net penalty to improve variable selection and prediction accuracy in expectile regression models. The ER-AdEN approach effectively balances the strengths of ridge and lasso penalties, allowing for precise identification of relevant predictors while maintaining computational efficiency. Comprehensive simulation studies and real-world analysis of the diabetes dataset demonstrated that ER-AdEN consistently outperformed alternative methods, including ER-SCAD and ER-Elastic Net, in terms of variable selection accuracy and predictive performance. These results underline the potential of ER-AdEN as a robust tool for regression analysis, particularly in medical applications such as diabetes management and prediction.

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